# SAMPLE PAPER TEST 01 FOR BOARD EXAM 2025

**SUBJECT: MATHEMATICS (041)** **MAX. MARKS : 80**
**CLASS : XII** **DURATION: 3 HRS**

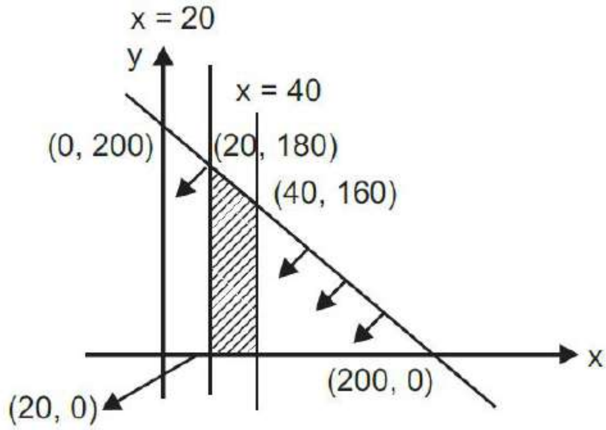
**General Instructions:**

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

## SECTION – A

**Questions 1 to 20 carry 1 mark each.**

**1.** For an L.P.P. the objective function is Z = 400x + 300y, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.




Find the coordinates at which the objective function is maximum.

(a) (20, 0) (b) (40, 0) (c) (40, 160) (d) (20, 180)

**2.** If A = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and A = B², thenx equals:

(a) ± 1 (b) – 1 (c) 1 (d) 2

**3.** If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is:

(a) 1 (b) 2 (c) 3 (d) 4

**4.** The cofactor of (-1) in the matrix $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ is:

(a) 1 (b) 2 (c) -1 (d) 0

**5.** The integrating factor of the differential equation $\frac{dy}{dx}(x\log x)+y=2\log x$ is

(a) $e^x$ (b) log x (c) log(log x) (d) x

**6.** $\int \sin^3(2x+1)dx=?$

(a) $\frac{1}{2}\cos(2x+1)+\frac{1}{3}\cos^3(2x+1)+C$ (b) $-\frac{1}{2}\cos(2x+1)+\frac{1}{6}\cos^3(2x+1)+C$

(c) $\frac{1}{8}\sin^4(2x+1)+C$ (d) none of these

**7.** The value of $\int_0^a \frac{\sqrt{a}}{\sqrt{x}+\sqrt{a-x}}dx$ is:

(a) a/2 (b) a (c) $a^2$ (d) 0

**8.** The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is:

(a) 2 (b) 3 (c) 5 (d) 0

**9.** A function $f(x) = \begin{cases} \frac{\sin x}{x}+\cos x, & x\neq 0 \\ 2k, & x=0 \end{cases}$ is continuous at $x = 0$ for

(*a*) $k = 1$ (*b*) $k = 2$ (*c*) $k = 1/2$ (*d*) $k = 3/2$

**10.** The direction ratios of the line 6x – 2 = 3y + 1 = 2z – 2 are:

(a) 6, 3, 2 (b) 1, 1, 2 (c) 1, 2, 3 (d) 1, 3, 2

**11.** Two vector $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ are collinear if:

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $a_1/b_1 = a_2/b_2 = a_3/b_3$

(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

**12.** Two-line $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy plane has coordinates

(a) (2, 4, 7) (b) (-2, 4, 7) (c) (2, -4, -7) (d) (2, -4, 7)

**13.** Feasible region is the set of points which satisfy

(a) the objective functions (b) some of the given constraints

(c) all of the given constraints (d) none of these

**14.** If the position vector $\vec{a}$ of the point (5, n) is such that $|\vec{a}|$ = 13, then the value(s) of n can be

(a) ±12 (b) ±8 (c) Only 12 (d) Only 8

**15.** If for any two events A and B, P(A) = 4/5 and P(A ∩ B) = 7/10, then P(B/A) is equals to:

(a) 1/10 (b) 1/8 (c) 7/8 (d) 17/20

**16.** *A* and *B* are invertible matrices of the same order such that $|(AB)^{-1}| = 8$, If $|A| = 2$, then $|B|$ is

(*a*) 16 (*b*) 4 (*c*) 6 (*d*) 1/16

**17.** If f (x) = x $\tan^{-1}$ x, then f ′ (1) =

(a) $1+\frac{\pi}{4}$ (b) $\frac{1}{2}+\frac{\pi}{4}$ (c) $\frac{1}{2}-\frac{\pi}{4}$ (d) 2

**18.** If F(x) = $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ then F(x) F(y) is equal to

(a) F(x) (b) F(xy) (c) F(x + y) (d) F(x – y)

**ASSERTION-REASON BASED QUESTIONS**

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**19. Assertion (A):** Lines $\frac{x+2}{-2}=\frac{y-1}{3}=\frac{z-z}{1}$ and $\frac{x-3}{-3}=\frac{y}{-2}=\frac{z+1}{2}$ are coplanar.

**Reason (R):** Let line $l_1$ passes through the point $(x_1, y_1, z_1)$ and parallel to the vector whose direction ratios are $a_1$, $b_1$ and $c_1$; and let line $l_2$ passes through the point $(x_2, y_2, z_2)$ and parallel to the vector whose direction ratios are $a_2$, $b_2$ and $c_2$.

Then both lines $l_1$ and $l_2$ are coplanar if and only if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

**20. Assertion (A):** $\sin^{-1}(\sin(2\pi/3)) = 2\pi/3$

**Reason (R):** $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in [(-\pi)/2, \pi/2]$

## <u>SECTION – B</u>

**Questions 21 to 25 carry 2 marks each.**

**21.** Prove that the points A, B and C with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively are collinear if and only if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{b}=\vec{0}$

**22.** Check whether the relation R in the set Z of integers defined as R = {(a, b) : a + b is "divisible by 2"} is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e., [0]. 3

**OR**

Write the domain and range (principle value branch) of the following functions:
$f(x) = \tan^{-1} x$

**23.** Find the area of a parallelogram whose adjacent side are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

**OR**

Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

**24.** Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

**25.** If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$.

## SECTION – C

**Questions 26 to 31 carry 3 marks each.**

**26.** Evaluate: $\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x}\right) dx$

**27.** Evaluate: $\int \frac{3x+1}{(x-1)^2(x+3)} dx$

**28.** Evaluate: $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$

**OR**

Find: $\int \frac{x^4}{(x-1)(x^2+1)} dx$

**29.** Solve: $\frac{dy}{dx} = (x+y+1)^2$

**OR**

Find the general solution of the differential equation: $xdy - ydx = \sqrt{x^2+y^2}\,dx$

**30.** Solve the following Linear Programming Problem graphically :

Maximize: P = 70x + 40y

Subject to: $3x + 2y \le 9$,

$3x + y \le 9$

$x \ge 0,\ y \ge 0$

**31.** Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X.

**OR**

There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head. What is probability that it was the two headed coin?

## SECTION – D

**Questions 32 to 35 carry 5 marks each.**

**32.** Show that each of the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by R = {(a, b): |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.

**33.** Using integration, find the area bounded by the lines x + 2y = 2, y - x = 1 and 2x + y = 7.

**34.** If A = $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find $A^{-1}$ and use it to solve the following system of the equations :

x + 2y – 3z = 6, 3x + 2y – 2z = 3, 2x – y + z = 2

**OR**

The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

**35.** Find the vector equation of the line passing through (1, 2, – 4) and perpendicular to the two lines: $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$

**OR**

Find the shortest distance between the lines whose vector equations are:
$\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

## SECTION – E(Case Study Based Questions)

**Questions 36 to 38 carry 4 marks each.**

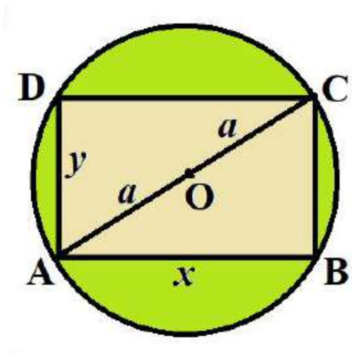
**36. Case-Study 1:**

Sonam wants to prepare a sweet box for Diwali at home. For making lower part of box, she takes a square piece of cardboard of side 18 cm.
Now, x cm be the length of each side of the square cardboard which is to be cut off from corner of the square piece of side 18 cm.

Based on the above information, answer the following questions :
(i) Express Volume of the open box formed by folding up the cutting corner in terms of x and find the value of x for which $\frac{dV}{dx}=0$. (2)
(ii) Sonam is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum? (2)

**37. Case-Study 2:**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Find the perimeter of rectangle in terms of any one side and radius of circle. (1)
(ii) Find critical points to maximize the perimeter of rectangle? (1)
(iii) Check for maximum or minimum value of perimeter at critical point. (2)

**OR**

(iii) If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region. (2)

**38. Case-Study 3:**

In a school, teacher asks a question to three students Kanabh, Raj and Shubi respectively. The probability of solving the question by Kanabh, Raj and Shubi are 40%, 15% and 50% respectively. The probability of making error by Kanabh, Raj and Shubi are 1.5%, 2% and 2.5%.

Based on the given information, answer the following questions:
(i) Find the probability that Shubi solved the question and committed an error. (2)
(ii) Find the total probability of committing an error is solving the question. (2)

# SAMPLE PAPER TEST 01 FOR BOARD EXAM 2025
# (ANSWERS)

**SUBJECT: MATHEMATICS (041)** **MAX. MARKS : 80**
**CLASS : XII** **DURATION: 3 HRS**

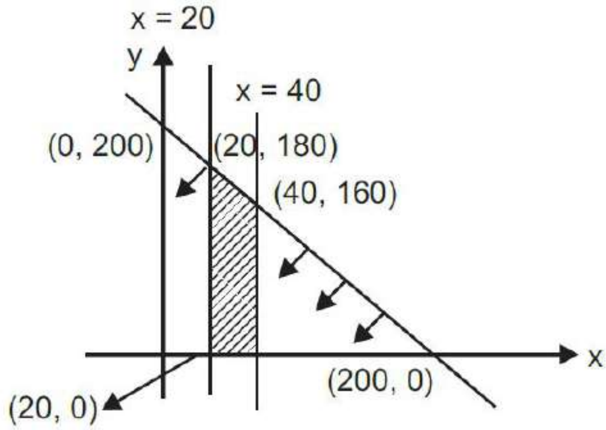
**General Instructions:**

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

## SECTION – A

**Questions 1 to 20 carry 1 mark each.**

**1.** For an L.P.P. the objective function is Z = 400x + 300y, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.




Find the coordinates at which the objective function is maximum.

(a) (20, 0) (b) (40, 0) (c) (40, 160) (d) (20, 180)

Ans: (c) (40, 160)

Value of z at each corner point

z at (20, 0), z = 400 × 20 + 300 × 0 = 8000

z at (40, 0) = 400 × 40 + 300 × 0 = 16000

z at (40, 160) = 400 × 40 + 300 × 160 = 16000 + 48000 = 64000

z at (20, 180) = 400 × 20 + 300 × 180 = 8000 + 54000 = 62000

max z = 64000 for x = 40, y = 160

**2.** If A = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and A = B², thenx equals:

(a) ± 1 (b) – 1 (c) 1 (d) 2

Ans: (c) 1

**3.** If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is:

(a) 1 (b) 2 (c) 3 (d) 4

Ans: (d) 4

**4.** The cofactor of (-1) in the matrix $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ is:

(a) 1 (b) 2 (c) -1 (d) 0

Ans: (c) -1

Cofactor of (-1) = $(-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ = (-1) × 1 = -1

**5.** The integrating factor of the differential equation $\frac{dy}{dx}(x\log x) + y = 2\log x$ is

(a) $e^x$ (b) log x (c) log(log x) (d) x

Ans: (b) log x

**6.** $\int \sin^3(2x+1)dx = ?$

(a) $\frac{1}{2}\cos(2x+1) + \frac{1}{3}\cos^3(2x+1) + C$ (b) $-\frac{1}{2}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) + C$

(c) $\frac{1}{8}\sin^4(2x+1) + C$ (d) none of these

Ans: (b) $-\frac{1}{2}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) + C$

**7.** The value of $\int_0^a \frac{\sqrt{a}}{\sqrt{x}+\sqrt{a-x}}dx$ is:

(a) a/2 (b) a (c) $a^2$ (d) 0

Ans: (a) a/2

$$I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx \quad ...(i)$$

$$= \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-(a-x)}}dx$$

$$= \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \quad ...(ii)$$

Adding (i) and (ii), we get $2I = \int_0^a 1.dx \Rightarrow 2I = a \text{ or } I = \frac{a}{2}$

**8.** The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is:

(a) 2 (b) 3 (c) 5 (d) 0

Ans: (b) 3

$$\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right) = 3\left(\frac{dy}{dx}\right)^2 \frac{d^2y}{dx^2}$$

order is 2 and degree is 1

∴ required answer 2 + 1 = 3

**9.** A function $f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & x \neq 0 \\ 2k, & x = 0 \end{cases}$ is continuous at $x = 0$ for

(*a*) $k = 1$ (*b*) $k = 2$ (*c*) $k = 1/2$ (*d*) $k = 3/2$

Ans: (*a*) $k = 1$

$$\lim_{x \to 0}\left(\frac{\sin x}{x} + \cos x\right) = 1 + 1 = 2 = 2k \Rightarrow k = 1$$

**10.** The direction ratios of the line 6x – 2 = 3y + 1 = 2z – 2 are:

(a) 6, 3, 2 (b) 1, 1, 2 (c) 1, 2, 3 (d) 1, 3, 2

Ans: (c) 1, 2, 3

Given the equation of a line is

6x – 2 = 3y + 1 = 2z – 2

$$\Rightarrow 6\left(x - \frac{1}{3}\right) = 6\left(y + \frac{1}{3}\right) = 2(z-1) \Rightarrow \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{3}}{2} = \frac{z-1}{3}$$

This shows that the given line passes through (1/3, -1/3, 1), and has direction ratios 1, 2, and 3.

**11.** Two vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if:

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $a_1/b_1 = a_2/b_2 = a_3/b_3$

(c) $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

Ans: (b) $a_1/b_1 = a_2/b_2 = a_3/b_3$

**12.** Two-line $\dfrac{x-3}{1} = \dfrac{y+1}{3} = \dfrac{z-6}{-1}$ and $\dfrac{x+5}{7} = \dfrac{y-2}{-6} = \dfrac{z-3}{4}$ intersect at the point R. The reflection of R in the xy plane has coordinates

(a) (2, 4, 7) (b) (-2, 4, 7) (c) (2, -4, -7) (d) (2, -4, 7)

Ans: (c) (2, -4, -7)

**13.** Feasible region is the set of points which satisfy

(a) the objective functions (b) some of the given constraints

(c) all of the given constraints (d) none of these

Ans: (c) all of the given constraints

**14.** If the position vector $\vec{a}$ of the point (5, n) is such that $|\vec{a}| = 13$, then the value(s) of n can be

(a) ±12 (b) ±8 (c) Only 12 (d) Only 8

Ans:

$\vec{a} = 5\hat{i} + n\hat{j}$

$\therefore |\mathbf{a}| = \sqrt{25 + n^2} = 13 \Rightarrow 25 + n^2 = 169 \Rightarrow n^2 = 169 - 25 = 144 \Rightarrow n = \pm 12$

**15.** If for any two events A and B, P(A) = 4/5 and P(A ∩ B) = 7/10, then P(B/A) is equals to:

(a) 1/10 (b) 1/8 (c) 7/8 (d) 17/20

Ans: (c) 7/8

**16.** *A* and *B* are invertible matrices of the same order such that $|(AB)^{-1}| = 8$, If $|A| = 2$, then $|B|$ is

(*a*) 16 (*b*) 4 (*c*) 6 (*d*) 1/16

Ans: (*d*) 1/16

$$|(AB)^{-1}| = \frac{1}{|AB|} = \frac{1}{|A||B|} \Rightarrow 8 = \frac{1}{2|B|} \Rightarrow B = \frac{1}{16}$$

**17.** If f (x) = x $\tan^{-1}$ x, then f ′ (1) =

(a) $1+\frac{\pi}{4}$ (b) $\frac{1}{2}+\frac{\pi}{4}$ (c) $\frac{1}{2}-\frac{\pi}{4}$ (d) 2

Ans: (b) $\frac{1}{2}+\frac{\pi}{4}$

Differentiating w.r.t. $x$, we get $f'(x) = x\times\frac{1}{1+x^2}+\tan^{-1}x$

$\Rightarrow f'(x) = \frac{x}{1+x^2}+\tan^{-1}x$

Put $x = 1$, then $f'(x) = \frac{1}{1+(1)^2}+\tan^{-1}(1) = \frac{1}{1+1}+\frac{\pi}{4}=\frac{1}{2}+\frac{\pi}{4}$

**18.** If F(x) = $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ then F(x) F(y) is equal to

(a) F(x) (b) F(xy) (c) F(x + y) (d) F(x – y)

Ans: (c) F(x + y)

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}=\begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$$

## ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

**19. Assertion (A):** Lines $\frac{x+2}{-2}=\frac{y-1}{3}=\frac{z-z}{1}$ and $\frac{x-3}{-3}=\frac{y}{-2}=\frac{z+1}{2}$ are coplanar.

**Reason (R):** Let line $l_1$ passes through the point $(x_1, y_1, z_1)$ and parallel to the vector whose direction ratios are $a_1$, $b_1$ and $c_1$; and let line $l_2$ passes through the point $(x_2, y_2, z_2)$ and parallel to the vector whose direction ratios are $a_2$, $b_2$ and $c_2$.

Then both lines $l_1$ and $l_2$ are coplanar if and only if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

**Ans:** (a) Both A and R are true and R is the correct explanation of A.

**20. Assertion (A):** $\sin^{-1}(\sin(2\pi/3)) = 2\pi/3$

**Reason (R):** $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in [(-\pi)/2, \pi/2]$

Ans: (d) A is false but R is true.

The principal value branch of $\sin^{-1}x$ is $[(-\pi)/2, \pi/2]$

Let $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$\sin^{-1}(\sin\theta) = \sin^{-1}x = \theta$

$\sin^{-1}(\sin\theta) = \theta$, if $\theta \in [(-\pi)/2, \pi/2]$

Hence R is true.

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)\neq\frac{2\pi}{3}$, since $\frac{2\pi}{3}\notin\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Hence A is false.

## SECTION – B

**Questions 21 to 25 carry 2 marks each.**

**21.** Prove that the points A, B and C with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively are collinear if and only if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{b}=\vec{0}$

Ans:

The points $A$, $B$ and $C$ are collinear

$\Rightarrow$ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel vectors.

$\Rightarrow \overrightarrow{AB}\times\overrightarrow{BC}=\vec{0}$

$\Rightarrow (\vec{b}-\vec{a})\times(\vec{c}-\vec{b})=\vec{0} \Leftrightarrow (\vec{b}-\vec{a})\times\vec{c}-(\vec{b}-\vec{a})\times\vec{b}=\vec{0}$

$\Rightarrow (\vec{b}\times\vec{c}-\vec{a}\times\vec{c})-(\vec{b}\times\vec{b}-\vec{a}\times\vec{b})=\vec{0}$

$\Rightarrow (\vec{b}\times\vec{c}+\vec{c}\times\vec{a})-(\vec{0}-\vec{a}\times\vec{b})=\vec{0}\ [\because \vec{a}\times\vec{c}=-(\vec{c}\times\vec{a})]$

$\Rightarrow \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

**22.** Check whether the relation R in the set Z of integers defined as R = {(a, b) : a + b is “divisible by 2”} is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e., [0]. 3

Ans: Reflexive: (a, a) : a + a = 2a which is even

∴ divisible by 2

∴ (a, a) ∈ R ∀ a ∈ Z. Hence R is reflexive.

Symmetric: If (a, b) ∈ R, then a + b is “divisible by 2”

Let a + b = 2m ⇒ b + a = 2m ...[∵ b + a = a + b

⇒ (b, a) ∈ R ∀ a, b ∈ z

Hence R is symmetric.

Transitive: If (a, b) ∈ R and (b, c) ∈ R

Let a + b = 2m ...(i)

b + c = 2n ...(ii)

Adding (i) and (ii), we have a + b + b + c = 2m + 2n

a + 2b + c = 2m + 2n ⇒ a + c = 2m + 2n – 2b ⇒ a + c = 2(m + n – b)

⇒ a + c = 2k ...where[k = m + n – b ⇒ (a, c) ∈ R

Hence R is transitive. Equivalence class containing 0 i.e. [0] = {......–4, –2, 0, 2, 4, ......}

**OR**

Write the domain and range (principle value branch) of the following functions:

f (x) = $\tan^{-1}$ x

Ans: f(x) = $\tan^{-1}$x

Domain = Real number and Range = $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

**23.** Find the area of a parallelogram whose adjacent side are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

Ans:

$$\vec{a} = \hat{i}-\hat{j}+3\hat{k}$$

$$\vec{b} = 2\hat{i}-7\hat{j}+\hat{k}$$

Required area of $||^{gm}$

$$= |\vec{a}\times\vec{b}| = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix} = i(-1+21)-\hat{j}(1-6)+\hat{k}(-7+2)$$

$$= |20\hat{i}+5\hat{j}-5\hat{k}| = |\sqrt{400+25+25}| = 15\sqrt{2}\ \text{unit}^2$$

**OR**

Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

Ans: Vector equation of the line passing through (1, 2, 3) and (–3, 4, 3) is

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) \quad \text{where } \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{b} = -3\hat{i} + 4\hat{j} + 3\hat{k}$$

$$\Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda[(-3-1)\hat{i} + (4-2)\hat{j} + (3-3)\hat{k}]$$

$$\Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-4\hat{i} + 2\hat{j})$$

Equation of $z$-axis is $\vec{r} = \mu\hat{k}$

Since $(-4\hat{i} + 2\hat{j}) \cdot \hat{k} = 0$

$\therefore$ **Line (*i*) is $\perp$ to $z$-axis.**

**24.** Show that the function f(x) = $x^3 - 3x^2 + 6x - 100$ is increasing on R.

Ans: We have, f(x) = $x^3 - 3x^2 + 6x - 100$ ...(i)

Differentiating (i) w.r.t. x, we get

$f'(x) = 3x^2 - 6x + 6 = 3(x^2 - 2x + 1) + 3 = 3(x-1)^2 + 3 > 0$

($\because$ For all values of x, $(x-1)^2$ is always positive)

$\therefore f'(x) > 0$

So, f(x) is an increasing function on R.

**25.** If $y = 500e^{7x} + 600e^{-7x}$, show that $\dfrac{d^2y}{dx^2} = 49y$.

Ans: Given that $y = 500e^{7x} + 600e^{-7x}$

$$\Rightarrow \frac{dy}{dx} = 7 \times 500e^{7x} - 7 \times 600e^{-7x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 49 \times 500e^{7x} + 49 \times 600e^{-7x} = 49(500e^{7x} + 600e^{-7x})$$

$$\Rightarrow \frac{d^2y}{dx^2} = 49y.$$

## SECTION – C

**Questions 26 to 31 carry 3 marks each.**

**26.** Evaluate: $\displaystyle\int e^x\left(\frac{\sin 4x - 4}{1 - \cos 4x}\right)dx$

Ans:

$$\int e^x\left\{\frac{2\sin 2x\cos 2x - 4}{2\sin^2 2x}\right\}dx = \int e^x\{\cot 2x - 2\operatorname{cosec}^2 2x\}dx;$$

Let $f(x) = \cot 2x$, $f'(x) = -2\operatorname{cosec}^2 2x$

Using $\int e^x\{f(x) + f'(x)dx = e^x f(x) + C$, we get

$$\int e^x\left(\frac{\sin 4x - 4}{1 - \cos 4x}\right)dx = e^x \cot 2x + C$$

**27.** Evaluate: $\displaystyle\int \frac{3x+1}{(x-1)^2(x+3)}dx$

Ans:

Using partial fraction

$$\frac{3x+1}{(x-1)^2(x+3)} = \frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+3)} \quad ...(i)$$

On comparing $3x + 1 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$

Putting $x = 1$, we get $3 \times (1) + 1 = 0 + B(1+3) + 0$

$\Rightarrow 4 = 4B \Rightarrow B = 1$

Putting $x = -3$, we get

$3 \times (-3) + 1 = A(-3-1)(-3+3) + B(-3+3) + C(-3-1)2$

$\Rightarrow -8 = 16C \Rightarrow C = -\frac{1}{2}$

We have $-3A + C = 1 \Rightarrow -3A-\frac{1}{2} = 1$

$\Rightarrow -3A = \frac{3}{2} \Rightarrow A = \frac{1}{2}$

Equation (i) can be written as

$$\frac{3x+1}{(x+3)(x-1)^2} = \frac{1}{2(x-1)}+\frac{1}{(x-1)^2}-\frac{1}{2(x+3)}$$

$$\Rightarrow I = \int\frac{3x+1}{(x+3)(x-1)^2}dx = \frac{1}{2}\int\frac{1}{(x-1)}dx+\int\frac{1}{(x-1)^2}dx-\frac{1}{2}\int\frac{1}{(x+3)}dx$$

$$\Rightarrow I = \frac{1}{2}\log|x-1|-\frac{1}{(x-1)}-\frac{1}{2}\log|x+3|+C$$

**28.** Evaluate: $\int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx$

Ans:

$$I = \int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx \Rightarrow I = \int_0^{2\pi}\frac{1}{1+e^{\sin(2\pi-x)}}dx$$

$$\Rightarrow I = \int_0^{2\pi}\frac{1}{1+e^{-\sin x}}dx = \int_0^{2\pi}\frac{e^{\sin x}}{1+e^{\sin x}}dx$$

$$\Rightarrow 2I = \int_0^{2\pi}\frac{1+e^{\sin x}}{1+e^{\sin \pi}}dx \Rightarrow 2I = \int_0^{2\pi}1dx = [x]_0^{2\pi}$$

$$\Rightarrow 2I = 2\pi-0 \Rightarrow I = \int_0^{2\pi}\frac{1}{1+e^{\sin x}} = \pi$$

**OR**

Find: $\int\frac{x^4}{(x-1)(x^2+1)}dx$

Ans:

$$\int\frac{x^4}{(x-1)(x^2+1)}dx = \int\frac{x^4}{x^3-x^2+x-1}dx$$

$$\int\frac{x^4}{(x-1)(x^2+1)}dx = \int\left[x+1+\frac{1}{(x-1)(x^2+1)}\right]dx = \frac{x^2}{2}+x+\int\frac{1}{(x-1)(x^2+1)}dx$$

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$\Rightarrow 1 = A(x^2+1) + (Bx+C)(x-1)$

$\Rightarrow 1 = (A+B)x^2 + (A-C) + x(C-B)$

$\therefore \quad A + B = 0 \Rightarrow A = -B$

$C - B = 0 \Rightarrow C = B \Rightarrow A - C = 1$

$\Rightarrow -2C = 1 \Rightarrow C = -\frac{1}{2} \Rightarrow B = -\frac{1}{2}$ and $A = \frac{1}{2}$

$$\int \frac{1}{(x-1)(x^2+1)}dx = \frac{1}{2}\int\left[\frac{+1}{x-1} + \frac{-x-1}{x^2+1}\right]dx = \frac{1}{2}\left[\int\frac{dx}{x-1} - \int\frac{x}{x^2+1}dx - \int\frac{dx}{x^2+1}\right]$$

$$= \frac{1}{2}\left[\log|x-1| - \frac{1}{2}\log|x^2+1| - \tan^{-1}x\right] + c_1 = \frac{1}{2}\log\left|\frac{x-1}{\sqrt{x^2+1}}\right| - \frac{1}{2}\tan^{-1}x + c$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log\left|\frac{x-1}{\sqrt{x^2+1}}\right| - \frac{1}{2}\tan^{-1}x + c$$

**29.** Solve: $\frac{dy}{dx} = (x+y+1)^2$

**Ans:**

We have $\frac{dy}{dx} = (x+y+1)^2 \quad$ ... (i)

Put $x + y + 1 = t$

Differentiating both sides w.r.t. $x$, we get $1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

Substituting the value of $\frac{dy}{dx}$ and $(x+y+1)$ in (i), we get

$\frac{dt}{dx} - 1 = t^2 \Rightarrow \frac{dt}{dx} = t^2 + 1$

Separating the variables, we get $\frac{1}{t^2+1}dt = dx \Rightarrow \int\frac{1}{t^2+1}dt = \int dx$

$\Rightarrow \tan^{-1}t = x + C \Rightarrow \tan^{-1}(x+y+1) = x + C$ [Integrating both sides]

**OR**

Find the general solution of the differential equation: $xdy - ydx = \sqrt{x^2+y^2}dx$

Ans:

Given differential equation is $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$\Rightarrow x\,dy = \left(\sqrt{x^2+y^2} + y\right)dx \Rightarrow \frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$ ...(i)

The given differential equation is homogenous with zero degree

So, put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From eq. (i), we get $v + x\frac{dv}{dx} = \frac{\sqrt{x^2+(vx)^2}+vx}{x}$

$\Rightarrow x\frac{dv}{dx} = \frac{\sqrt{x^2(1+v^2)}+vx}{x} - v \Rightarrow x\frac{dv}{dx} = \sqrt{1+v^2} + v - v$

$\Rightarrow x\frac{dv}{dx} = \sqrt{1+v^2} \Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$

Integrating both sides, we get $\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$

$\Rightarrow \quad \log \left|v+\sqrt{v^2+1}\right| = \log x + \log c \Rightarrow \quad \log \left|v+\sqrt{v^2+1}\right| = \log (cx)$

$\Rightarrow \quad v + \sqrt{v^2+1} = cx$

Put $v = \frac{y}{x}$, we get $\frac{y}{x} + \sqrt{\left(\frac{y}{x}\right)^2 + 1} = cx \Rightarrow \quad y + \sqrt{x^2+y^2} = cx^2$ ...(ii)

Given, $x = 1$ when $y = 0$

$\therefore \quad 0+\sqrt{1+0} = c \times 1 \Rightarrow \quad c = 1$

Put $c = 1$ in eq (ii), we get $y + \sqrt{x^2+y^2} = x^2$

$\Rightarrow \sqrt{x^2+y^2} = x^2 - y \Rightarrow \quad (x^2 + y^2) = (x^2 - y)^2$

**30.** Solve the following Linear Programming Problem graphically :

Maximize: $P = 70x + 40y$

Subject to: $3x + 2y \leq 9$,

$3x + y \leq 9$

$x \geq 0, y \geq 0$

Ans: Maximize:$P = 70x + 40y$

$3x + 2y \leq 9$, $3x + y \leq 9$

$3x + 2y = 9$

| $x$ | 0 | 3 |
|---|---|---|
| $y$ | 9/2 | 0 |

$3x + y = 9$

| $x$ | 0 | 3 |
|---|---|---|
| $y$ | 9 | 0 |

$x \geq 0, y \geq 0$

Feasible area is OABO

$P = 70x + 40y$

At (0, 0), $P = 70 \times 0 + 40 \times 0 = 0$

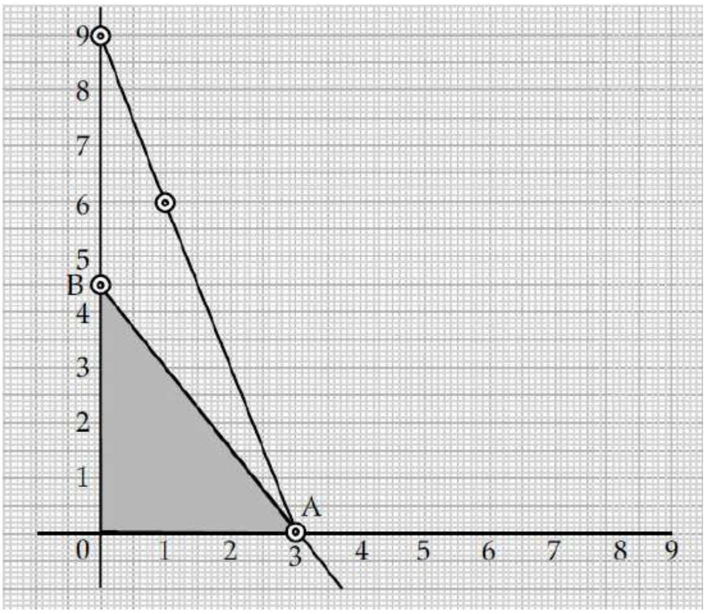
At (3, 0), $P = 70 \times 3 + 40 \times 0 = 210$

At (0,92), $P = 70 \times 0 + 40 \times 92 = 180$

Maximise at (3, 0)

$x = 3$ and $y = 0$

Maximum value = 210



**31.** Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X.

Ans: First 7 natural numbers are 1, 2, 3, 4, 5, 6, 7.

$$S = \begin{Bmatrix} (1,2)\,(1,3)\,(1,4)\,(1,5)\,(1,6)\,(1,7) \\ (2,1)\,(2,3)\,(2,4)\,(2,5)\,(2,6)\,(2,7) \\ (3,1)\,(3,2)\,(3,4)\,(3,5)\,(3,6)\,(3,7) \\ (4,1)\,(4,2)\,(4,3)\,(4,5)\,(4,6)\,(4,7) \\ (5,1)\,(5,2)\,(5,3)\,(5,4)\,(5,6)\,(5,7) \\ (6,1)\,(6,2)\,(6,3)\,(6,4)\,(6,5)\,(6,7) \\ (7,1)\,(7,2)\,(7,3)\,(7,4)\,(7,5)\,(7,6) \end{Bmatrix} \text{ i.e. 42 ways}$$

$$P(X=1)=\frac{12}{42}=\frac{2}{7}, P(X=2)=\frac{10}{42}=\frac{5}{21}, P(X=3)=\frac{8}{42}=\frac{4}{21}$$

$$P(X=4)=\frac{6}{42}=\frac{1}{7}, P(X=5)=\frac{4}{42}=\frac{2}{21}, P(X=6)=\frac{2}{42}=\frac{1}{21}$$

∴ Probability distribution is

| **X** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **P(X)** | $\frac{2}{7}$ | $\frac{5}{21}$ | $\frac{4}{21}$ | $\frac{1}{7}$ | $\frac{2}{21}$ | $\frac{1}{21}$ |

**OR**

There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head. What is probability that it was the two headed coin?

Ans: Let $E_1$ : Two headed coin is chosen

$E_2$ : Coin chosen is biased

$E_3$ : Coin chosen is unbiased

A : Coin shows head

$$P(E_1)=\frac{1}{3}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{1}{3}$$

$$P(A/E_1)=1, P(A/E_2)=\frac{75}{100}=\frac{3}{4}, P(A/E_3)=\frac{1}{2}$$

Using Baye's theorem,

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot(A|E_2)+P(E_3)\cdot(A|E_3)}$$

$$=\frac{\frac{1}{3}\times 1}{\left(\frac{1}{3}\times 1\right)\times\left(\frac{1}{3}\times\frac{3}{4}\right)\times\left(\frac{1}{3}\times\frac{1}{2}\right)}=\frac{\frac{1}{3}}{\frac{1}{3}\times\frac{3}{12}\times\frac{1}{6}}=\frac{\frac{1}{3}}{\frac{4+3+2}{12}}=\frac{1}{3}\times\frac{12}{9}=\frac{4}{9}$$

## SECTION – D

**Questions 32 to 35 carry 5 marks each.**

**32.** Show that each of the relation R in the set $A=\{x\in Z: 0\le x\le 12\}$, given by R = {(a, b): |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.

Ans: $A=\{x\in Z: 0\le x\le 12\}=\{0,1,2,3,4,5,6,7,8,9,10,11,12\}$ and

R = {(a, b): |a – b| is a multiple of 4}

For any element $a$ ∈ A, we have $(a, a)$ ∈ R ⇒ |a – a| = 0 is a multiple of 4.

$\therefore$ R is reflexive.

Now, let $(a, b) \in$ R $\Rightarrow$|a – b| is a multiple of 4.

$\Rightarrow$|–(a – b)| is a multiple of 4

$\Rightarrow$|b – a| is a multiple of 4.

$\Rightarrow$ $(b, a) \in$ R

$\therefore$ R is symmetric.

Now, let $(a, b)$, $(b, c) \in$ R.

$\Rightarrow$|a – b| is a multiple of 4 and |b – c| is a multiple of 4.

$\Rightarrow$(a – b) is a multiple of 4 and (b – c) is a multiple of 4.

$\Rightarrow$(a – b + b – c) is a multiple of 4

$\Rightarrow$(a – c) is a multiple of 4

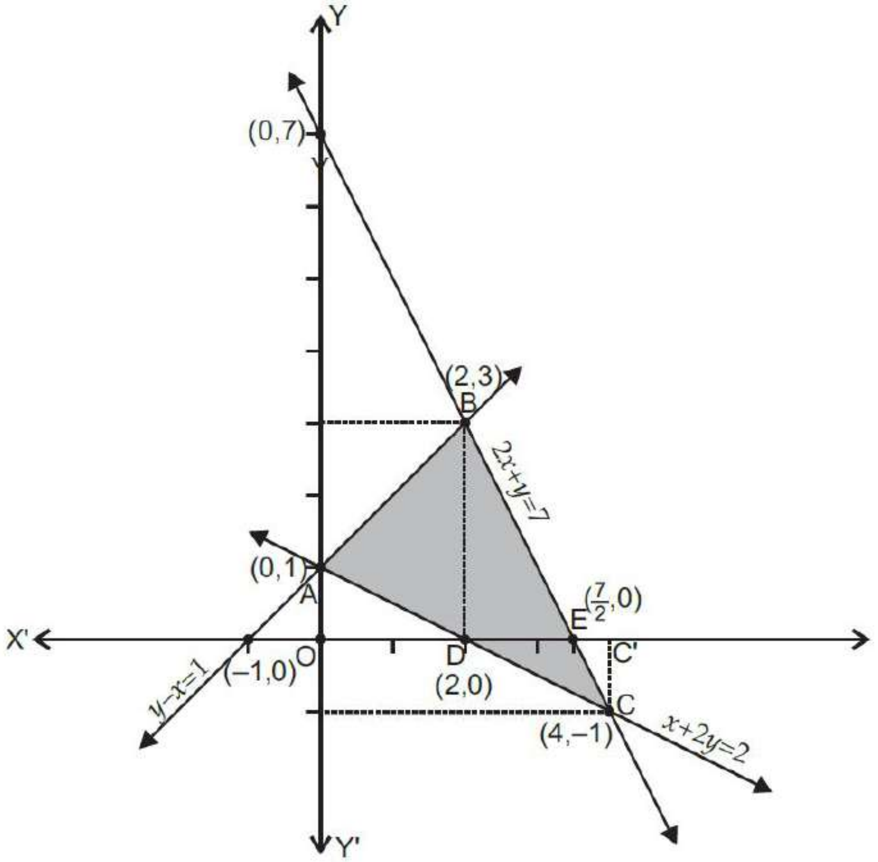
$\Rightarrow$|a – c| is a multiple of 4

$\Rightarrow$ $(a, c) \in$R

$\therefore$ R is transitive.

Hence, R is an equivalence relation.
The set of elements related to 1 is {1, 5, 9} since
|1 – 1| = 0 is a multiple of 4
|5 – 1| = 4 is a multiple of 4
|9 – 1| = 8 is a multiple of 4

**33.** Using integration, find the area bounded by the lines x + 2y = 2, y - x = 1 and 2x + y = 7.
Ans: Given, $x + 2y = 2$ ...(*i*)
$y – x = 1$ ...(*ii*)
$2x + y = 7$ ...(*iii*)
On plotting these lines, we have



Area of required region

$$= \int_{-1}^{3} \frac{7-y}{2}dy - \int_{-1}^{1} (2-2y)dy - \int_{1}^{3} (y-1)dy$$

$$= \frac{1}{2}\left[7y - \frac{y^2}{2}\right]_{-1}^{3} - [2y - y^2]_{-1}^{1} - \left[\frac{y^2}{2} - y\right]_{1}^{3}$$

$$= \frac{1}{2}\left(21 - \frac{9}{2} + 7 + \frac{1}{2}\right) - (2 - 1 + 2 + 1) - \left(\frac{9}{2} - 3 - \frac{1}{2} + 1\right)$$

$$= 12 - 4 - 2 = 6 \text{ sq. units}$$

**34.** If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find $A^{-1}$ and use it to solve the following system of the equations :

$x + 2y - 3z = 6$, $3x + 2y - 2z = 3$, $2x - y + z = 2$

Ans:

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix} = 1(2-2) - 2(3+4) - 3(-3-4) = -14 + 21 = 7 \neq 0$$

$\therefore\ A^{-1}$ exists

Now, $A_{11} = 0, A_{12} = -7, A_{13} = -7, A_{21} = 1, A_{22} = 7,$
$A_{23} = 5, A_{31} = 2, A_{32} = -7, A_{33} = -4$

$$\therefore\ \text{adj } A = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

The given system of equations is
$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

The system of equations can be written as $AX = B$

where $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

$\because\ A^{-1}$ exists, so system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\Rightarrow x = 1, y = -5, z = -5$

**OR**

The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Ans: Let the first , second and third number be x, y, z respectively.

Then, according to the given condition, we have

x + y + z = 6
y + 3z = 11
x + z = 2y or x − 2y + z = 0
This system of equations can be written as AX = B, where

$$A=\begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \ \& \ B=\begin{bmatrix}6\\11\\0\end{bmatrix}$$

A = 1(1 + 6) – 0 + 1(3 − 1) = 9
⇒ |A| ≠ 0
∴ The system of equation is consistent and has a unique solution.
Now, we find adj(A)

$A_{11}=7, A_{12}=3, A_{13}=-1,\ A_{21}=-3, A_{22}=0, A_{23}=3,$

$A_{31}=2, A_{32}=-3, A_{33}=1$

Hence, $adj(A)=\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}$

Thus, $A^{-1}=\dfrac{1}{|A|}adj(A)\ =\dfrac{1}{9}\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}$

Since, AX=B
∴ X = $A^{-1}$B

$$\Rightarrow X=\frac{1}{9}\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}\begin{bmatrix}6\\11\\0\end{bmatrix} \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{9}\begin{bmatrix}9\\18\\27\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

⇒ x = 1, y = 2, z = 3

**35.** Find the vector equation of the line passing through (1, 2, – 4) and perpendicular to the two lines: $\dfrac{x-8}{3}=\dfrac{y+19}{-16}=\dfrac{z-10}{7}$ and $\dfrac{x-15}{3}=\dfrac{y-29}{8}=\dfrac{z-5}{-5}$

Ans:

Equation of any line through the point (1, 2, – 4) is

$\dfrac{x-1}{a}=\dfrac{y-2}{b}=\dfrac{z+4}{c}$ ...(*i*) where *a*, *b* and *c* are direction ratios of line (*i*).

Now the line (*i*) is perpendicular to the lines $\dfrac{x-8}{3}=\dfrac{y+19}{-16}=\dfrac{z-10}{7}$ and $\dfrac{x-15}{3}=\dfrac{y-29}{8}=\dfrac{z-5}{-5}$ having direction ratios 3, – 16, 7 and 3, 8, – 5 respectively.

∴ $3a-16b+7c=0$ ...(*ii*)

$3a+8b-5c=0$ ...(*iii*)

Solving (*ii*) and (*iii*) by cross-multiplication method, we have

$$\frac{a}{80-56}=\frac{b}{21+15}=\frac{c}{24+48} \Rightarrow \frac{a}{24}=\frac{b}{36}=\frac{c}{72} \Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{6}$$

Let $\dfrac{a}{2}=\dfrac{b}{3}=\dfrac{c}{6}=\lambda \Rightarrow a=2\lambda, b=3\lambda$ and $c=6\lambda$

The equation of required line which passes through point (1, 2, –4) and parallel to vector $2\hat{i}+3\hat{j}+6\hat{k}$ is $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$.

**OR**

Find the shortest distance between the lines whose vector equations are:
$\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$
Ans:
Comparing the given equations with equations $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$
We get $\vec{a_1}=\hat{i}+\hat{j}, \vec{b_1}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{a_2}=2\hat{i}+\hat{j}-\hat{k}, \vec{b_2}=3\hat{i}-5\hat{j}+2\hat{k}$

Therefore, $\vec{a_2}-\vec{a_1}=(\hat{i}-\hat{k})$ and

$$\vec{b_1}\times\vec{b_2}=(2\hat{i}-\hat{j}+\hat{k})\times(3\hat{i}-5\hat{j}+2\hat{k})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix}=3\hat{i}-\hat{j}-7\hat{k}$$

$$\left|\vec{b_1}\times\vec{b_2}\right|=\sqrt{9+1+49}=\sqrt{59}$$

Hence, the shortest distance between the given lines is given by

$$d=\left|\frac{(\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1})}{\left|\vec{b_1}\times\vec{b_2}\right|}\right|=\left|\frac{3-0+7}{\sqrt{59}}\right|=\frac{10}{\sqrt{59}}\text{ units.}$$

## SECTION – E(Case Study Based Questions)

**Questions 36 to 38 carry 4 marks each.**

**36. Case-Study 1:**

Sonam wants to prepare a sweet box for Diwali at home. For making lower part of box, she takes a square piece of cardboard of side 18 cm.
Now, x cm be the length of each side of the square cardboard which is to be cut off from corner of the square piece of side 18 cm.

Based on the above information, answer the following questions :
(i) Express Volume of the open box formed by folding up the cutting corner in terms of x and find the value of x for which $\frac{dV}{dx}=0$.
(ii) Sonam is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?
Ans: (i) height of open box = x cm
Length of open box = 18 – 2x
and width of open box = 18 – 2x
∴ Volume (V) of the open box = x × (18 – 2x) × (18 – 2x)
$\Rightarrow V = x(18-2x)^2 \Rightarrow \frac{dV}{dx}=x\cdot 2(18-2x)(-2)+(18-2x)^2$
$=(18-2x)(-4x+18-2x)=(18-2x)(18-6x)$
Now, $\frac{dV}{dx}=0 \Rightarrow 18-2x=0$ or $18-6x=0 \Rightarrow x=9$ or $3$

(ii) We have, V = $x(18 - 2x)^2$ and $\frac{dV}{dx} = (18 - 2x)(18 - 6x)$

$\Rightarrow \frac{d^2V}{dx^2} = (18 - 2x)(-6) + (18 - 6x)(-2)$

$= (-2)[54 - 6x + 18 - 6x]$

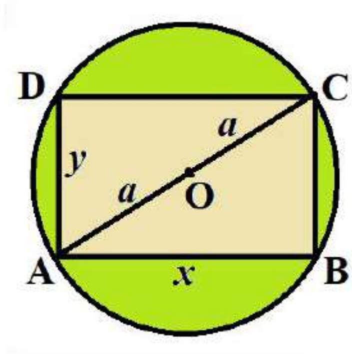
$= (-2)[72 - 12x] = 24x - 144$

For x = 3, $\frac{d^2V}{dx^2} < 0$ and for x = 9, $\frac{d^2V}{dx^2} > 0$

So, volume will be maximum when x = 3.

**37. Case-Study 2:**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)




(i) Find the perimeter of rectangle in terms of any one side and radius of circle.

(ii) Find critical points to maximize the perimeter of rectangle?

(iii) Check for maximum or minimum value of perimeter at critical point.

**OR**

(iii) If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region.

Ans: (i) Let 'y' be the breadth and 'x' be the length of rectangle and 'a' is radius of given circle.

From figure, $4a^2 = x^2 + y^2 \Rightarrow y^2 = 4a^2 - x^2$

$\Rightarrow y = \sqrt{4a^2 - x^2}$

Perimeter (P) = 2x + 2y = $2\left(x + \sqrt{4a^2 - x^2}\right)$

(ii) We know that P = 2x + 2y = $2\left(x + \sqrt{4a^2 - x^2}\right)$

Critical points to maximize perimeter, $\frac{dP}{dx} = 0$

$\Rightarrow \frac{dP}{dx} = 2\left[1 + \frac{1}{2\sqrt{4a^2 - x^2}}(-2x)\right] = 0 \Rightarrow 2\left(\frac{\sqrt{4a^2 - x^2} - x}{\sqrt{4a^2 - x^2}}\right) = 0 \Rightarrow \sqrt{4a^2 - x^2} - x = 0$

$\Rightarrow \sqrt{4a^2 - x^2} = x \Rightarrow 4a^2 - x^2 = x^2 \Rightarrow 2x^2 = 4a^2 \Rightarrow x^2 = 2a^2 \Rightarrow x = \pm\sqrt{2}a$

But $x = -\sqrt{2}a$ is not possible as length is never negative

$\therefore x = \sqrt{2}a \Rightarrow y = \sqrt{2}a$

Hence, critical point is $(\sqrt{2}a, \sqrt{2}a)$

(iii) We have, $\frac{dP}{dx} = 2\left[1 - \frac{x}{\sqrt{4a^2 - x^2}}\right]$

$$\Rightarrow \frac{d^2P}{dx^2} = -2\left[\frac{\sqrt{4a^2-x^2}-(x)\dfrac{-2x}{2\sqrt{4a^2-x^2}}}{\sqrt{4a^2-x^2}}\right] = -2\left[\frac{\sqrt{4a^2-x^2}+\dfrac{x^2}{\sqrt{4a^2-x^2}}}{\sqrt{4a^2-x^2}}\right]$$

$$\Rightarrow \frac{d^2P}{dx^2} = -2\left[\frac{4a^2-x^2+x^2}{(4a^2-x^2)^{3/2}}\right] = -2\left[\frac{4a^2}{(4a^2-x^2)^{3/2}}\right]$$

$$\Rightarrow \left[\frac{d^2P}{dx^2}\right]_{x=a\sqrt{2}} = -2\left[\frac{4a^2}{(4a^2-2a^2)^{3/2}}\right] = \frac{-2}{2\sqrt{2}a} < 0$$

Hence, Perimeter is maximum at a critical point.

**OR**

From the above results know that x = y = $\sqrt{2}a$ and $a$ = radius

Here, x = y = $10\sqrt{2}$

Perimeter = P = 4 x side = $40\sqrt{2}$ cm

**38. Case-Study 3:**

In a school, teacher asks a question to three students Kanabh, Raj and Shubi respectively. The probability of solving the question by Kanabh, Raj and Shubi are 40%, 15% and 50% respectively. The probability of making error by Kanabh, Raj and Shubi are 1.5%, 2% and 2.5%.


Based on the given information, answer the following questions:

(i) Find the probability that Shubi solved the question and committed an error.

(ii) Find the total probability of committing an error is solving the question.

**Ans:** According to given case,

Let $E_1$ = Solved by Kanabh, $E_2$ = Solved by Raj and $E_3$ = Solved in Shubi

Also let A = question has some error.

∴ $P(E_1) = 40/100$, $P(E_2) = 15/100$, $P(E_3) = 50/100$

$P(A/E_1) = 1.5/100$, $P(A/E_2) = 2/100$, $P(A/E_3) = 2.5/100$,

(i) The probability that Shubi solved the question and committed an error i.e.,

$$P(E_3 \cap A) = P(A \cap E_3) = P\left(\frac{A}{E_3}\right) \times P(E_3) = \frac{2.5}{100} \times \frac{50}{100} = 0.0125$$

(ii) The total probability of committing an error is solving the question

$$P(A) = P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right) + P(E_3) \times P\left(\frac{A}{E_3}\right)$$

$$= \frac{40}{100} \times \frac{1.5}{100} + \frac{15}{100} \times \frac{2}{100} + \frac{50}{100} \times \frac{2.5}{100} = 0.006 + 0.003 + 0.0125 = 0.0215.$$