SAMPLE PAPER TEST 02 FOR BOARD EXAM 2025

SUBJECT: MATHEMATICS (041) | **MAX. MARKS : 80**
CLASS : XII | **DURATION: 3 HRS**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is

(a) $20\pi^2$ sq. units (b) 25π sq. units (c) 20π sq. units (d) $16\pi^2$ sq. units

2. The magnitude of the vector $6\hat{i}-2\hat{j}+3\hat{k}$ is:

(a) 1 (b) 5 (c) 7 (d) 12

3. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to

(a) $4\pi^2$ sq units (b) 4π sq units (c) 2π sq units (d) $2\sqrt{2}\pi$ sq units

4. If $|\vec{a}| = 5$, $|\vec{b}| = 13$ and $|\vec{a}\times\vec{b}| = 25$, then $\vec{a}.\vec{b}$ is equal to

(a) 12 (b) 5 (c) 13 (d) 60

5. If for non zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\times\vec{b}$ is a unit vector and $|\vec{a}|=|\vec{b}|=\sqrt{2}$, then angle θ between vectors $\vec{a}$ and $\vec{b}$ is

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (d) $-\pi/2$

6. If A and B are two events such that P(A) = 1/2, P(B) = 1/3 and P(A/B) = 1/4, then P(A' ∩ B') equals

(a) 1/12 (b) 3/4 (c) 1/4 (d) 3/16

7. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is:

(a) 27/32 (b) 5/32 (c) 31/32 (d) 1/32

8. $\int \frac{\sec x}{\sec x-\tan x}dx$ equals:

(a) secx – tanx + c (b) secx + tanx + c (c) tanx – secx + c (d) – (secx + tanx) + c

9. A point that lies on the line $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{1-z}{7}$ is:

(a) (1, -3, 1) (b) (-2, 4, 7) (c) (-1, 3, 1) (d) (2, -4, -7)

10. The general solution of the differential equation $\frac{dy}{dx} = 2^{-y}$ is:

(a) 2y = x log 2 + C log 2 (b) 2y = x log 3 – C log 3
(c) y = x log 2 – C log 2 (d) None of these

11. $\int \cos^3 x.e^{\log(\sin x)}dx$ is equal to

(a) $-\frac{\cos^4 x}{4}+C$ (b) $-\frac{\sin^4 x}{4}+C$ (c) $\frac{e^{\sin x}}{4}+C$ (d) none of these

12. If A and B are invertible matrices, then which of the following is not correct.
(a) adj A = |A|. A^{-1} (b) det (A^{-1}) = [det (A)]$^{-1}$
(c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A + B)^{-1} = B^{-1} + A^{-1}$

13. Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when
(a) x > 2 (b) 1 < x < 2 (c) x = 2 (d) x > 3

14. If one root of the equation $\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 7$ is x = -9, then the other two roots are:

(a) 6, 3 (b) 6, -3 (c) -2, -7 (d) 2, 6

15. Let A be a non-singular matrix of order (3 × 3). Then |adj.A| is equal to
(a) |A| (b) $|A|^2$ (c) $|A|^3$ (d) 3|A|

16. The order and the degree of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ are:

(a) 1, 1 (b) 2, 1 (c) 1, 2 (d) 3, 1

17. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to:

(a) 1 (b) 1/2 (c) 1/3 (d) 1/4

18. $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx = ?$

(a) $(x+1)e^{x+\frac{1}{x}}+C$ (b) $xe^{x+\frac{1}{x}}+C$

(c) $-xe^{x+\frac{1}{x}}+C$ (d) $(x-1)e^{x+\frac{1}{x}}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion(A): Determinant of a skew-symmetric matrix of order 3 is zero.
Reason(R): For any matrix A, |A'| = |A| and |-A| = |A|.

20. Assertion (A): If manufacturer can sell x items at a price of Rs. $\left(5-\frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5}+500\right)$. Then, the number of items he should sell to earn maximum profit is 240 items.

Reason (R): The profit for selling x items is given by $\frac{24}{5}x-\frac{x^2}{100}-300$.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Solve $(1 + x^2) \sec^2 y dy + 2x \tan y dx = 0$, given that $y = \pi/4$ when $x = 1$.

22. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}$ in simplest form.

23. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then find the probability that both drawn balls are black.

24. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$, $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$ show that $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

25. Find the values of x and y from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix}+\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

OR

If $A=\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. Solve the following differential equation: $\frac{dy}{dx}=x^3 \operatorname{cosec} y$, given that $y(0) = 0$.

OR

Find the particular solution of the differential equation $(1 + e^{2x})\, dy + (1 + y^2)\, e^x\, dx = 0$, given that when $x = 0$, $y = 1$.

27. Find the value of $\int_2^4 \frac{x^2+x}{\sqrt{2x+1}}dx$.

OR

Find the value of $\int_1^2 \frac{dx}{x(1+\log x)^2}$.

28. Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

29. Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$

OR

Three vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$, if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

30. Find the area of the following region using integration: $\{(x,y) : y^2 \le 2x \text{ and } y \ge x - 4\}$

31. Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax+1, & if \quad x \le 3 \\ bx+3, & if \quad x > 3 \end{cases}$ is continuous at $x = 3$.

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Solve the following Linear Programming Problem graphically:
Maximise z = 8x + 9y subject to the constraints: $2x + 3y \le 6$, $3x - 2y \le 6$, $y \le 1$; $x, y \ge 0$

33. Define the relation R in the set $N \times N$ as follows:
For (a, b), (c, d) ∈ $N \times N$, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation in $N \times N$.

OR

Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \dfrac{x}{1+|x|}, x \in R$ is one-one and onto function.

34. Evaluate: $\displaystyle\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)dx$

35. Find the shortest distance between the lines whose vector equations are
$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} + (2s+1)\hat{k}$

OR

Find the equation of a line passing through the point (1, 2, –4) and perpendicular to two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case-Study 1:
Anil is the owner of a high rise residential society having 50 apartments. When he set rent at Rs.10000/month, all apartments are rented. If he increases rent by Rs.250/ month, one fewer apartment is rented. The maintenance cost for each occupied unit is Rs.500/month. Anil represented the rent price per apartment by P and the number of rented apartments represented by N.

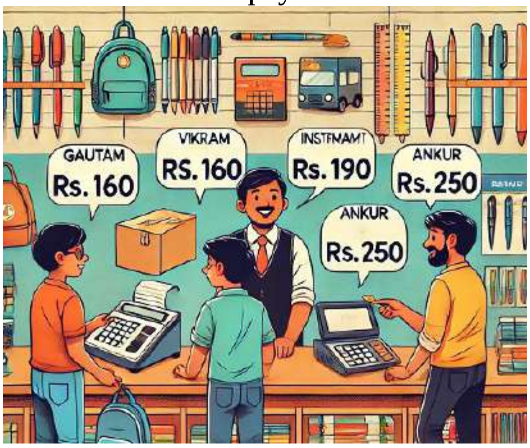
Based on the above information answer the following questions.
(a) If P = 10500, then find N [1]
(b) If P = 11,000, then find the profit. [1]
(c) Find the rent that maximizes the total amount of profit. [2]

37. Case-Study 2:
Read the following passage and answer the questions given below.

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?
(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

38. Case-Study 3:
Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of Rs. 160. From the same shop. Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of Rs. 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of Rs. 250.



Based on the above information, answer the following questions:
(I) Convert the given above situation into a matrix equation of the form AX =B.
(II) Find |A|.
(III) Find A^{-1}.

OR

(III) Determine P $=A^2 - 5A$.

SAMPLE PAPER TEST 02 FOR BOARD EXAM 2025
(ANSWERS)

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SECTION – A
Questions 1 to 20 carry 1 mark each.

1. The area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is
(a) $20\pi^2$ sq. units (b) 25π sq. units (c) 20π sq. units (d) $16\pi^2$ sq. units
Ans: (c) 20π sq. units
The area of the standard ellipse is given by ; πab. Here, a = 5 and b = 4
Therefore, the area of curve is $\pi(5)(4) = 20\pi$.

2. The magnitude of the vector $6\hat{i}-2\hat{j}+3\hat{k}$ is:
(a) 1 (b) 5 (c) 7 (d) 12
Ans: (c) 7

3. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to
(a) $4\pi^2$ sq units (b) 4π sq units (c) 2π sq units (d) $2\sqrt{2}\pi$ sq units
Ans: (c) 2π sq units

4. If $|\vec{a}| = 5$, $|\vec{b}| = 13$ and $|\vec{a}\times\vec{b}| = 25$, then $\vec{a}.\vec{b}$ is equal to
(a) 12 (b) 5 (c) 13 (d) 60
Ans: (d) 60

5. If for non zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\times\vec{b}$ is a unit vector and $|\vec{a}|=|\vec{b}|=\sqrt{2}$, then angle θ between vectors $\vec{a}$ and $\vec{b}$ is
(a) π/2 (b) π/3 (c) π/6 (d) - π/2
Ans: (c) π/6

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

6. If A and B are two events such that P(A) = 1/2, P(B) = 1/3 and P(A/B) = 1/4, then P(A' ∩ B') equals
(a) 1/12 (b) 3/4 (c) 1/4 (d) 3/16
Ans: (c) 1/4

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)} \Rightarrow \frac{1}{4}=\frac{P(A\cap B)}{1/3} \Rightarrow P(A\cap B)=\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$$

$$\text{Now}P(A'\cap B')=1-P(A\cup B)$$
$$=1-[P(A)+P(B)-P(A\cap B)]$$
$$=1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{12}\right]=1-\left[\frac{5}{6}-\frac{1}{12}\right]=1-\frac{9}{12}=\frac{3}{12}=\frac{1}{4}$$

7. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is:

(a) 27/32 (b) 5/32 (c) 31/32 (d) 1/32

Ans: (c) 31/32

Probability of the event that at least one head comes up = $1-\left(\frac{1}{2}\right)^5=1-\frac{1}{32}=\frac{31}{32}$

8. $\int\frac{\sec x}{\sec x-\tan x}dx$ equals:

(a) secx – tanx + c (b) secx + tanx + c (c) tanx – secx + c (d) – (secx + tanx) + c

Ans: (b) secx + tanx + c

9. A point that lies on the line $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{1-z}{7}$ is:

(a) (1, -3, 1) (b) (-2, 4, 7) (c) (-1, 3, 1) (d) (2, -4, -7)

Ans: (a) (1, -3, 1)

The equation of the Line can be written as $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{z-1}{-7}$

So, it passes through (1, -3, 1).

10. The general solution of the differential equation $\frac{dy}{dx}=2^{-y}$ is:

(a) 2y = x log 2 + C log 2 (b) 2y = x log 3 – C log 3

(c) y = x log 2 – C log 2 (d) None of these

Ans: (a) 2y = x log 2 + C log 2

Given, $\frac{dy}{dx}=2^{-y}$

$$\Rightarrow \frac{dy}{2^{-y}}=dx \Rightarrow \int 2^y dx=\int dx$$

$$\Rightarrow \frac{2^y}{\log 2}=x+C \Rightarrow 2y=x\log 2+C\log 2$$

11. $\int\cos^3 x.e^{\log(\sin x)}dx$ is equal to

(a) $-\frac{\cos^4 x}{4}+C$ (b) $-\frac{\sin^4 x}{4}+C$ (c) $\frac{e^{\sin x}}{4}+C$ (d) none of these

Ans: (a) $-\frac{\cos^4 x}{4}+C$

12. If A and B are invertible matrices, then which of the following is not correct.

(a) adj A = |A|. A^{-1} (b) det (A^{-1}) = $[\det(A)]^{-1}$

(c) $(AB)^{-1}=B^{-1}A^{-1}$ (d) $(A+B)^{-1}=B^{-1}+A^{-1}$

Ans: (d) $(A + B)^{-1} = B^{-1} + A^{-1}$

13. Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when

(a) $x > 2$ (b) $1 < x < 2$ (c) $x = 2$ (d) $x > 3$

Ans: (b) $1 < x < 2$

14. If one root of the equation $\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 7$ is x = -9, then the other two roots are:

(a) 6, 3 (b) 6, -3 (c) -2, -7 (d) 2, 6

Ans: (c) -2, -7

$\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 7(7x - 6) - 6(14 - 2x) + x(6 - x^2)$

$= -x^3 + 67x - 126$

$= (x + 9)(-x^2 + 9x - 14)$

$= (x + 9)(-x - 2)(x + 7)$

Hence the other two roots are -2 and -7.

15. Let A be a non-singular matrix of order (3 × 3). Then |adj.A| is equal to

(a) $|A|$ (b) $|A|^2$ (c) $|A|^3$ (d) $3|A|$

Ans: (b) $|A|^2$

If A is a matrix of order n × n then $|\text{adj } A| = |A|^{n-1}$

16. The order and the degree of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ are:

(a) 1, 1 (b) 2, 1 (c) 1, 2 (d) 3, 1

Ans: (b) 2, 1

The highest order is 2 and the degree of the highest order is 1.

Hence, the order is 2 and the degree is 1.

17. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to:

(a) 1 (b) 1/2 (c) 1/3 (d) 1/4

Ans: (a) 1

18. $\int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx$ = ?

(a) $(x+1)e^{x+\frac{1}{x}} + C$ (b) $xe^{x+\frac{1}{x}} + C$

(c) $-xe^{x+\frac{1}{x}} + C$ (d) $(x-1)e^{x+\frac{1}{x}}$

Ans: (b) $xe^{x+\frac{1}{x}} + C$

$$\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$$
$$=\int e^{x+\frac{1}{x}}dx+\int x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$$
$$=\int e^{x+\frac{1}{x}}dx+xe^{x+\frac{1}{x}}-\int\frac{d}{dx}(x)e^{x+\frac{1}{x}}dx$$
$$=\int e^{x+\frac{1}{x}}dx+xe^{x+\frac{1}{x}}-\int e^{x+\frac{1}{x}}dx\ \left(\because\int\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx=e^{x+\frac{1}{x}}\right]$$
$$=\int e^{x+\frac{1}{x}}dx+xe^{x+\frac{1}{x}}-\int ex^{x+\frac{1}{x}}dx=xe^{x+\frac{1}{x}}+c$$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion(A): Determinant of a skew-symmetric matrix of order 3 is zero.
Reason(R): For any matrix A, |A'| = |A| and |-A| = |A|.
Ans. (c) A is true but R is false.
Assertion: Determinant of a skew-symmetric matrix of odd order is zero.
∴ Assertion is true.
Reason: For any matrix A, |A'| = A
and |-A] = |A| [when A is of even order]
and |-A| = -|A| [when A is of odd order]
∴ Reason is false.

20. Assertion (A): If manufacturer can sell x items at a price of Rs. $\left(5-\frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5}+500\right)$. Then, the number of items he should sell to earn maximum profit is 240 items.

Reason (R): The profit for selling x items is given by $\frac{24}{5}x-\frac{x^2}{100}-300$.

Ans. (c) A is true but R is false.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Solve $(1 + x^2)\sec^2 y dy + 2x \tan y dx = 0$, given that $y = \pi/4$ when $x = 1$.
Ans:
Here, $(1 + x^2)\sec^2 y\, dy + 2x \tan y\, dx = 0$,
Given that, $y=\frac{\pi}{4}$ when x = 1
$\Rightarrow (1 + x^2)\sec^2 y\, dy + 2x\tan y\, dx = 0 \Rightarrow \frac{\sec^2 y}{\tan y}dy+\frac{2x}{1+x^2}dx=0$
$\Rightarrow \int\frac{\sec^2 y}{\tan y}dy+\int\frac{2x}{1+x^2}dx=0 \Rightarrow \log\tan y + \log(1 + x^2) = \log c$
for $y = \frac{\pi}{4}$, x, = 1
We have, 0 + log 2 = log c,
c = 2, Therefore,the required particular solution is $\tan y\left(1+x^2\right)=2$

22. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}$ in simplest form.

Ans:

We have, $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$

$$= \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right] = \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right] = \tan^{-1}\left[\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$$

23. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then find the probability that both drawn balls are black.

Ans: Let E and F denote respectively the events that first and second ball drawn are black. We have to find P(E ∩ F).

Now, $P(E) = \frac{10}{15}$, $P(F|E) = \frac{9}{14}$

By multiplication rule of probability, we have

$$P(E \cap F) = P(E)\cdot P(F|E) = \frac{10}{15}\times\frac{9}{14} = \frac{3}{7}$$

24. If $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$, $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$ show that $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

Ans:

$\vec{a}+\vec{b} = 4\hat{i}+\hat{j}-\hat{k}$

and $\vec{a}-\vec{b} = -2\hat{i}+3\hat{j}-5\hat{k}$

Consider $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = (4\hat{i}+\hat{j}-\hat{k})\cdot(-2\hat{i}+3\hat{j}-5\hat{k}) = -8+3+5 = 0$

As dot product is zero

$\therefore$ $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

25. Find the values of *x* and *y* from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Ans: $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

or $2x + 3 = 7$ and $2y - 4 = 14$

or $2x = 7 - 3$ and $2y = 18$

or $x = \frac{4}{2}$ and $y = \frac{18}{2}$

i.e. $x = 2$ and $y = 9$.

OR

If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find k so that $A^2 = kA - 2I$

Ans: Given than $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix} = k\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix} - 2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}9-8 & -6+4\\ 12-8 & -8+4\end{bmatrix} = \begin{bmatrix}3k & -2k\\ 4k & -2k\end{bmatrix} - \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix} = \begin{bmatrix}3k-2 & -2k\\ 4k & -2k-2\end{bmatrix}$$

By definition of equality of matrix as the given matrices are equal, their corresponding elements are equal. Comparing the corresponding elements, we get

$3k - 2 = 1 \Rightarrow k = 1$

$-2k = -2 \Rightarrow k = 1$

$4k = 4 \Rightarrow k = 1$

$-4 = -2k - 2 \Rightarrow k = 1$

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. Solve the following differential equation: $\frac{dy}{dx} = x^3 \cos ec y$, given that y(0) = 0.

Ans:

Given differential equation is $\frac{dy}{dx} = x^3.\text{cosec } y$

$\therefore \quad \frac{dy}{\text{cosec } y} = x^3.dx$

Integrating both sides, we get $\int \sin y \, dy = \int x^3 \, dx$

$\Rightarrow -\cos y = \frac{x^4}{4} + c \qquad \Rightarrow \quad -\cos y = \frac{0}{4} + c$

$\Rightarrow -1 = c$

Putting the value of c in (i), we get

$-\cos y = \frac{x^4}{4} - 1 \qquad \therefore \ \cos y = \left(1 - \frac{x^4}{4}\right)$

OR

Find the particular solution of the differential equation $(1 + e^{2x})\, dy + (1 + y^2)\, e^x\, dx = 0$, given that when $x = 0$, $y = 1$.

Ans:

$$\int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}} dx \qquad \ldots(i)$$

For $\int \frac{e^x}{1+e^{2x}} dx = \int \frac{1}{1+t^2} dt = \tan^{-1} t = \tan^{-1} e^x$ | Let $e^x = t \Rightarrow e^x dx = dt$

From (i), we get

$$\tan^{-1} y = -\tan^{-1} e^x + C \qquad \ldots(ii)$$

When $x = 0, y = 1 \quad \Rightarrow \tan^{-1} 1 = -\tan^{-1} 1 + C \quad \Rightarrow C = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} \quad (e^0 = 1)$

Substituting in (ii), we get

$\tan^{-1} y = -\tan^{-1} e^x + \frac{\pi}{2} \Rightarrow \tan^{-1} y + \tan^{-1} e^x = \frac{\pi}{2}$ is the required solution.

27. Find the value of $\int_2^4 \frac{x^2+x}{\sqrt{2x+1}}dx$.

Ans:

Using integration by parts, we get

$$\int_2^4 \frac{(x^2+x)}{\sqrt{2x+1}}dx = \left[(x^2+x)\cdot\sqrt{2x+1}\right]_2^4 - \int_2^4 (2x+1)\cdot\sqrt{2x+1}\,dx$$

$$= (60-6\sqrt{5}) - \int_2^4 (2x+1)^{3/2}dx$$

$$= (60-6\sqrt{5}) - \frac{1}{5}\cdot\left[(2x+1)^{5/2}\right]_2^4$$

$$= (60-6\sqrt{5}) - \left(\frac{243}{5} - 5\sqrt{5}\right) = \left(\frac{57}{5} - \sqrt{5}\right) = \left(\frac{57-5\sqrt{5}}{5}\right)$$

OR

Find the value of $\int_1^2 \frac{dx}{x(1+\log x)^2}$.

Ans: Let $I = \int_1^2 \frac{dx}{x(1+\log x)^2}$

Put $1+\log x = t \Rightarrow \frac{dx}{x} = dt$

When $x = 1$, $t = 1$ and when $x = 2$, $t = 1 + \log 2$

$$\therefore \quad I = \int_1^{1+\log 2} \frac{dt}{t^2} = \left[\frac{-1}{t}\right]_1^{1+\log 2} = -\left[\frac{1}{1+\log 2} - 1\right] = -\left[\frac{1-1-\log 2}{1+\log 2}\right] = \frac{\log 2}{1+\log 2}$$

28. Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

Ans:

$$u = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) \text{ and } v = \sin^{-1}(2x\sqrt{1-x^2})$$

Let $x = \sin\theta$

$$u = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\frac{1}{\cos\theta} = \theta$$

$$\therefore \quad u = \sin^{-1}x$$

$$\Rightarrow v = \sin^{-1}(2\sin\theta(\sqrt{1-\sin^2\theta}) = \sin^{-1}(2\sin\theta\cos\theta)$$

$$\Rightarrow v = \sin^{-1}\sin 2\theta = 2\theta \Rightarrow v = 2\sin^{-1}x$$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(2\sin^{-1}x) = \frac{2}{\sqrt{1-x^2}} \Rightarrow \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{2}$$

29. Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$

Ans: P (0, 2, 3)

Line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$

General point on the line is [(5λ – 3), (2λ + 1), (3λ – 4)]

Direction ratio of the perpendicular line

[(5λ – 3), (2λ – 1), (3λ – 7)]

∴ 5(5λ – 3) + 2(2λ – 1) + 3(3λ – 7) = 0

⇒ 25λ – 15 + 4λ – 2 + 9λ – 21 = 0 ⇒ 38λ – 38 = 1

⇒ λ = 1

∴ foot of perpendicular line is [(5 – 3), (2 + 1), (3 – 4)

= (2, 3, – 1)

OR

Three vectors $\vec{a},\vec{b},\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, if $|\vec{a}|$ = 3, $|\vec{b}|$ = 4 and $|\vec{c}|$ = 2.

Ans:

$$\vec{a}+\vec{b}+\vec{c}=0$$

$$(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$$

$$\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{a}+\vec{b}.\vec{c}+\vec{b}.\vec{b}+\vec{c}.\vec{a}+\vec{c}.\vec{b}+\vec{c}.\vec{c}=0$$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$$

$$\Rightarrow 3^2+4^2+2^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$$

$$\Rightarrow 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=-29 \Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-\frac{29}{2}$$

30. Find the area of the following region using integration: {(x,y) : $y^2 \le 2x$ and $y \ge x-4$}

Ans: Given: $y^2 = 2x$ ...(1)

$y = x - 4$ ...(2)

Required area is OABCO

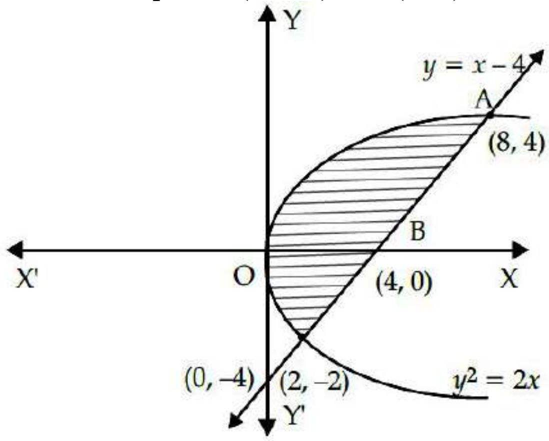
from (1) and (2), $(x-4)^2 = 2x$

⇒ $x^2 - 10x + 16 = 0$

⇒ (x – 8) (x – 2) = 0

⇒ x = 8 and x = 2

∴ Intersection points (2, – 2) and (8, 4)



$$\text{Required Area} = \left[\frac{y^2}{2}+4y-\frac{y^3}{6}\right]_{-2}^{4} = \left(8+16-\frac{32}{3}-2+8-\frac{4}{3}\right) = 30-12$$

$$= 18 \text{ unit}^2$$

31. Find the relationship between a and b so that the function f defined by $f(x)=\begin{cases} ax+1, & if \quad x\le 3 \\ bx+3, & if \quad x>3 \end{cases}$ is continuous at $x = 3$.

Ans. Here, $f(x)=\begin{cases} ax+1, & if \quad x\le 3 \\ bx+3, & if \quad x>3 \end{cases}$

$$LHL = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(ax+1)$$

Putting $x = 3 - h$ has $x \to 3^-$ when $h \to 0$

$$\therefore \lim_{x\to 3^-} f(x) = \lim_{h\to 0}(a(3-h)+1) = \lim_{h\to 0}(3a-ah+1) = 3a+1$$

$$RHL = \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(bx+3)$$

Putting $x = 3 + h$ as $x \to 3^+$ when $h \to 0$

$$\therefore \lim_{x\to 3^+} f(x) = \lim_{h\to 0}(b(3+h)+3) = \lim_{h\to 0}(3b+bh+3) = 3b+3$$

Also, f(3) =3 a +1 [since $f(x) = ax + 1$]

Since, $f(x)$ is continuous at $x = 3$.

$\therefore$ LHL = RHL = f(3)

$$\Rightarrow 3a+1 = 3b+3 \Rightarrow 3a = 3b+2 \Rightarrow a = b+\frac{2}{3}$$

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Solve the following Linear Programming Problem graphically:

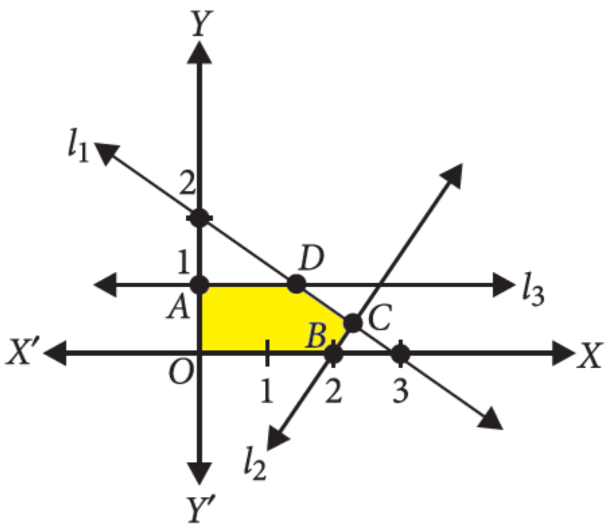
Maximise z = 8x + 9y subject to the constraints: $2x + 3y \le 6$, $3x - 2y \le 6$, $y \le 1$; $x, y \ge 0$

Ans:

Let $l_1 : 2x + 3y = 6$, $l_2 : 3x - 2y = 6$, $l_3 : y = 1$; $x = 0$, $y = 0$

Solving l_1 and l_3, we get D (1.5, 1)

Solving l_1 and l_2, we get C$\left(\frac{30}{13},\frac{6}{13}\right)$



Shaded portion OADCB is the feasible region,

where coordinates of the corner points are O(0, 0),

A(0, 1), D(1.5, 1), C$\left(\frac{30}{13},\frac{6}{13}\right)$, B(2, 0).

The value of the objective function at these points are:

Corner points	Value of the objective function z = 8x + 9y
O (0, 0)	$8 \times 0 + 9 \times 0 = 0$
A (0, 1)	$8 \times 0 + 9 \times 1 = 9$
D (1.5, 1)	$8 \times 1.5 + 9 \times 1 = 21$
$C\left(\frac{30}{13}, \frac{6}{13}\right)$	$8\times\frac{30}{13}+9\times\frac{6}{13}$ = 22.6 (Maximum)
B (2, 0)	$8 \times 2 + 9 \times 0 = 16$

The maximum value of z is 22.6, which is at $C\left(\frac{30}{13}, \frac{6}{13}\right)$

33. Define the relation R in the set $N \times N$ as follows:
For (a, b), (c, d) $\in N \times N$, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation in $N \times N$.
Ans: Let $(a, b) \in N \times N$. Then we have
ab = ba (by commutative property of multiplication of natural numbers)
$\Longrightarrow (a, b)R(a, b)$
Hence, R is reflexive.
Let $(a, b), (c, d) \in N \times N$ such that (a, b) R (c, d). Then ad = bc
$\Longrightarrow cb = da$ (by commutative property of multiplication of natural numbers
$\Longrightarrow (c, d)R(a, b)$
Hence, R is symmetric.
Let $(a, b), (c, d), (e, f) \in N \times N$ such that (a, b) R (c, d) and (c, d) R (e, f).
Then ad = bc, cf = de
$\Longrightarrow adcf = bcde$
$\Longrightarrow af = be$
$\Longrightarrow (a, b)R(e, f)$
Hence, R is transitive.
Since, R is reflexive, symmetric and transitive, R is an equivalence relation on $N \times N$.

OR

Show that the function $f: R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one-one and onto function.

Ans: It is given that $f: R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$

Suppose, $f(x) = f(y)$, where $x, y \in R \Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$
It can be observed that if x is positive and y is negative, then we have

$$\frac{x}{1+x} = \frac{y}{1-y} \Rightarrow 2xy = x - y$$

Since, x is positive and y is negative, then $x > y \Rightarrow x - y > 0$
But, $2xy$ is negative. Then, $2xy \neq x - y$.
Thus, the case of x being positive and y being negative can be ruled out.
Under a similar argument, x being negative and y being positive can also be ruled out. Therefore, x and y have to be either positive or negative.

When x and y are both positive, we have $f(x) = f(y) \Rightarrow \frac{x}{1+x} = \frac{y}{1+y} \Rightarrow x + xy = y + xy \Rightarrow x = y$

When x and y are both negative, we have $f(x) = f(y) \Rightarrow \frac{x}{1-x} = \frac{y}{1-y} \Rightarrow x - xy = y - xy \Rightarrow x = y$

Therefore, f is one-one. Now, let $y \in R$ such that $-1 < y < 1$.

If y is negative, then there exists $x=\frac{y}{1+y}\in R$ such that

$$f(x)=f\left(\frac{y}{1+y}\right)=\frac{\left(\frac{y}{1+y}\right)}{1+\left|\frac{y}{1+y}\right|}=\frac{\frac{y}{1+y}}{1+\left(\frac{-y}{1+y}\right)}=\frac{y}{1+y-y}=y$$

If y is positive, then there exists $x=\frac{y}{1-y}\in R$ such that

$$f(x)=f\left(\frac{y}{1-y}\right)=\frac{\left(\frac{y}{1-y}\right)}{1+\left|\frac{y}{1-y}\right|}=\frac{\frac{y}{1-y}}{1+\left(\frac{y}{1-y}\right)}=\frac{y}{1-y+y}=y$$

Therefore, f is onto. Hence, f is one-one and onto.

34. Evaluate: $\int_0^{\pi/2}\sin 2x\tan^{-1}(\sin x)dx$

Ans:

$$I=\int_0^{\pi/2}\sin 2x\tan^{-1}(\sin x)dx=\int_0^{\pi/2}2\sin x\cos x\tan^{-1}(\sin x)dx$$

Let $\sin x=t \Rightarrow \cos x\,dx=dt$

when $x=0, t=0$

$x=\frac{\pi}{2}, t=1$

$$\therefore\ I=\int_0^1 2t\tan^{-1}t\,dt=2\left[\tan^{-1}t\int t\,dt-\int\left[\frac{d}{dx}\tan^{-1}t\right]\int t\,dt\right].dt$$

$$=2\left[\frac{t^2}{2}\tan^{-1}t-\int\frac{t^2}{2(1+t^2)}dt\right]_0^1=\left[t^2\tan^{-1}t\right]_0^1-\int_0^1\frac{t^2}{(1+t^2)}dt$$

$$=\tan^{-1}1-\left[\int_0^1 1dt-\int_0^1\frac{1}{1+t^2}dt\right]=\frac{\pi}{4}-\left[t-\tan^{-1}t\right]_0^1=\frac{\pi}{4}-1+\frac{\pi}{4}=\frac{\pi}{2}-1$$

35. Find the shortest distance between the lines whose vector equations are $\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}+(2s+1)\hat{k}$

Ans:

Lines are $\vec{r}=(\hat{i}-2\hat{j}+3\hat{k})+t(-\hat{i}+\hat{j}-2\hat{k})$

and $\vec{r}=(\hat{i}-\hat{j}-\hat{k})+s(\hat{i}+2\hat{j}-2\hat{k})$

The shortest distance $=\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$.

Here $\vec{a}_1=\hat{i}-2\hat{j}+3\hat{k},\ \vec{b}_1=-\hat{i}+\hat{j}-2\hat{k};$

$\vec{a}_2=\hat{i}-\hat{j}-\hat{k},\ \vec{b}_2=\hat{i}+2\hat{j}-2\hat{k}$

$\vec{a}_2-\vec{a}_1=\hat{i}-\hat{j}-\hat{k}-\hat{i}+2\hat{j}-3\hat{k}=\hat{j}-4\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&1&-2\\1&2&-2\end{vmatrix}=2\hat{i}-4\hat{j}-3\hat{k}\Rightarrow\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{4+16+9}=\sqrt{29}$$

The shortest distance $= \left|\frac{(\hat{j}-4\hat{k})\cdot(2\hat{i}-4\hat{j}-3\hat{k})}{\sqrt{29}}\right| = \left|\frac{-4+12}{\sqrt{29}}\right| = \frac{8}{\sqrt{29}}$ units

OR

Find the equation of a line passing through the point (1, 2, –4) and perpendicular to two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Ans:

Let line through the point (1, 2, –4) be

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda'\vec{m},\ \lambda' \text{ is a scalar} \quad ...(i)$$

Line (i) is perpendicular to lines

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and $$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$$

$\therefore\ (3\hat{i} - 16\hat{j} + 7\hat{k})\cdot\vec{m} = 0$ and $(3\hat{i} + 8\hat{j} - 5\hat{k})\cdot\vec{m} = 0$

$$\Rightarrow \vec{m} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$\therefore$ From (i) line is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda'(24\hat{i} + 36\hat{j} + 72\hat{k})$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda''(2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ where } \lambda'' = 12\lambda'. \text{ is a scalar}$$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case-Study 1:

Anil is the owner of a high rise residential society having 50 apartments. When he set rent at Rs.10000/month, all apartments are rented. If he increases rent by Rs.250/ month, one fewer apartment is rented. The maintenance cost for each occupied unit is Rs.500/month. Anil represented the rent price per apartment by P and the number of rented apartments represented by N.

Based on the above information answer the following questions.

(a) If P = 10500, then find N [1]

(b) If P = 11,000, then find the profit. [1]

(c) Find the rent that maximizes the total amount of profit. [2]

Ans: (a) If P is the rent price per apartment and N is the number of rented apartment, the profit is given by NP – 500 N = N(P – 500) [∵ Rs. 500/month is the maintenance charges for each occupied unit]

Clearly, if P = 10500, then 10500 = 10000 + 250 x ⇒ x = 2 ⇒ N = 48

(b) If P = 11000, then 11000 = 10000 + 250 x ⇒ x = 4 and so profit P(4) = 250(50 – 4) (38 + 4) = Rs. 4,83,000
(c) If x be the number of non-rented apartments, then N = 50 – x and P = 10000 + 250 x
Thus, profit = N(P – 500) = (50 – x) (10000 + 250 x – 500) = (50 – x) (9500 + 250 x) = 250(50 – x) (38 + x)
Now, P′(x) = 250[50 – x – (38 + x)] = 250[12 – 2x]
For maxima/minima, put P′(x) = 0
⇒ 12 – 2x = 0 ⇒ x = 6
Thus, price per apartment is, P = 10000 + 1500 = 11500
Hence, the rent that maximizes the profit is Rs. 11500.

37. Case-Study 2:
Read the following passage and answer the questions given below.

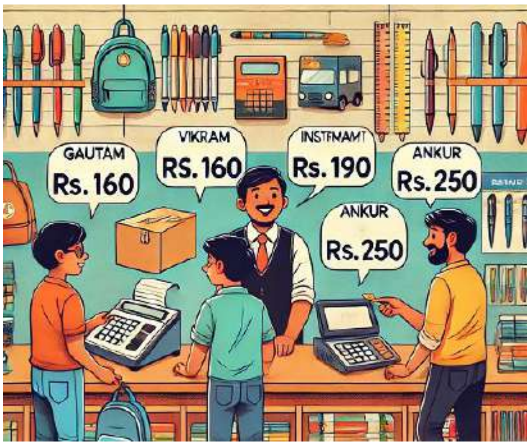
There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?
(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?
Ans:
(i)P(Shell fired from exactly one of them hits the plane)
= P[(Shell from A hits the plane and Shell from B does not hit the plane) or (Shell from A does not hit the plane and Shell from B hits the plane)]
= 0.3 × 0.8 + 0.7 × 0.2 = 0.38
(ii)P(Shell fired from B hit the plane/Exactly one of them hit the plane)

$$= \frac{\text{P(Shell fired from B hit the plane} \cap \text{Exactly one of them hit the plane)}}{\text{P(Exactly one of them hit the plane)}}$$

$$= \frac{\text{P(Shell from only B hit the plane)}}{\text{P(Exactly one of them hit the plane)}} = \frac{0.14}{0.38} = \frac{7}{19}$$

38. Case-Study 3:
Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of Rs. 160. From the same shop. Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of Rs. 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of Rs. 250.



Based on the above information, answer the following questions:
(I) Convert the given above situation into a matrix equation of the form AX =B.
(II) Find |A|.
(III) Find A^{-1}.

OR

(III) Determine P $=A^2 - 5A$.

Ans:	Pen	Bags	Instrument
Gautam	5	3	1
Vikram	2	1	3
Ankur	1	2	4

(i) $\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$

where x = cost of Pen
y = cost of Bag
z = cost of Instrument

(ii) $|A| = \begin{vmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix}$ = 5(4 – 6) – 3(8 – 3) + 1(4 – 1) = – 10 – 15 + 3 = – 22

(iii) $C_{11} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 4-6 = -2, C_{12} = -\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = -(8-3) = -5, C_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4-1 = 3$

$C_{21} = -\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = -(12-2) = -10, C_{22} = \begin{vmatrix} 5 & 1 \\ 1 & 4 \end{vmatrix} = 20-1 = 19, C_{23} = -\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10-3) = -7$

$C_{31} = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 9-1 = 8, C_{32} = -\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} = -(15-2) = -13, C_{33} = \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} = 5-6 = -1$

$$Adj(A) = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}^T = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{Adj(A)}{|A|} = -\frac{1}{22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix} = \frac{1}{22}\begin{bmatrix} 2 & 10 & -8 \\ 5 & -19 & 13 \\ -3 & 7 & 1 \end{bmatrix}$$

OR

(iii) $P = A^2 - 5A$

$$=\begin{bmatrix}5&3&1\\2&1&3\\1&2&4\end{bmatrix}\begin{bmatrix}5&3&1\\2&1&3\\1&2&4\end{bmatrix}-5\begin{bmatrix}5&3&1\\2&1&3\\1&2&4\end{bmatrix}=\begin{bmatrix}25+6+1&15+3+2&5+9+4\\10+2+3&6+1+6&2+3+12\\5+4+4&3+2+8&1+6+16\end{bmatrix}-\begin{bmatrix}25&15&5\\10&5&15\\5&10&20\end{bmatrix}$$

$$=\begin{bmatrix}32&20&18\\15&13&17\\13&13&23\end{bmatrix}-\begin{bmatrix}25&15&5\\10&5&15\\5&10&20\end{bmatrix}=\begin{bmatrix}7&5&13\\5&8&2\\8&3&3\end{bmatrix}$$