

**PRACTICE PAPER 09****CHAPTER 08 INTRODUCTION TO TRIGONOMETRY****SUBJECT: MATHEMATICS****MAX. MARKS : 40****CLASS : X****DURATION : 1½ hrs****General Instructions:**

- (i). All questions are compulsory.
- (ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
- (iii). **Section A** comprises of 10 MCQs of 1 mark each. **Section B** comprises of 4 questions of 2 marks each. **Section C** comprises of 3 questions of 3 marks each. **Section D** comprises of 1 question of 5 marks each and **Section E** comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

**SECTION – A****Questions 1 to 10 carry 1 mark each.**

1.  $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$  is equal to:  
(a) -1 (b) 1 (c) 0 (d) 2
2. In  $\triangle ABC$  right angled at B,  $\sin A = \frac{7}{25}$ , then the value of  $\cos C$  is .....  
(a)  $\frac{7}{25}$  (b)  $\frac{24}{25}$  (c)  $\frac{7}{24}$  (d)  $\frac{24}{7}$
3. If  $5 \tan \theta = 4$ , then the value of  $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta}$  is  
(a) 1/6 (b) 1/7 (c) 1/4 (d) 1/5
4. If  $\operatorname{cosec} A = 13/12$ , then the value of  $\frac{2 \sin A - 3 \cos A}{4 \sin A - 9 \cos A}$   
(a) 4 (b) 5 (c) 6 (d) 3
5. Given that  $\sin \alpha = 1/2$  and  $\cos \beta = 1/2$ , then the value of  $(\beta - \alpha)$  is  
(a)  $0^\circ$  (b)  $30^\circ$  (c)  $60^\circ$  (d)  $90^\circ$
6. If  $\tan \theta = 1$ , then the value of  $\sec \theta + \operatorname{cosec} \theta$  is:  
(a)  $3\sqrt{2}$  (b)  $4\sqrt{2}$  (c)  $2\sqrt{2}$  (d)  $\sqrt{2}$
7. If  $\sin 2A = \frac{1}{2} \tan^2 45^\circ$  where A is an acute angle, then the value of A is  
(a)  $60^\circ$  (b)  $45^\circ$  (c)  $30^\circ$  (d)  $15^\circ$
8. If  $\theta$  is an acute angle and  $\tan \theta + \cot \theta = 2$ , then the value of  $\sin^3 \theta + \cos^3 \theta$  is  
(a) 1 (b)  $\frac{1}{2}$  (c)  $\frac{\sqrt{2}}{2}$  (d)  $\sqrt{2}$

**In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.



(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** In a right  $\triangle ABC$ , right angled at B, if  $\tan A = 1$ , then  $2 \sin A \cdot \cos A = 1$ .

**Reason (R):**  $\tan 45^\circ = 1$  and  $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$

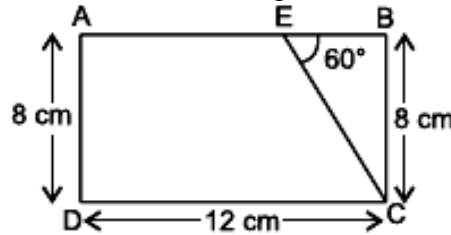
10. **Assertion (A):**  $\sin(A + B) = \sin A + \sin B$

**Reason (R):** For any value of  $\theta$ ,  $1 + \tan^2\theta = \sec^2\theta$

### SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the given figure, ABCD is a rectangle with AD = 8 cm and CD = 12 cm. Line segment CE is drawn, making an angle of  $60^\circ$  with AB, intersecting AB at E. Find the length of CE and BE.



12. If  $\sin(A + B) = \sqrt{3}/2$  and  $\sin(A - B) = \frac{1}{2}$ ,  $0 \leq A + B \leq 90^\circ$  and  $A > B$ , then find A and B.

13. Evaluate:  $3 \cos^2 60^\circ \sec^2 30^\circ - 2 \sin^2 30^\circ \tan^2 60^\circ$ .

14. Simplify:  $\frac{\tan^2 \theta}{1 + \tan^2 \theta} + \frac{\cot^2 \theta}{1 + \cot^2 \theta}$

OR

If  $7 \sin^2 A + 3 \cos^2 A = 4$ , then find  $\tan A$

### SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If  $\operatorname{cosec} \theta + \cot \theta = p$ , then prove that  $\cos \theta = \frac{p^2 - 1}{p^2 + 1}$

16. Prove that  $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \sec \theta + \tan \theta$

OR

If  $\sin \theta + \cos \theta = \sqrt{3}$ , then prove that  $\tan \theta + \cot \theta = 1$ .

17. Prove that:  $\frac{\cos^2 \theta}{1 - \tan \theta} + \frac{\sin^2 \theta}{1 - \cot \theta} = 1 + \sin \theta \cos \theta$

OR

If  $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ , show that  $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$ .

### SECTION – D

Questions 18 carry 5 marks.

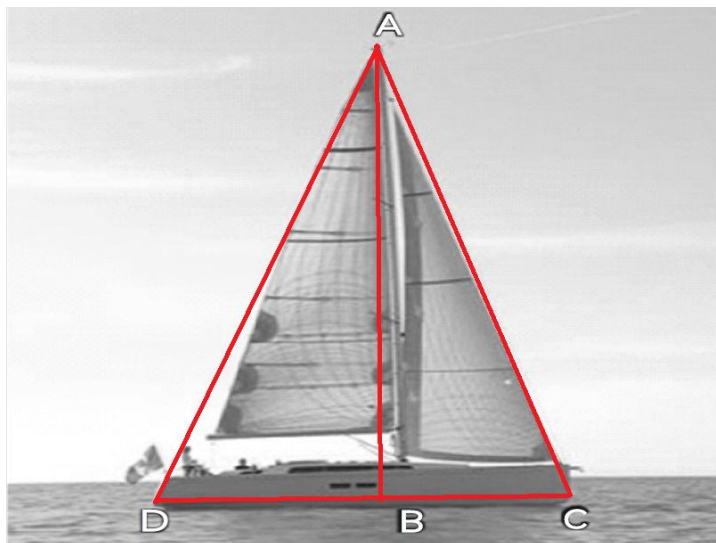
18. (a) Prove that  $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$  [3]

(b) If  $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$  and  $x \sin \theta = y \cos \theta$  then find  $x^2 + y^2$ . [2]

## SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A sailing boat with triangular masts is shown below. Two right triangles can be observed. Triangles ABC and ABD, both right-angled at B. The distance  $BC = 1$  m and  $BD = 2$  m and height  $AB = 4$  m.



Based on the given information, answer the following questions:

- (a) Find the value of  $\sec D$ . [1]
  - (b) Find the value of  $\operatorname{cosec} C$ . [1]
  - (c) Find the value of  $\tan D + \cot C$ . [1]
  - (d) Find the value of  $\sin^2 C + \cos^2 D$ . [1]
20. Varanasi is a city of temples, including the gold-plated Vishwanath temple of Lord Shiva; the Bharat Mata, or Mother India, temple that boasts a huge three dimensional relief map of the Indian subcontinent carved out of marble; and the hundreds of small temples that dot the waterways and alleys. It is a city of scholars, home to one of Asia's largest universities. It is also a city of legends. The figure below shows one such temple along the banks of the sacred river “Ganges” or “Ganga”. A person sitting at point marked A looks at the top of a nearby temple and imagines that a right angled triangle ABC can be drawn as shown in the figure below.



Based on the above information, answer the following questions. (Take  $\sqrt{3} = 1.732$ )

- (a) Find the value of  $\sin A$ . [1]
- (b) Find the value of  $\sin C$ . [1]
- (c) Find the value of  $\tan A - \cot C$ . [1]
- (d) Find the value of  $\operatorname{cosec}^2 C$ . [1]



**PRACTICE PAPER 09****CHAPTER 08 INTRODUCTION TO TRIGONOMETRY (ANSWERS)****SUBJECT: MATHEMATICS****MAX. MARKS : 40****CLASS : X****DURATION : 1½ hrs****General Instructions:**

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- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

**SECTION – A****Questions 1 to 10 carry 1 mark each.****1.**  $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$  is equal to:

- (a) -1 (b) 1 (c) 0 (d) 2

Ans. (b) 1

$$(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$$

$$= \tan^2\theta \times \cot^2\theta$$

$$= \tan^2\theta \times 1/\tan^2\theta = 1$$

**2.** In  $\triangle ABC$  right angled at B,  $\sin A = \frac{7}{25}$ , then the value of  $\cos C$  is .....

- (a)
- $\frac{7}{25}$
- (b)
- $\frac{24}{25}$
- (c)
- $\frac{7}{24}$
- (d)
- $\frac{24}{7}$

Ans: (a)  $\frac{7}{25}$ **3.** If  $5 \tan \theta = 4$ , then the value of  $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta}$  is

- (a) 1/6 (b) 1/7 (c) 1/4 (d) 1/5

Ans: (a) 1/6

**4.** If  $\operatorname{cosec} A = 13/12$ , then the value of  $\frac{2 \sin A - 3 \cos A}{4 \sin A - 9 \cos A}$ 

- (a) 4 (b) 5 (c) 6 (d) 3

Ans: (d) 3

Given  $\operatorname{cosec} A = 13/12$ ,

$$\sin A = \frac{12}{13}, \cos A = \frac{5}{13}$$

$$\text{Now, } \frac{2 \sin A - 3 \cos A}{4 \sin A - 9 \cos A} = \frac{2\left(\frac{12}{13}\right) - 3\left(\frac{5}{13}\right)}{4\left(\frac{12}{13}\right) - 9\left(\frac{5}{13}\right)} = \frac{24 - 15}{48 - 45} = \frac{9}{3} = 3$$

**5.** Given that  $\sin \alpha = 1/2$  and  $\cos \beta = 1/2$ , then the value of  $(\beta - \alpha)$  is

- (a)
- $0^\circ$
- (b)
- $30^\circ$
- (c)
- $60^\circ$
- (d)
- $90^\circ$

Ans: (b)  $30^\circ$ 

6. If  $\tan \theta = 1$ , then the value of  $\sec \theta + \operatorname{cosec} \theta$  is:  
 (a)  $3\sqrt{2}$  (b)  $4\sqrt{2}$  (c)  $2\sqrt{2}$  (d)  $\sqrt{2}$

Ans: (c)  $2\sqrt{2}$

Given,  $\tan \theta = 1$ , we have  $\theta = 45^\circ$

So,  $\sec \theta + \operatorname{cosec} \theta = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$ .

7. If  $\sin 2A = \frac{1}{2} \tan^2 45^\circ$  where A is an acute angle, then the value of A is

- (a)  $60^\circ$  (b)  $45^\circ$  (c)  $30^\circ$  (d)  $15^\circ$

Ans: (d)  $15^\circ$

$$\sin 2A = \frac{1}{2} \tan^2 45^\circ = \frac{1}{2} \times 1^2 = \frac{1}{2} = \sin 30^\circ \Rightarrow 2A = 30^\circ \Rightarrow A = 15^\circ$$

8. If  $\theta$  is an acute angle and  $\tan \theta + \cot \theta = 2$ , then the value of  $\sin^3 \theta + \cos^3 \theta$  is

- (a) 1 (b)  $\frac{1}{2}$  (c)  $\frac{\sqrt{2}}{2}$  (d)  $\sqrt{2}$

Ans: (c)  $\frac{\sqrt{2}}{2}$

$$\tan \theta + \cot \theta = 2 \Rightarrow \tan \theta + \frac{1}{\tan \theta} = 2 \Rightarrow \tan^2 \theta - 2 \tan \theta + 1 = 0$$

$$\Rightarrow (\tan \theta - 1)^2 = 0 \Rightarrow \tan \theta = 1 = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

$$\text{Now, } \sin^3 \theta + \cos^3 \theta = \sin^3 45^\circ + \cos^3 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).  
 (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).  
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9. **Assertion (A):** In a right  $\triangle ABC$ , right angled at B, if  $\tan A = 1$ , then  $2 \sin A \cdot \cos A = 1$ .

**Reason (R):**  $\tan 45^\circ = 1$  and  $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$

Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. **Assertion (A):**  $\sin(A + B) = \sin A + \sin B$

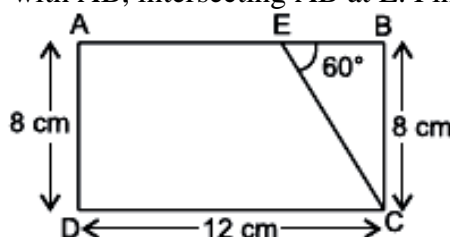
**Reason (R):** For any value of  $\theta$ ,  $1 + \tan^2 \theta = \sec^2 \theta$

Ans. (d) Assertion (A) is false but reason (R) is true.

## SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the given figure, ABCD is a rectangle with AD = 8 cm and CD = 12 cm. Line segment CE is drawn, making an angle of  $60^\circ$  with AB, intersecting AB at E. Find the length of CE and BE.



Ans: In  $\Delta CBE$ , we have  $\tan 60^\circ = \frac{CB}{BE}$

$$\Rightarrow \sqrt{3} = \frac{8}{BE} \Rightarrow BE = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3} \text{ cm}$$

$$\text{and } \sin 60^\circ = \frac{CB}{CE} \Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{CE} \Rightarrow CE = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3} \text{ cm}$$

12. If  $\sin(A + B) = \sqrt{3}/2$  and  $\sin(A - B) = \frac{1}{2}$ ,  $0 \leq A + B \leq 90^\circ$  and  $A > B$ , then find A and B.

Ans:  $\sin(A + B) = \sqrt{3}/2 = \sin 60^\circ$

$$\Rightarrow A + B = 60^\circ \dots\dots(i)$$

$$\sin(A - B) = 1/2 = \sin 30^\circ$$

$$\Rightarrow A - B = 30^\circ \dots\dots(ii)$$

Solving eq. (i) and (ii),  $A = 45^\circ$  and  $B = 15^\circ$

13. Evaluate:  $3 \cos^2 60^\circ \sec^2 30^\circ - 2 \sin^2 30^\circ \tan^2 60^\circ$ .

Ans:  $3 \cos^2 60^\circ \sec^2 30^\circ - 2 \sin^2 30^\circ \tan^2 60^\circ$

$$= 3 \left(\frac{1}{2}\right)^2 \left(\frac{2}{\sqrt{3}}\right)^2 - 2 \left(\frac{1}{2}\right)^2 (\sqrt{3})^2 = \frac{3}{4} \times \frac{4}{3} - 2 \times \frac{1}{4} \times 3 = 1 - \frac{3}{2} = -\frac{1}{2}$$

14. Simplify:  $\frac{\tan^2 \theta}{1 + \tan^2 \theta} + \frac{\cot^2 \theta}{1 + \cot^2 \theta}$

$$\begin{aligned} \text{Ans: } \frac{\tan^2 \theta}{1 + \tan^2 \theta} + \frac{\cot^2 \theta}{1 + \cot^2 \theta} &= \frac{\tan^2 \theta}{\sec^2 \theta} + \frac{\cot^2 \theta}{\operatorname{cosec}^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{1} + \frac{\cos^2 \theta}{\sin^2 \theta} \times \frac{\sin^2 \theta}{1} = \sin^2 \theta + \cos^2 \theta = 1 \end{aligned}$$

**OR**

If  $7 \sin^2 A + 3 \cos^2 A = 4$ , then find  $\tan A$

Ans: Given,  $7 \sin^2 A + 3 \cos^2 A = 4$

Dividing both sides by  $\cos^2 A$ , we get

$$7 \tan^2 A + 3 = 4 \sec^2 A \quad [\because \sec^2 \theta = 1 + \tan^2 \theta]$$

$$\Rightarrow 7 \tan^2 A + 3 = 4(1 + \tan^2 A)$$

$$\Rightarrow 7 \tan^2 A + 3 = 4 + 4 \tan^2 A$$

$$\Rightarrow 3 \tan^2 A = 1 \Rightarrow \tan^2 A = 1/3 \Rightarrow \tan A = 1/\sqrt{3}$$

## SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If  $\operatorname{cosec} \theta + \cot \theta = p$ , then prove that  $\cos \theta = \frac{p^2 - 1}{p^2 + 1}$

Ans: Given  $\operatorname{cosec} \theta + \cot \theta = p \dots\dots (1)$

$$\Rightarrow (\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta) = 1 \Rightarrow (\operatorname{cosec} \theta - \cot \theta)p = 1$$

$$\Rightarrow \operatorname{cosec} \theta - \cot \theta = \frac{1}{p} \dots\dots (2)$$

Adding (1) and (2), we get

$$\operatorname{cosec} \theta = \frac{p + \frac{1}{p}}{2} = \frac{p^2 + 1}{2p}; \cot \theta = \frac{p - \frac{1}{p}}{2} = \frac{p^2 - 1}{2p}$$



$$\text{Now, } \cos \theta = \frac{\cot \theta}{\operatorname{cosec} \theta} = \frac{\frac{p^2-1}{2p}}{\frac{p^2+1}{2p}} = \frac{p^2-1}{p^2+1}$$

16. Prove that  $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \sec \theta + \tan \theta$

Ans: LHS =  $\frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta}$  (Dividing numerator and denominator by  $\cos \theta$ )

$$\begin{aligned} &= \frac{\tan \theta + \sec \theta - 1}{\tan \theta + 1 - \sec \theta} \\ &= \frac{\tan \theta + \sec \theta - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta + 1 - \sec \theta} \\ &= \frac{(\sec \theta + \tan \theta)(1 - \sec \theta + \tan \theta)}{\tan \theta + 1 - \sec \theta} = \sec \theta + \tan \theta = \text{RHS} \end{aligned}$$

**OR**

If  $\sin \theta + \cos \theta = \sqrt{3}$ , then prove that  $\tan \theta + \cot \theta = 1$ .

Ans:  $\sin \theta + \cos \theta = \sqrt{3} \Rightarrow (\sin \theta + \cos \theta)^2 = 3$

$$\begin{aligned} \Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta &= 3 \\ \Rightarrow 1 + 2 \sin \theta \cos \theta &= 3 \Rightarrow 2 \sin \theta \cos \theta = 2 \\ \Rightarrow \sin \theta \cos \theta &= 1 = \sin^2 \theta + \cos^2 \theta \\ \Rightarrow 1 &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \tan \theta + \cot \theta \Rightarrow \tan \theta + \cot \theta = 1 \end{aligned}$$

17. Prove that:  $\frac{\cos^2 \theta}{1 - \tan \theta} + \frac{\sin^2 \theta}{1 - \cot \theta} = 1 + \sin \theta \cos \theta$

Ans: LHS =  $\frac{\cos^2 \theta}{1 - \tan \theta} + \frac{\sin^2 \theta}{1 - \cot \theta}$

$$\begin{aligned} &= \frac{\cos^3 \theta}{\cos \theta - \sin \theta} - \frac{\sin^3 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\cos^3 \theta - \sin^3 \theta}{\cos \theta - \sin \theta} = \frac{(\cos \theta - \sin \theta)(\cos^2 \theta + \sin^2 \theta + \cos \theta \sin \theta)}{\cos \theta - \sin \theta} \\ &= \cos^2 \theta + \sin^2 \theta + \cos \theta \sin \theta = 1 + \sin \theta \cos \theta = \text{RHS} \end{aligned}$$

**OR**

If  $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ , show that  $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$ .

Ans: Given,  $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$

Squaring both sides, we get

$$\begin{aligned} (\cos \theta + \sin \theta)^2 &= (\sqrt{2} \cos \theta)^2 \\ \Rightarrow \cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta &= 2 \cos^2 \theta \\ \Rightarrow 2 \sin \theta \cos \theta &= \cos^2 \theta - \sin^2 \theta \\ \Rightarrow 2 \sin \theta \cos \theta &= (\cos \theta - \sin \theta)(\cos \theta + \sin \theta) \\ \Rightarrow 2 \sin \theta \cos \theta &= (\cos \theta - \sin \theta)(\sqrt{2} \cos \theta) \\ \Rightarrow \sqrt{2} \sin \theta &= \cos \theta - \sin \theta \Rightarrow \cos \theta - \sin \theta = \sqrt{2} \sin \theta \end{aligned}$$

## SECTION – D

Questions 18 carry 5 marks.

18. (a) Prove that  $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$  [3]



(b) If  $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$  and  $x\sin\theta = y\cos\theta$  then find  $x^2 + y^2$ . [2]

Ans: (a) L.H.S =  $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$   
 $= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$   
 $= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2\sin A \times 1/\sin A + 2\cos A \times 1/\cos A$   
 Since,  $(\sin^2 A + \cos^2 A = 1)$   
 $(\sec^2 A = 1 + \tan^2 A, \operatorname{cosec}^2 A = 1 + \cot^2 A)$   
 $= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$   
 $= 7 + \tan^2 A + \cot^2 A = \text{RHS}$

(b)

We have,  $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$\Rightarrow x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$\Rightarrow x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$$

$$\Rightarrow x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$$

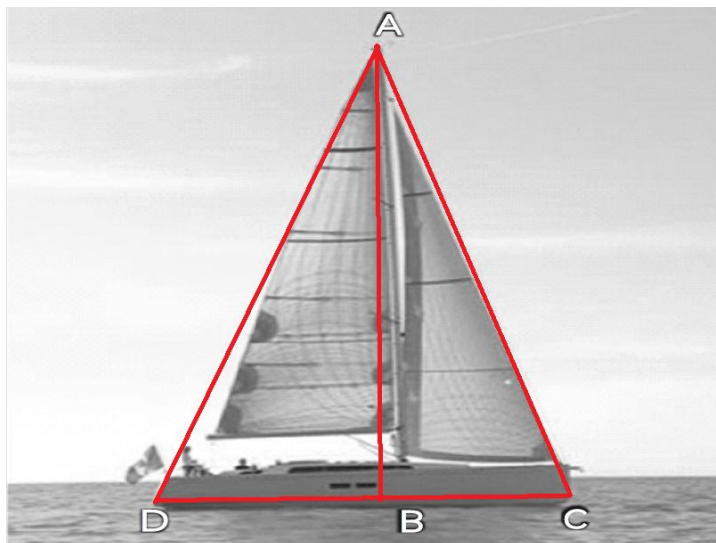
$$\text{Now, } x\sin\theta = y\cos\theta \Rightarrow \cos\theta\sin\theta = y\cos\theta \Rightarrow y = \sin\theta$$

$$\text{Hence, } x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$$

### SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A sailing boat with triangular masts is shown below. Two right triangles can be observed. Triangles ABC and ABD, both right-angled at B. The distance BC = 1 m and BD = 2 m and height AB = 4 m.



Based on the given information, answer the following questions:

- (a) Find the value of  $\sec D$ . [1]  
 (b) Find the value of  $\operatorname{cosec} C$ . [1]  
 (c) Find the value of  $\tan D + \cot C$ . [1]  
 (d) Find the value of  $\sin^2 C + \cos^2 D$ . [1]

Ans. (a) In  $\triangle ABD$ ,  $\sec D = AD/BD$

by using Pythagoras theorem in right triangle ABD.

$$AD^2 = BD^2 + AB^2 = 2^2 + 4^2 = 20$$

$$\Rightarrow AD = \sqrt{20} = 2\sqrt{5}\text{m}$$

$$\therefore \sec D = AD/BD = 2\sqrt{5}/2 = \sqrt{5}$$

(b) In  $\triangle ABC$ ,  $\operatorname{cosec} C = AC/AB$

by using Pythagoras theorem in right triangle ABC.

$$AC^2 = AB^2 + BC^2 = 4^2 + 1^2 = 17$$

$$\Rightarrow AC = \sqrt{17}\text{ m}$$





$$\begin{aligned} \therefore \operatorname{cosec} C &= AC/AB = \sqrt{17}/4 \\ \text{(c) In } \triangle ABD, \tan D &= AB/BD = 4/2 = 2 \\ \text{In } \triangle ABC, \cot C &= BC/AB = 1/4 \\ \therefore \tan D + \cot C &= 2 + 1/4 = 9/4 \\ \text{(d) In } \triangle ABC, \sin C &= AB/AC = 4/\sqrt{17} \\ \text{In } \triangle ABD, \cos D &= BD/AD = 1/\sqrt{5} \\ \therefore \sin^2 C + \cos^2 D &= 16/17 + 1/5 = 97/85 \end{aligned}$$

20. Varanasi is a city of temples, including the gold-plated Vishwanath temple of Lord Shiva; the Bharat Mata, or Mother India, temple that boasts a huge three dimensional relief map of the Indian subcontinent carved out of marble; and the hundreds of small temples that dot the waterways and alleys. It is a city of scholars, home to one of Asia's largest universities. It is also a city of legends. The figure below shows one such temple along the banks of the sacred river “Ganges” or “Ganga”. A person sitting at point marked A looks at the top of a nearby temple and imagines that a right angled triangle ABC can be drawn as shown in the figure below.



Based on the above information, answer the following questions. (Take  $\sqrt{3} = 1.732$ )

- (a) Find the value of  $\sin A$ . [1]  
 (b) Find the value of  $\sin C$ . [1]  
 (c) Find the value of  $\tan A - \cot C$ . [1]  
 (d) Find the value of  $\operatorname{cosec}^2 C$ . [1]

Ans. (a) In  $\triangle ABC$ ,  $\sin A = BC/AC$

by using Pythagoras theorem in right triangle ABC.

$$AC^2 = AB^2 + BC^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$\Rightarrow AC = 13 \text{ m}$$

$$\therefore \sin A = BC/AC = 5/13$$

(b) In  $\triangle ABC$ ,  $\sin C = AB/AC$

$$\Rightarrow \sin C = AB/AC = 12/13$$

(c) In  $\triangle ABC$ ,  $\tan A = BC/AB = 5/12$

$$\Rightarrow \cot C = BC/AB = 5/12$$

Therefore,  $\tan A - \cot C = 0$

(d) In  $\triangle ABC$ ,  $\sin C = AB/AC = 12/13$

$$\operatorname{cosec} C = 1/\sin C = 13/12$$

Therefore,  $\operatorname{cosec}^2 C = 169/144$

