PRACTICE PAPER 16
CHAPTER 12 PROBABILITY

SUBJECT: MATHEMATICS — **MAX. MARKS : 40**
CLASS : XII — **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Let A and B be two given events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$. Then $P(A'/B')$ is

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$

2. Let A and B be two given independent events such that $P(A) = p$ and $P(B) = q$ and P(exactly one of A, B) $= \frac{2}{3}$, then value of $3p + 3q - 6pq$ is

(a) 2 (b) –2 (c) 4 (d) –4

3. If $P(A \cap B) = 70\%$ and $P(B) = 85\%$, then $P(A/B)$ is equal to

(a) $\frac{14}{17}$ (b) $\frac{17}{20}$ (c) $\frac{7}{8}$ (d) $\frac{1}{8}$

4. Two dice are thrown once. If it is known that the sum of the numbers on the dice was less than 6 the probability of getting a sum 3 is

(a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

5. The probability distribution of the discrete variable X is given as:

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of k is

(a) 8 (b) 16 (c) 32 (d) 48

6. The probability of A, B and C solving a problem are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Then the probability that the problem will be solved is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) none of these

7. Three persons A, B and C, fire a target in turn. Their probabilities of hitting the target are 0.2, 0.3 and 0.5 respectively, the probability that target is hit, is

(a) 0.993 (b) 0.94 (c) 0.72 (d) 0.90

8. Bag A contains 3 red and 5 black balls and bag B contains 2 red and 4 black balls. A ball is drawn from one of the bags. The probability that ball drawn is red is

(*a*) $\frac{17}{24}$ (*b*) $\frac{17}{48}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{1}{3}$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** Three dice are rolled simultaneously. Consider the event E 'getting a total of at least 5', F 'getting the same number on all three dice' and G 'getting a total of 15'. Then E, F and G are mutually independent events.
Reason (R): Three events A, B and C are said to be mutually independent, if
$P(A \cap B) = P(A)\,P(B)$
$P(A \cap C) = P(A)\,P(C)$
$P(B \cap C) = P(B)\,P(C)$
and $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$
If at least one of the above conditions is not true for the given events, then events are not independent.

10. **Assertion (A):** If A and B are two mutually exclusive events with $P(\overline{A}) = 5/6$ and $P(B) = 1/3$. Then $P(A|B)$ is equal to 1/4.
Reason (R): If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.6$ and $P(A|B) = 0.2$ then the value of $P(A|\overline{B})$ is 0.2.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$ .State whether A and B are independent?

12. The probability of simultaneous occurrence of at least one of the two events A and B is p. If the probability that exactly one of A, B occurs is q, then prove that $P(A') + P(B') = 2 - 2p + q$.

13. If $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$, find $(A \cap B)$, if (*i*) A and B are mutually exclusive (*ii*) A and B are independent

14. A black and a red die are rolled together. Find the conditional probability of obtaining the sum8, given that the red die resulted in a number less than 4.

OR

Two cards are drawn from a well shuffled pack of 52 cards one after the other without replacement. Find the probability that one of them is a queen and the other is a king of opposite colour.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. A bag contains (2n + 1) coins, out of which n coins have head on both the sides and rest are fair coins. A coin is selected at random and is tossed, if it results in a head with probability $\frac{31}{42}$, find n.

16. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (*i*) the youngest is a girl? (*ii*) atleast one is a girl?

17. The random variable X has a probability distribution P(X) of the following form, where k is some number :

$$P(X)=\begin{cases} k \;\; if \;\; x=0 \\ 2k \;\; if \;\; x=1 \\ 3k, \;\; if \;\; x=2 \\ 0, otherwise \end{cases}$$

(a) Determine the value of k.

(b) Find P (X < 2), P (X ≤ 2), P(X ≥ 2).

OR

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

SECTION – D

Questions 18 carry 5 marks.

18. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

OR

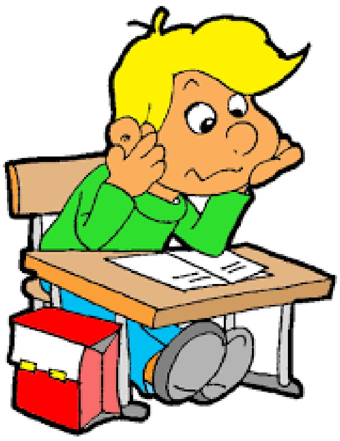
Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution.

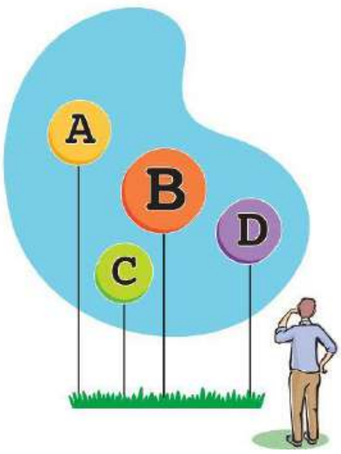
SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

In a test, you either guesses or copies or knows the answer to a multiple-choice question with four choice. The probability that you make a guess is 1/3, you copy the answer is 1/6. The probability that your answer is correct, given that you guess it, is 1/8. And also, the probability that you answer is correct, given that you copy it, is 1/4.





(i) The probability that you know the answer. [1]

(ii) Find the probability that your answer is correct given that you guess it and the probability that your answer is correct given that you know the answer . [1]

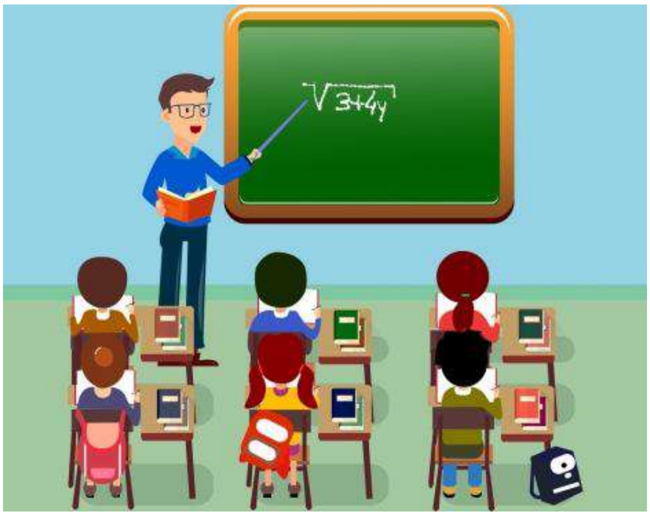
(iii) Find the probability that you know the answer given that you correctly answered it. [2]

OR

(iii) Find the total probability of correctly answered the question. [2]

20. Case-Study 2: Read the following passage and answer the questions given below.

Final exams are approaching, so Mr. Kumar decided to check the preparation of the few weak students in the class. He chooses four students A, B, C and D then a problem in mathematics is given to those four students A, B, C, D. Their chances of solving the problem, respectively, are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{2}{3}$.



Based on the given information answer the following questions. What is the probability that:

(i) the problem will be solved? (2)

(ii) at most one of them solve the problem? (2)

PRACTICE PAPER 16

CHAPTER 12 PROBABILITY (ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**

CLASS : XII | **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Let A and B be two given events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$. Then $P(A'/B')$ is

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$

Ans. (c) $\frac{3}{8}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$\Rightarrow P(A \cap B) = 0.5 \times 0.2 = 0.1$

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - (0.6 + 0.2 - 0.1)}{1 - 0.2} = \frac{0.3}{0.8} = \frac{3}{8}$$

2. Let A and B be two given independent events such that $P(A) = p$ and $P(B) = q$ and P(exactly one of A, B) = $\frac{2}{3}$, then value of $3p + 3q - 6pq$ is

(a) 2 (b) –2 (c) 4 (d) –4

Ans. (a) 2

$P(A)\, P(\overline{B}) + P(\overline{A})\, P(B) = \frac{2}{3}$

$\Rightarrow p \cdot (1 - q) + (1 - p)q = \frac{2}{3}$

$\Rightarrow p - pq + q - pq = \frac{2}{3} \Rightarrow 3p + 3q - 6pq = 2$

3. If $P(A \cap B) = 70\%$ and $P(B) = 85\%$, then $P(A/B)$ is equal to

(a) $\frac{14}{17}$ (b) $\frac{17}{20}$ (c) $\frac{7}{8}$ (d) $\frac{1}{8}$

Ans. (a) $\frac{14}{17}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{70}{100} \times \frac{100}{85} = \frac{14}{17}$$

4. Two dice are thrown once. If it is known that the sum of the numbers on the dice was less than 6 the probability of getting a sum 3 is (*a*) $\frac{1}{18}$ (*b*) $\frac{5}{18}$ (*c*) $\frac{1}{5}$ (*d*) $\frac{2}{5}$

Ans. (*c*) $\frac{1}{5}$

Favourable cases for sum less than 6 are 10 and favourable for a total of 3 is 2.
Required Probability = 2/10 = 1/5

5. The probability distribution of the discrete variable X is given as:

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of k is

(*a*) 8 (*b*) 16 (*c*) 32 (*d*) 48

Ans. (*c*) 32

$\sum P(X) = 1 \Rightarrow \frac{5}{k}+\frac{7}{k}+\frac{9}{k}+\frac{11}{k}=1 \Rightarrow \frac{32}{k}=1 \Rightarrow k = 32$

6. The probability of A, B and C solving a problem are $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{4}$ respectively. Then the probability that the problem will be solved is

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{4}$ (*d*) none of these

Ans. (*b*) $\frac{3}{4}$

P(problem solved) = 1 – P(more solves) = $1 - \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=1-\frac{1}{4}=\frac{3}{4}$

7. Three persons A, B and C, fire a target in turn. Their probabilities of hitting the target are 0.2, 0.3 and 0.5 respectively, the probability that target is hit, is

(a) 0.993 (b) 0.94 (c) 0.72 (d) 0.90

Ans. (c) 0.72

P(target hit) = P(at least one hits the target)

= 1 – P(none hits) = 1 – P($\overline{A}\,\overline{B}\,\overline{C}$) = 1 – 0.8 × 0.7 × 0.5

= 1 – 0.28 = 0.72

8. Bag A contains 3 red and 5 black balls and bag B contains 2 red and 4 black balls. A ball is drawn from one of the bags. The probability that ball drawn is red is

(*a*) $\frac{17}{24}$ (*b*) $\frac{17}{48}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{1}{3}$

Ans. (*b*) $\frac{17}{48}$

$P(\text{red}) = P(A) \cdot P(R/A) + P(B) . P(R/B)$

$= \frac{1}{2}\times\frac{3}{8}+\frac{1}{2}\times\frac{2}{6}=\frac{3}{16}+\frac{1}{6}=\frac{17}{48}.$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): Three dice are rolled simultaneously. Consider the event E 'getting a total of at least 5', F 'getting the same number on all three dice' and G 'getting a total of 15'. Then E, F and G are mutually independent events.

Reason (R): Three events A, B and C are said to be mutually independent, if

$P(A \cap B) = P(A) P(B)$

$P(A \cap C) = P(A) P(C)$

$P(B \cap C) = P(B) P(C)$

and $P(A \cap B \cap C) = P(A) P(B) P(C)$

If at least one of the above conditions is not true for the given events, then events are not independent.

Ans. (a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).

We have, P(A) = 1/5 and P(A or B)= 1/2

We know that, for independent events

$P(A \cup B) = P(A) + P(B) – P(A) P(B)$

$$\Rightarrow \frac{1}{2} = \frac{1}{5} + p - \frac{1}{5}p \Rightarrow \frac{1}{2} = \frac{1}{5} + \frac{4}{5}p \Rightarrow p = \frac{3}{8}$$

10. Assertion (A): If A and B are two mutually exclusive events with $P(\overline{A})$ = 5/6 and P(B) = 1/3. Then P(A|B) is equal to 1/4.

Reason (R): If A and B are two events such that P(A) = 0.2, P(B) = 0.6 and P(A|B) = 0.2 then the value of $P(A|\overline{B})$ is 0.2.

Ans. (b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).

$$P(A/\overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})} = \frac{P(A)}{P(\overline{B})}$$ [Since, given A and B are two mutually exclusive events]

$$P(A/B) = \frac{1-\frac{5}{6}}{1-\frac{1}{3}} = \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{4}$$

Now, for independent events, $P(A|\overline{B}) = P(A) = 0.2$

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Events *A* and *B* are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$ .State whether *A* and *B* are independent?

Ans.

$$P(\overline{A} \cup \overline{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$$

$$\Rightarrow \frac{1}{4} = 1 - P(A \cap B) \Rightarrow P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$$

Also $P(A) \times P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$

As $P(A \cap B) \neq P(A) \times P(B)$, so they are not independent.

12. The probability of simultaneous occurrence of at least one of the two events *A* and *B* is p. If the probability that exactly one of *A*, *B* occurs is q, then prove that $P(A') + P(B') = 2 – 2p + q$.

Ans. $P(A') + P(B') = 1 – P(A) + 1 – P(B) = 2 – [P(A) + P(B)] = 2 – [(P(A \cup B) + P(A \cap B)]$ ...(*i*)

$P(\text{exactly one}) = q = P(A – B) + P(B – A) = P(A \cup B) – P(A \cap B) = p – P(A \cap B)$

$\Rightarrow P(A \cap B) = p – q$

Substituting in (*i*), we get

$2 – [p + p – q] = 2 – 2p + q$.

13. If $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$, find $(A \cap B)$, if (*i*) A and B are mutually exclusive (*ii*) A and B are independent

Ans. (*i*) $P(A \cap B) = 0$, if A and B are mutually exclusive

(*ii*) $P(A \cap B) = P(A) \cdot P(B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$, if A and B are independent

14. A black and a red die are rolled together. Find the conditional probability of obtaining the sum8, given that the red die resulted in a number less than 4.

Ans. When two dice (one black and another red) are rolled, the sample space S = 6 ×6 = 36 (equally likely sample events)

Let E: set of events having 8 as the sum of the observations, F: set of events in which red die resulted in a (in any one die) number less than 4

∴E={(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)} ⇒ n(E) = 5

⇒E ∩ F = ((5, 3), (6, 2))

⇒ n(E ∩ F) = 2

Now, P(E) = Number of favourable outcomes/Total number of outcomes=5/36

Similarly, P(F) = 18/36 = 1/2 and P(E ∩ F) = 2/36 = 1/18

∴ Required probability = P(E/F) = P(E ∩ F) / P(F) = $\frac{\frac{1}{18}}{\frac{1}{2}} = \frac{1}{18} \times \frac{2}{1} = \frac{1}{9}$

OR

Two cards are drawn from a well shuffled pack of 52 cards one after the other without replacement. Find the probability that one of them is a queen and the other is a king of opposite colour.

Ans. Probability of drawing an ace in the first draw = 4/52

Probability of drawing a queen of opposite shade in the second draw = 2/51

Probability of drawing a queen in the first draw = 4/52

Probability of drawing an ace of opposite shade in the second draw = 2/51

P(a king and other a queen of opposite colour or a queen and other a king of opposite colour)

$= \frac{4}{52} \times \frac{2}{51} + \frac{4}{52} \times \frac{2}{51} = \frac{4}{663}$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. A bag contains (2n + 1) coins, out of which n coins have head on both the sides and rest are fair coins. A coin is selected at random and is tossed, if it results in a head with probability $\frac{31}{42}$, find n.

Ans. Total coins (2n + 1)

A: two headed coin; P(A) = $\frac{n}{2n+1}$

B: fair coin; P(B) = $\frac{n+1}{2n+1}$

E: selected coin is tossed and results in head.

∴ P(E/A) = 1; P(E/B) = $\frac{1}{2}$,

Using theorem of total probability

∴ P(head) = P(A) · P(E/A) + P(B) · P(E/B)

$\frac{n+1}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2} = \frac{2n+n+1}{2(2n+1)} = \frac{3n+1}{2(2n+1)}$

Given, $\frac{3n+1}{2(2n+1)} = \frac{31}{42} \Rightarrow 63n + 21 = 62n + 31 \Rightarrow n = 10$

16. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (*i*) the youngest is a girl? (*ii*) atleast one is a girl?

Ans: A family has 2 children, then Sample space = $S = \{BB, BG, GB, GG\}$, where B stands for Boy and G for Girl.

(*i*) Let A and B be two event such that

A = Both are girls = $\{GG\}$

B = the youngest is a girl = $\{BG, GG\}$

Now, $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{\frac{1}{4}}{\frac{2}{4}} = \dfrac{1}{2}$

(*ii*) Let C be event such that

C = at least one is a girl = $\{BG, GB, GG\}$

Now, $P(A/C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{\frac{1}{4}}{\frac{3}{4}} = \dfrac{1}{3}$

17. The random variable X has a probability distribution P(X) of the following form, where k is some number :

$$P(X) = \begin{cases} k \ \ if \ \ x = 0 \\ 2k \ \ if \ \ x = 1 \\ 3k, \ if \ \ x = 2 \\ 0, otherwise \end{cases}$$

(a) Determine the value of k.

(b) Find P (X < 2), P (X ≤ 2), P(X ≥ 2).

Ans. Given distribution of X is

X	0	1	2	otherwise
P(X)	k	$2k$	$3k$	0

(a) Since, $\sum P(X) =$ 1, therefore $P(0) + P(1) + P(2) + P$ (otherwise) = 1

$\Rightarrow k + 2k + 3k + 0 = 1 \Rightarrow 6k = 1 \Rightarrow k = \dfrac{1}{6}$

(b) $P(X = 2) = P(0) + P(1) = k + 2k = 3k = 3 \times \dfrac{1}{6} = \dfrac{1}{2}$

$P(X \le 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 6 \times \dfrac{1}{6} = 1$

and $P(X \ge 2) = P(2) + P(\text{otherwise}) = 3k + 0 = 3k = 3 \times \dfrac{1}{6} = \dfrac{1}{2}$

OR

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

Ans. Let X denotes the random variable which denotes the number of tails when a biased coin is tossed twice.

So, X may have value 0, 1 or 2.

Since, the coin is biased in which head is 3 times as likely to occur as a tail.

$\therefore P\{H\} = \dfrac{3}{4}$ and $P\{T\} = \dfrac{1}{4}$

$P(X = 1) = P\{HH\} = \left(\dfrac{3}{4}\right)^2 = \dfrac{9}{16}$

$P(X = 1) = P$ (one tail and one head) $= P\{HT,TH\} = P\{HT\} + P\{TH\} + P\{H\}P\{T\} + P\{T\}P\{H\}$

$= \frac{3}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}=\frac{3}{16}+\frac{3}{16}=\frac{6}{16}=\frac{3}{8}$

$P(X = 2) = P$ (two tails) $= P\{TT\} = P\{T\}\,P\{T\} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Therefore, the required probability distribution is as follows

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

SECTION – D

Questions 18 carry 5 marks.

18. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

Ans. Let E_1, E_2 , E_3 ,E_4 and A be event defined as

E_1 = the lost card is a spade card.

E_2 = the lost card is a heart card.

E_3 = the lost card is a club card.

E_4 = the lost card is diamond card.

and A = Drawing three spade cards from the remaining cards.

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

$P(A/E_1) = \frac{{}^{12}C_3}{{}^{51}C_3} = \frac{220}{20825}$, $P(A/E_2) = \frac{{}^{13}C_3}{{}^{51}C_3} = \frac{286}{20825}$

$P(A/E_3) = \frac{{}^{13}C_3}{{}^{51}C_3} = \frac{286}{20825}$ $P(A/E_4) = \frac{{}^{13}C_3}{{}^{51}C_3} = \frac{286}{20825}$

By using Baye's theorem, we have

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)+P(E_4)P(A/E_4)}$$

$$= \frac{\frac{1}{4}\times\frac{220}{20825}}{\frac{1}{4}\times\frac{220}{20825}+\frac{1}{4}\times\frac{286}{20825}+\frac{1}{4}\times\frac{286}{20825}+\frac{1}{4}\times\frac{286}{20825}} = \frac{220}{220+286+286+286} = \frac{220}{1078} = \frac{10}{49}$$

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution.

Ans. First six positive integers are 1, 2, 3, 4, 5, 6

If two numbers are selected at random from above six numbers then sample space S is given by

$$S = \{(1,2)\,(1,3),(1,4),(1,5),(1,6),(2,1),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,4),(3,5),$$
$$(3,6),(4,1),(4,2),(4,3),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,6),(6,1),(6,2),(6,3)$$
$$(6,4)\,(6,5)\}$$

$n(s) = 30$.

Here, X is random variable, which may have value 2, 3, 4, 5 or 6.

Therefore, required probability distribution is given as

$P(X = 2)$ = Probability of event getting (1, 2), (2, 1) $= \frac{2}{30}$

$P(X = 3)$ = Probability of event getting (1, 3), (2, 3), (3, 1), (3, 2) = $\frac{4}{30}$

$P(X = 4)$ = Probability of event getting (1, 4), (2, 4), (3, 4), (4, 1), (4, 2), (4, 3) = $\frac{6}{30}$

$P(X = 5)$ = Probability of event getting (1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4) = $\frac{8}{30}$

$P(X = 6)$ = Probability of event getting (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5) = $\frac{10}{30}$

It is represented in tabular form as

X	2	3	4	5	6
$P(X)$	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$

Required mean = $E(x) = \sum p_i x_i = 2\times\frac{2}{30}+3\times\frac{4}{30}+4\times\frac{6}{30}+5\times\frac{8}{30}+6\times\frac{10}{30}$

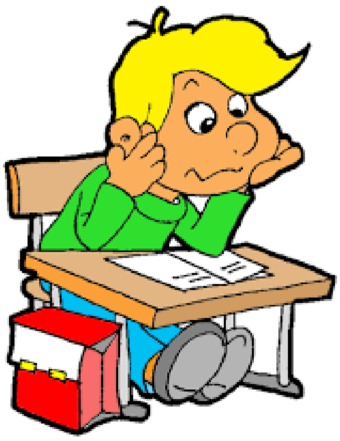
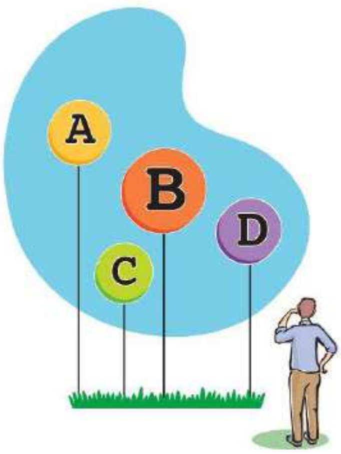
$= \frac{4+12+24+40+60}{30} = \frac{140}{30} = \frac{14}{3} = 4\frac{2}{3}$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

In a test, you either guesses or copies or knows the answer to a multiple-choice question with four choice. The probability that you make a guess is 1/3, you copy the answer is 1/6. The probability that your answer is correct, given that you guess it, is 1/8. And also, the probability that you answer is correct, given that you copy it, is 1/4.

(i) The probability that you know the answer. [1]

(ii) Find the probability that your answer is correct given that you guess it and the probability that your answer is correct given that you know the answer . [1]

(iii) Find the probability that you know the answer given that you correctly answered it. [2]

OR

(iii) Find the total probability of correctly answered the question. [2]

Ans: (i) Let E1 be the event that he guesses

E2 be the event that he copies

E3 be the event that he knows the answer.

Let A be the event that he answered correctly.

Given, $P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{6}, P(E_3) = \frac{1}{2}$

$$P(E_3) = 1 - \left(\frac{1}{3} + \frac{1}{6}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

(ii) $P(A/E_1) = \frac{1}{8}, P(A/E_3) = 1$

(iii) $P(E_3/A) = \frac{P(E_3) \times P(A/E_3)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2) + P(E_3) \times P(A/E_3)}$

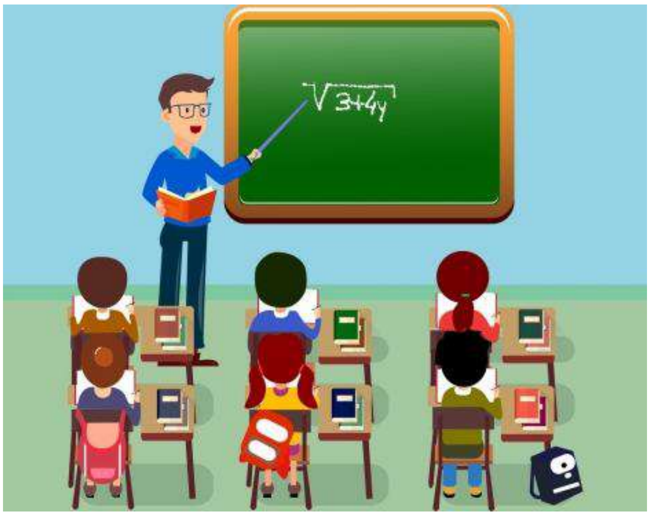
$$\Rightarrow P(E_3/A) = \frac{\frac{1}{2} \times 1}{\frac{1}{3} \times \frac{1}{8} + \frac{1}{6} \times \frac{1}{4} + \frac{1}{2} \times 1} = \frac{6}{7}$$

OR

$$P(A) = P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2) + P(E_3) \times P(A/E_3) = \frac{1}{3} \times \frac{1}{8} + \frac{1}{6} \times \frac{1}{4} + \frac{1}{2} \times 1 = \frac{7}{12}$$

20. Case-Study 2: Read the following passage and answer the questions given below.

Final exams are approaching, so Mr. Kumar decided to check the preparation of the few weak students in the class. He chooses four students A, B, C and D then a problem in mathematics is given to those four students A, B, C, D. Their chances of solving the problem, respectively, are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{2}{3}$.



Based on the given information answer the following questions. What is the probability that:

(i) the problem will be solved?

(ii) at most one of them solve the problem?

Ans: Let E be the event = A solves the problem,

F be the event = B solves the problem,

G be the event = C solves the problem,

H be the event = D solves the problem,

$$P(E)=\frac{1}{3} \Rightarrow P(\bar{E})=\frac{2}{3}$$

$$P(F)=\frac{1}{4} \Rightarrow P(\bar{F})=\frac{3}{4}$$

$$P(G)=\frac{1}{5} \Rightarrow P(\bar{G})=\frac{4}{5}$$

$$P(H)=\frac{2}{3} \Rightarrow P(\bar{H})=\frac{1}{3}$$

(i) The required probability $= P(E\cup F\cup G\cup H) = 1-P(\overline{E})\times P(\overline{F})\times P(\overline{G})\times P(\overline{H})$

$$=1-\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}=\frac{13}{15}$$

(ii) The required probability

$$= P(\overline{E})\times P(\overline{F})\times P(\overline{G})\times P(\overline{H})+P(E)\times P(\overline{F})$$
$$\times P(\overline{G})\times P(\overline{H})+P(\overline{E})\times P(F)\times P(\overline{G})\times P(\overline{H})$$
$$+P(\overline{E})\times P(\overline{F})\times P(G)\times P(\overline{H})+P(\overline{E})\times P(\overline{F})\times P(\overline{G})\times P(H)$$
$$=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}+\frac{1}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}+\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}\times\frac{1}{3}+\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}\times\frac{1}{3}+\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{2}{3}$$
$$=\frac{2}{15}+\frac{1}{15}+\frac{2}{45}+\frac{1}{30}+\frac{4}{15}=\frac{7}{15}+\frac{1}{30}+\frac{2}{45}=\frac{42+3+4}{90}=\frac{49}{90}$$