PRACTICE PAPER 14
CHAPTER 11 THREE DIMENSIONAL GEOMETRY

SUBJECT: MATHEMATICS — **MAX. MARKS : 40**
CLASS : XII — **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. The lines $\frac{1-x}{2}=\frac{y-1}{3}=\frac{z}{1}$ and $\frac{2x-3}{2p}=\frac{y}{-1}=\frac{z-4}{7}$ are perpendicular to each other for p equal to:
(a) −1/2 (b) 1/2 (c) 2 (d) 3

2. The cartesian equation of a line passing through the point with position vector $\vec{a}=\hat{i}-\hat{j}$ and parallel to the line $\vec{r}=\hat{i}+\hat{k}+\mu(2\hat{i}-\hat{j})$ is:
(a) $\frac{x-2}{1}=\frac{y+1}{0}=\frac{z}{1}$ (b) $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{0}$ (c) $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z}{0}$ (d) $\frac{x-1}{2}=\frac{y}{-1}=\frac{z-1}{0}$

3. The direction ratios of the line 3x + 1 = 6y − 2 = 1 – z are:
(a) 3, 6, 1 (b) 3, 6, –1 (c) 2, 1, 6 (d) 2, 1, –6

4. If a line in space makes angle α, β, γ with the positive direction of coordinate axes then the value of cos 2α + cos 2β + cos 2γ is:
(a) 0 (b) –1 (c) 2 (d) 3

5. The direction ratios of three lines *l*, m, n are given below:
2, 3, 4; –7, 2, 2; 4, 7, 7
Which of the two pairs of lines are perpendicular?
(a) *l* and m; m and n (b) *l* and n; m and n
(c) *l* and m; *l* and n (d) *l* and m; m and n; *l* and n

6. A line m passes through the point (–4, 2, –3) and is parallel to line n, given by:
$$\frac{-x-2}{4}=\frac{y+3}{-2}=\frac{2z-6}{3}$$
The vector equation of line m is given by: $\vec{r}=(-4\hat{i}+2\hat{j}-3\hat{k})+\lambda(p\hat{i}+q\hat{j}+r\hat{k})$, where λ ∈ R.
Which of the following could be the possible values for p, q and r?
(a) p = 4, q = (–2), r = 3 (b) p = (–4), q = (–2), r = 3
(c) p = (–2), q = 3, r = (–6) (d) p = 8, q = 4, r = (–3)

7. The lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(6\hat{i}+9\hat{j}-18\hat{k})$ (where λ and μ are scalars) are:
(a) coincident (b) skew (c) intersecting (d) parallel

8. If the direction cosines of a line are $<\frac{1}{c},\frac{1}{c},\frac{1}{c}>$ then:
(a) 0 < c < 1 (b) c > 2 (c) c = ± √2 (d) c = ± √3

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.
Reason (R): The sum of squares of the direction cosines of a line is 1.

10. Assertion (A): The vector equation of the line passing through the points (6, –4, 5) and (3, 4, 1) is $\vec{r} = (6\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + 8\hat{j} + 4\hat{k})$
Reason (R): The vector equation of the line passing through the points $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Using direction ratios, show that the points (2, 3, 4), (–1, –2, 1) and (5, 8, 7) are collinear.

12. The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

13. Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

14. If a line makes angles 90° and 60° respectively with the positive directions of x and y axes, find the angle which it makes with the positive direction of z-axis.

OR

Find the angle between the following pair of lines: $\frac{2-x}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$ and check whether the lines are parallel or perpendicular.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the equation of a line passing through the point $P(2, -1, 3)$ and perpendicular to the lines:
$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

16. Find the shortest distance between the lines whose vector equations are:
$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

OR

A line passes through (2, –1, 3) and is perpendicular to the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. Obtain its equation in vector and cartesian form.

17. Find the length and the foot of the perpendicular drawn from the point (2, –1, 5) on the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$

SECTION – D

Questions 18 carry 5 marks.

18. Find the co-ordinates of the foot of the r and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$. Also, find the image of P in this line.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

A cricket match is organised between the clubs A and B for which a team from each club is chosen. Remaining players of club A and club B are respectively sitting on the lines represented by the equations $\frac{x-3}{1}=\frac{y-2}{2}=\frac{z+4}{2}$ and $\frac{x-5}{3}=\frac{y+2}{2}=\frac{z}{6}$ and to cheer the team of their own clubs.

(a) Find the vector equation of the line on which players of club A can be seated.

(b) Write the direction cosines of the line on which players of club B are seated.

(c) If the line on which players of club A are seated, is perpendicular to the line, whose cartesian equation is $\frac{x-3}{3}=\frac{y-7}{4}=\frac{kz-7}{1}$, then find the value of k.

(d) Find the angle between the lines on which players of clubs A and B are seated.

20. Case-Study 2: Read the following passage and answer the questions given below.

In the given figure, the vector equation of lines in the extreme left side are $\vec{r}=(\hat{i}-2\hat{j})+\lambda(\hat{i}+3\hat{j}+\hat{k})$ and $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+3\hat{j}+\hat{k})$. There are 8 lines between the two extreme lines and the distance between any two consecutive lines are equal.

If the cartesian equation of finishing line is 7 – x = 3y – 2 = 1 – z and position vector of winning player (position shown in figure below) is $2\hat{i}+3\hat{j}-3\hat{k}$.

Based on the given information, answer the following questions:

(a) Find the distance between any two consecutive lines. (2)

(b) Find the perpendicular distance between the position of winning player and the finishing line.(2)

PRACTICE PAPER 14
CHAPTER 11 THREE DIMENSIONAL GEOMETRY (ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**
CLASS : XII | **DURATION : 1½ hrs**

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SECTION – A
Questions 1 to 10 carry 1 mark each.

1. The lines $\frac{1-x}{2}=\frac{y-1}{3}=\frac{z}{1}$ and $\frac{2x-3}{2p}=\frac{y}{-1}=\frac{z-4}{7}$ are perpendicular to each other for p equal to:
(a) −1/2 (b) 1/2 (c) 2 (d) 3
Ans. (c) 2

2. The cartesian equation of a line passing through the point with position vector $\vec{a}=\hat{i}-\hat{j}$ and parallel to the line $\vec{r}=\hat{i}+\hat{k}+\mu(2\hat{i}-\hat{j})$ is:
(a) $\frac{x-2}{1}=\frac{y+1}{0}=\frac{z}{1}$ (b) $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{0}$ (c) $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z}{0}$ (d) $\frac{x-1}{2}=\frac{y}{-1}=\frac{z-1}{0}$
Ans. (b) $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{0}$

3. The direction ratios of the line 3x + 1 = 6y − 2 = 1 – z are:
(a) 3, 6, 1 (b) 3, 6, –1 (c) 2, 1, 6 (d) 2, 1, –6
Ans. (d) 2, 1, –6

4. If a line in space makes angle α, β, γ with the positive direction of coordinate axes then the value of cos 2α + cos 2β + cos 2γ is:
(a) 0 (b) –1 (c) 2 (d) 3
Ans. (b) –1

5. The direction ratios of three lines *l*, m, n are given below:
2, 3, 4; –7, 2, 2; 4, 7, 7
Which of the two pairs of lines are perpendicular?
(a) *l* and m; m and n (b) *l* and n; m and n
(c) *l* and m; *l* and n (d) *l* and m; m and n; *l* and n
Ans. (a) *l* and m; m and n

6. A line m passes through the point (–4, 2, –3) and is parallel to line n, given by:
$\frac{-x-2}{4}=\frac{y+3}{-2}=\frac{2z-6}{3}$
The vector equation of line m is given by: $\vec{r}=(-4\hat{i}+2\hat{j}-3\hat{k})+\lambda(p\hat{i}+q\hat{j}+r\hat{k})$, where λ ∈ R.
Which of the following could be the possible values for p, q and r?
(a) p = 4, q = (–2), r = 3 (b) p = (–4), q = (–2), r = 3
(c) p = (–2), q = 3, r = (–6) (d) p = 8, q = 4, r = (–3)

Ans. (d) p = 8, q = 4, r = (–3)

7. The lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(6\hat{i} + 9\hat{j} - 18\hat{k})$ (where λ and μ are scalars) are:

(a) coincident (b) skew (c) intersecting (d) parallel

Ans. (d) parallel

8. If the direction cosines of a line are $< \frac{1}{c}, \frac{1}{c}, \frac{1}{c} >$ then:

(a) $0 < c < 1$ (b) $c > 2$ (c) $c = \pm\sqrt{2}$ (d) $c = \pm\sqrt{3}$

Ans. (d) $c = \pm\sqrt{3}$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Reason (R): The sum of squares of the direction cosines of a line is 1.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

10. **Assertion (A):** The vector equation of the line passing through the points (6, –4, 5) and (3, 4, 1) is $\vec{r} = (6\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + 8\hat{j} + 4\hat{k})$

Reason (R): The vector equation of the line passing through the points $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

Ans. (d) (A) is false but (R) is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Using direction ratios, show that the points (2, 3, 4), (–1, –2, 1) and (5, 8, 7) are collinear.

Ans. Let points be $A(2, 3, 4)$, $B(-1, -2, 1)$ and $C(5, 8, 7)$

Direction ratios of AB are 2 + 1, 3 + 2, 4 – 1, i.e. 3, 5, 3;

Direction ratios of BC are 5 + 1, 8 + 2, 7 – 1, i.e. 3, 5, 3

As $\frac{3}{3} = \frac{5}{5} = \frac{3}{3}$

$\Rightarrow$ AB is parallel to BC, B is common.

Hence, the given points are collinear.

12. The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

Ans. Line is $5x - 3 = 15y + 7 = -10z + 3$

$$\Rightarrow 5\left(x - \frac{3}{5}\right) = 15\left(y + \frac{7}{15}\right) = -10\left(z - \frac{3}{10}\right)$$

$$\Rightarrow \frac{x - \frac{3}{5}}{6} = \frac{y + \frac{7}{15}}{2} = \left(\frac{z - \frac{3}{10}}{-3}\right)$$

Direction ratios are 6, 2, –3

Dividing by $\sqrt{36 + 4 + 9} = 7$, we get

Direction cosines are $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}$

13. Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other.

Ans. Consider, line $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11} \Rightarrow \frac{-(x-1)}{3}=\frac{7(y-2)}{2\lambda}=\frac{5(z-2)}{11}$

$\Rightarrow \frac{x-1}{-105}=\frac{y-2}{10\lambda}=\frac{z-2}{77}$

Direction ratios of line are –105, 10λ, 77 ...(i)

Now consider, line $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5} \Rightarrow \frac{-7(x-1)}{3\lambda}=\frac{y-5}{1}=\frac{-(z-6)}{5}$

Direction ratios of line are 3λ, –7, 35 ...(*ii*)

If lines are perpendicular, then (–105) (3λ) + (10λ) (–7) + 77 × 35 = 0

$\Rightarrow -315\lambda - 70\lambda + 2695 = 0 \Rightarrow 385\lambda = 2695 \Rightarrow \lambda = 7.$

14. If a line makes angles 90° and 60° respectively with the positive directions of x and y axes, find the angle which it makes with the positive direction of z-axis.

Ans. Let the angle made by line with positive direction of *z*-axis be θ then,

We know that $\cos^2 90° + \cos^2 60° + \cos^2\theta = 1$

$\Rightarrow 0+\left(\frac{1}{2}\right)^2+\cos^2\theta = 1 \Rightarrow \frac{1}{4}+\cos^2\theta=1$

$\Rightarrow \cos^2\theta = 1-\frac{1}{4} \Rightarrow \cos^2\theta=\frac{3}{4} \Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 60°$ or $\frac{\pi}{3}$ if $\cos\theta=\frac{\sqrt{3}}{2}$ and $\theta = 150°$ or $\frac{5\pi}{6}$ if $\cos\theta = -\frac{\sqrt{3}}{2}$

OR

Find the angle between the following pair of lines: $\frac{2-x}{-2}=\frac{y-1}{7}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$ and check whether the lines are parallel or perpendicular.

Ans. dr's of lines are 2, 7, –3 and –1, 2, 4

As 2 × (– 1) + 7 × 2 – 3 × 4 = 0, so lines are perpendicular.

∴ Angle = 90°

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the equation of a line passing through the point *P*(2, – 1, 3) and perpendicular to the lines:

$\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$

Ans.

Let line through point (2, – 1, 3) is $\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\lambda'(a\hat{i}+b\hat{j}+c\hat{k})$ ...(*i*)

If line (*i*) is perpendicular to lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$

then $(a\hat{i}+b\hat{j}+c\hat{k})\cdot(2\hat{i}-2\hat{j}+\hat{k})=0 \Rightarrow 2a-2b+c=0$

and $(a\hat{i}+b\hat{j}+c\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})=0 \Rightarrow a+2b+2c=0$

$\Rightarrow \frac{a}{-4-2}=\frac{-b}{4-1}=\frac{c}{4+2}$ i.e. $\frac{a}{-6}=\frac{b}{-3}=\frac{c}{6} \Rightarrow a:b:c$ is $-6:-3:6$ or $2:1:-2$

From (*i*), line is $\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\lambda'(2\hat{i}+\hat{j}-2\hat{k})$

16. Find the shortest distance between the lines whose vector equations are:

$\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

Ans.

Comparing the given equations with equations $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$.

We get $\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

Therefore, $\vec{a}_2 - \vec{a}_1 = (\hat{i} - \hat{k})$ and $\vec{b}_1 \times \vec{b}_2 = (2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} - 5\hat{j} + 2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9+1+49} = \sqrt{59}$$

Hence, the shortest distance between the given lines is given by

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \left|\frac{3-0+7}{\sqrt{59}}\right| = \frac{10}{\sqrt{59}} \text{ units.}$$

OR

A line passes through (2, –1, 3) and is perpendicular to the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. Obtain its equation in vector and cartesian form.

Ans.

Let $\vec{b}$ be parallel vector of required line.

$\Rightarrow$ $\vec{b}$ is perpendicular to both given line.

$\Rightarrow$ $\vec{b} = (2\hat{i} - 2\hat{j} + \hat{k}) \times (\hat{i} + 2\hat{j} + 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = (-4-2)\hat{i} - (4-1)\hat{j} + (4+2)\hat{k} = -6\hat{i} - 3\hat{j} + 6\hat{k}.$$

Hence, the equation of line in vector form is

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(-6\hat{i} - 3\hat{j} + 6\hat{k}) \Rightarrow \vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) - 3\lambda(2\hat{i} + \hat{j} - 2\hat{k})$$

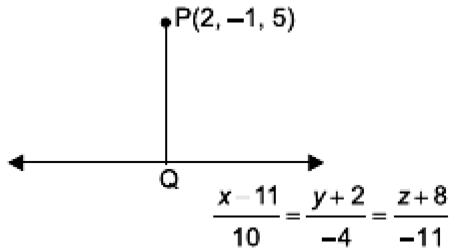
$$\Rightarrow \vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \mu(2\hat{i} + \hat{j} - 2\hat{k}) \qquad [\mu = -3\lambda]$$

Equation in cartesian form is $\dfrac{x-2}{2} = \dfrac{y+1}{1} = \dfrac{z-3}{-2}$

17. Find the length and the foot of the perpendicular drawn from the point (2, –1, 5) on the line $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11}$

Ans. General point on the line $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11} = \lambda$ is

$Q(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$ ...(*i*)



Direction ratios of PQ are $10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5$

i.e. $10\lambda + 9, -4\lambda - 1, -11\lambda - 13$

If PQ is perpendicular to the given line, then

$10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$

$\Rightarrow 237\lambda = -237 \quad \lambda = -1$

Substituting in (i), we get the foot of perpendicular as Q(1, 2, 3).

Length of perpendicular PQ = $\sqrt{(2-1)^2+(-1-2)^2+(5-3)^2} = \sqrt{1+9+4} = \sqrt{14}$

SECTION – D

Questions 18 carry 5 marks.

18. Find the co-ordinates of the foot of the r and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r}=(-\hat{i}+3\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}-\hat{k})$. Also, find the image of P in this line.

Ans: Any point on the line can be written in parametric form as $(2\lambda - 1, 3\lambda + 3, -\lambda + 1)$

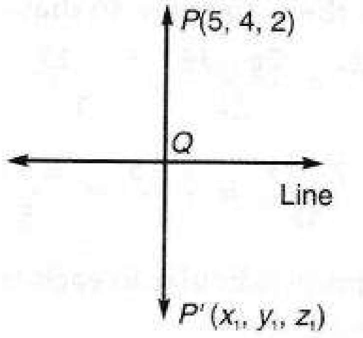
Assuming this as the foot of perpendicular from (5,4,2), we can equate the dot product of this vector and the line direction to zero.

$\therefore ((2\lambda-1-5)\hat{i}+(3\lambda+3-4)\hat{j}+(-\lambda+1-2)\hat{k})\cdot(2\hat{i}+3\hat{j}-\hat{k})=0$

$\Rightarrow (2\lambda-6)\times 2+(3\lambda-1)\times 3+(-\lambda-1)\times(-1)=0$

$\Rightarrow 4\lambda-12+9\lambda-3+\lambda+1=0$

$\Rightarrow 14\lambda-14=0 \ \Rightarrow \lambda=1$



The coordinates of the point are thus (1,6,0)

The length of the perpendicular can be found out by

$\sqrt{(5-1)^2+(4-6)^2+(2-0)^2} = \sqrt{16+4+4} = \sqrt{24}$

The foot of perpendicular would be the midpoint of P and the image of P in the line.

$\therefore (5\hat{i}+4\hat{j}+2\hat{k})+(x\hat{i}+y\hat{j}+z\hat{k})=2\times(\hat{i}+6\hat{j})$

$\Rightarrow x=2-5=-3, y=12-4=8, z=0-2=-2$

The image of point P is thus $(-3, 8, -2)$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

A cricket match is organised between the clubs A and B for which a team from each club is chosen. Remaining players of club A and club B are respectively sitting on the lines represented by the equations $\frac{x-3}{1}=\frac{y-2}{2}=\frac{z+4}{2}$ and $\frac{x-5}{3}=\frac{y+2}{2}=\frac{z}{6}$ and to cheer the team of their own clubs.

(a) Find the vector equation of the line on which players of club A can be seated.
(b) Write the direction cosines of the line on which players of club B are seated.
(c) If the line on which players of club A are seated, is perpendicular to the line, whose cartesian equation is $\frac{x-3}{3}=\frac{y-7}{4}=\frac{kz-7}{1}$, then find the value of k.
(d) Find the angle between the lines on which players of clubs A and B are seated.

Ans. (a) The vector equation of a line is given by $\vec{r}=\vec{a}+\lambda\vec{b}$

From the given cartesian form of equation of line we have $\vec{a}=3\hat{i}+2\hat{j}-4\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+2\hat{k}$

$\therefore$ Vector equation of given line is: $\vec{r}=(3\hat{i}+2\hat{j}-4\hat{k})+\lambda(\hat{i}+2\hat{j}+2\hat{k})$

(b) Equation of line B is $\frac{x-5}{3}=\frac{y+2}{2}=\frac{z}{6}$

So, direction ratios of this line are 3, 2, 6

So, direction cosines of this line are $\frac{3}{\sqrt{3^2+2^2+6^2}},\frac{2}{\sqrt{3^2+2^2+6^2}},\frac{6}{\sqrt{3^2+2^2+6^2}}=\frac{3}{7},\frac{2}{7},\frac{6}{7}$

(c) Given equation of line can also be written as $\frac{x-3}{3}=\frac{y-7}{4}=\frac{z-\frac{7}{k}}{\frac{1}{k}}$

So, direction ratios of this line are (3, 4, 1/k)
Also, direction ratios of line A are (1, 2, 2).
Since, the two lines are perpendicular,
$\therefore a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$
$\Rightarrow 3 \times 1 + 4 \times 2 + 1/k \times 2 = 0 \Rightarrow 2/k = -11 \Rightarrow k = -2/11$

(d) Angle between two lines is given as, $\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

Here, $(a_1, b_1, c_1) = (1, 2, 2)$ and $(a_2, b_2, c_2) = (3, 2, 6)$

$\therefore \cos\theta=\frac{3+4+12}{\sqrt{1+4+4}\sqrt{9+4+36}}=\frac{19}{3\times 7}=\frac{19}{21}\Rightarrow\theta=\cos^{-1}\left(\frac{19}{21}\right)$

20. Case-Study 2: Read the following passage and answer the questions given below.

In the given figure, the vector equation of lines in the extreme left side are $\vec{r}=(\hat{i}-2\hat{j})+\lambda(\hat{i}+3\hat{j}+\hat{k})$ and $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+3\hat{j}+\hat{k})$. There are 8 lines between the two extreme lines and the distance between any two consecutive lines are equal.

If the cartesian equation of finishing line is 7 – x = 3y – 2 = 1 – z and position vector of winning player (position shown in figure below) is $2\hat{i}+3\hat{j}-3\hat{k}$.

Based on the given information, answer the following questions:
(a) Find the distance between any two consecutive lines. (2)
(b) Find the perpendicular distance between the position of winning player and the finishing line.(2)

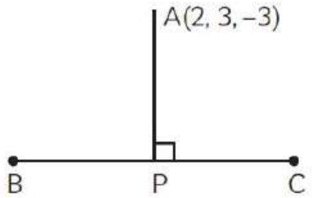
Ans. (a) Distance between two extreme lines, $\vec{r} = (\hat{i} - 2\hat{j}) + \lambda(\hat{i} + 3\hat{j} + \hat{k})$ and

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 3\hat{j} + \hat{k})$, $d = \left|\dfrac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|}\right| = \left|\dfrac{(\hat{i} + 3\hat{j} + \hat{k}) \times (3\hat{j})}{\sqrt{1+9+1}}\right| = \left|\dfrac{3\sqrt{2}}{\sqrt{11}}\right| = \dfrac{3\sqrt{2}}{\sqrt{11}}$ units

$\Rightarrow d = \dfrac{3\sqrt{22}}{11}$ units

$\therefore$ Distance between any two consecutive lines: $d = \dfrac{1}{9} \times \dfrac{3\sqrt{22}}{11} = \dfrac{\sqrt{22}}{33}$ units

(d) Let A and BC represent position of winning player and finishing line respectively.
Let 7 – x = 3y – 2 = 1 – z = λ
x = 7 – λ, y = (λ + 2)/3, z = 1 – λ



Let P be the point on the line such that AP ⊥ BC.
∴ P [7 – λ, (λ + 2)/3, 1 – λ]
Direction ratios of AP are 5 – λ, (λ − 7)/3, 4 – λ.
AP ⊥ BC and
Direction ratio of finishing line is (−1, 1/3, −1)
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
∴ λ − 5 + (1/9) (λ − 7) – 4 + λ = 0
⇒ λ = 88/19.

So, the distance between two points $\left(\dfrac{45}{19}, \dfrac{42}{19}, \dfrac{-69}{19}\right)$ and A(2, 3, –3) is:

$$= \sqrt{\left(2 - \frac{45}{19}\right)^2 + \left(3 - \frac{42}{19}\right)^2 + \left(-3 + \frac{69}{19}\right)^2} = \sqrt{\left(\frac{-7}{19}\right)^2 + \left(\frac{15}{19}\right)^2 + \left(\frac{12}{19}\right)^2}$$

$$= \frac{1}{19}\sqrt{49 + 225 + 144} = \frac{1}{19}\sqrt{418} = \frac{20.45}{19} = 1.08 \text{ units (approx.)}$$