PRACTICE PAPER 13
CHAPTER 10 VECTOR ALGEBRA

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XII **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The position vector of the point which divides the join of points with position vectors $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3 : 1 is

(a) $\frac{3\vec{a}-2\vec{b}}{2}$ (b) $\frac{7\vec{a}-8\vec{b}}{4}$ (c) $\frac{3\vec{a}}{4}$ (d) $\frac{5\vec{a}}{4}$

2. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a}.\vec{b}=2\sqrt{3}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{5\pi}{2}$ (d) $\frac{\pi}{6}$

3. The vectors from origin to the points A and B are $\vec{a}=2\hat{i}-3\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$,respectively, then the area of triangle OAB is

(a) 340 (b) $\sqrt{25}$ (c) $\frac{1}{2}\sqrt{229}$ (d) $\sqrt{229}$

4. If $|\vec{a}|=10, |\vec{b}|=2$ and $\vec{a}.\vec{b}=12$, then value of $|\vec{a}\times\vec{b}|$ is

(a) 5 (b) 10 (c) 14 (d) 16

5. The value of λ for which the two vectors $3\hat{i}-6\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+\lambda\hat{k}$ are parallel is

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$

6. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=2, |\vec{b}|=3, |\vec{c}|=5$ then value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is

(a) 0 (b) 1 (c) – 19 (d) 38

7. The area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$ is

(a) 15 (b) $15\sqrt{3}$ (c) $15\sqrt{2}$ (d) None of these

8. Area of a rectangle having vertices A, B, C and D with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$, respectively is

(a) 1/2 (b) 1 (c) 2 (d) 4

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If means $\vec{a} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + p\hat{k}$ are mutually perpendicular then p is –9.

Reason (R): For perpendicular vectors, $\vec{a}.\vec{b} = 0$

10. **Assertion (A):** If two vectors are inclined at an angle, so that their resultant is also a unit vector, then sin θ is $\frac{\sqrt{3}}{2}$.

Reason (R): If two vectors are inclined at an angle, so that their resultant is also a unit vector, then sin θ is 1/2.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}| = 13$ and $|\vec{a}| = 5$ find the value of $|\vec{b}|$.

12. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

13. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

14. Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find a unit vector perpendicular to each of the vectors $\vec{a} + 2\vec{b}$ and $2\vec{a} + \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

16. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular vectors.

17. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

SECTION – D

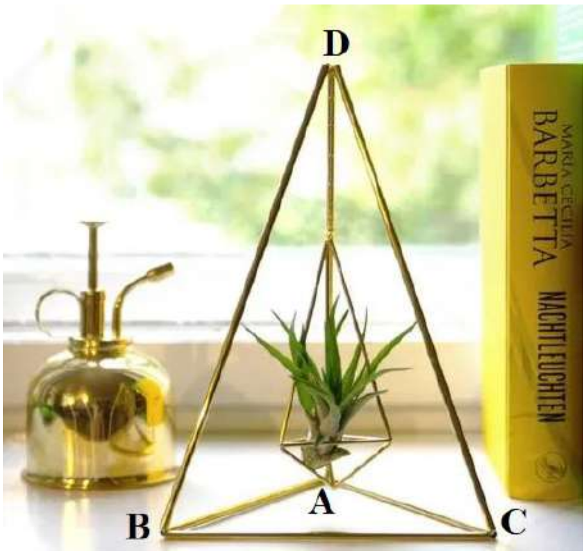
Questions 18 carry 5 marks.

18. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.
Aditi purchased an air plant holder which is in the shape of a tetrahedron. Let A, B, C and D are the coordinates of the air plant holder where A = (1, 1, 1), B = (2, 1, 3), C = (3, 2, 2) and D = (3, 3, 4).



Based on the above information, answer the following questions.

(i) Find the position vector $\overrightarrow{AB}$. (1)

(ii) Find the position vector $\overrightarrow{AC}$. (1)

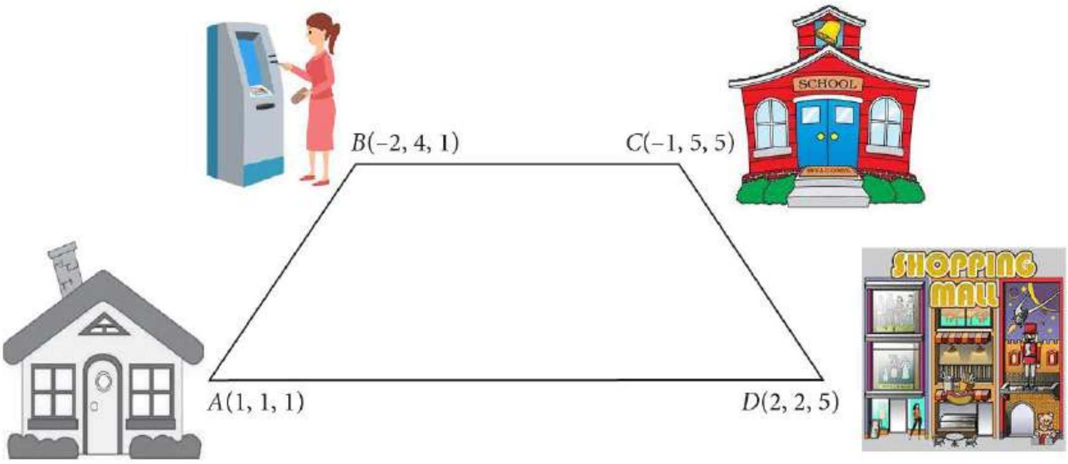
(iii) Find the unit vector along $\overrightarrow{AD}$ vector. (2)

OR

(iii) Find the area of ΔABC). (2)

20. Case-Study 2: Read the following passage and answer the questions given below.

Ritika starts walking from his house to shopping mall. Instead of going to the mall directly, she first goes to a ATM, from there to her daughter's school and then reaches the mall. In the diagram, A, B, C and D represent the coordinates of House, ATM, School and Mall respectively.



(i) Find the position vector $\overrightarrow{AB}$. (1)

(ii) Find the position vector $\overrightarrow{BC}$. (1)

(iii) Find the unit vector along $\overrightarrow{AD}$ vector. (2)

PRACTICE PAPER 13

CHAPTER 10 VECTOR ALGEBRA (ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**
CLASS : XII | **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The position vector of the point which divides the join of points with position vectors $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3 : 1 is

(a) $\frac{3\vec{a}-2\vec{b}}{2}$ (b) $\frac{7\vec{a}-8\vec{b}}{4}$ (c) $\frac{3\vec{a}}{4}$ (d) $\frac{5\vec{a}}{4}$

Ans. (d) $\frac{5\vec{a}}{4}$

2. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a}.\vec{b}=2\sqrt{3}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{5\pi}{2}$ (d) $\frac{\pi}{6}$

Ans. (a) $\frac{\pi}{3}$

3. The vectors from origin to the points A and B are $\vec{a}=2\hat{i}-3\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$,respectively, then the area of triangle OAB is

(a) 340 (b) $\sqrt{25}$ (c) $\frac{1}{2}\sqrt{229}$ (d) $\sqrt{229}$

Ans. (c) $\frac{1}{2}\sqrt{229}$

4. If $|\vec{a}|=10, |\vec{b}|=2$ and $\vec{a}.\vec{b}=12$, then value of $|\vec{a}\times\vec{b}|$ is

(a) 5 (b) 10 (c) 14 (d) 16

Ans. (d) 16

5. The value of λ for which the two vectors $3\hat{i}-6\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+\lambda\hat{k}$ are parallel is

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$

Ans. (b) $\frac{2}{3}$

6. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=2, |\vec{b}|=3, |\vec{c}|=5$ then value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is

(a) 0 (b) 1 (c) – 19 (d) 38

Ans. (c) – 19

7. The area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$ is

(a) 15 (b) $15\sqrt{3}$ (c) $15\sqrt{2}$ (d) None of these

Ans. (c) $15\sqrt{2}$

8. Area of a rectangle having vertices A, B, C and D with position vectors $-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ and $-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$, respectively is

(a) 1/2 (b) 1 (c) 2 (d) 4

Ans. (c) 2

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): If means $\vec{a} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + p\hat{k}$ are mutually perpendicular then p is –9.

Reason (R): For perpendicular vectors, $\vec{a}.\vec{b} = 0$

Ans. (a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).

10. Assertion (A): If two vectors are inclined at an angle, so that their resultant is also a unit vector, then sin θ is $\frac{\sqrt{3}}{2}$.

Reason (R): If two vectors are inclined at an angle, so that their resultant is also a unit vector, then sin θ is 1/2.

Ans. (c) Assertion (A) is true but reason (R) is false.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}| = 13$ and $|\vec{a}| = 5$ find the value of $|\vec{b}|$.

Ans: Given $|\vec{a}+\vec{b}| = 13$

$|\vec{a}+\vec{b}|^2 = 169 \Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 169$

$\Rightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = 169$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 = 169 \quad \left[\because \vec{a} \perp \vec{b} \Rightarrow \vec{a}.\vec{b} = 0\right]$

$\Rightarrow |\vec{b}|^2 = 169 - |\vec{a}|^2 = 169 - 25 = 144$

$\Rightarrow |\vec{b}| = 12$

12. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

Ans: Given that $\vec{a}+\vec{b}$ is also a unit vector

$\therefore |\vec{a}+\vec{b}| = 1$

$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b}).(\vec{a}+\vec{b})$

$\Rightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = 1^2 = 1$

$\Rightarrow 1+2\vec{a}.\vec{b}+1=1 \quad [\because |\vec{a}|=1, |\vec{b}|=1]$

$\Rightarrow 2\vec{a}.\vec{b}=-1 \Rightarrow \vec{a}.\vec{b}=-\frac{1}{2} \Rightarrow |\vec{a}||\vec{b}|\cos\theta=-\frac{1}{2}$

$\Rightarrow 1\times1\times\cos\theta=-\frac{1}{2} \Rightarrow \cos\theta=-\frac{1}{2} \Rightarrow \cos\theta=\cos\frac{2\pi}{3}$

$\Rightarrow \theta=\frac{2\pi}{3}$

13. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

Ans: Adjacent sides of parallelogram are given by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix}=20\hat{i}+5\hat{j}-5\hat{k}$

$\Rightarrow |\vec{a}\times\vec{b}|=\sqrt{400+25+25}=\sqrt{450}=15\sqrt{2}$

Hence, the area of the given parallelogram is $15\sqrt{2}$ sq. units.

14. Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices.

Ans: We have $\overrightarrow{AB}=\hat{j}+2\hat{k}$ and $\overrightarrow{AC}=\hat{i}+2\hat{j}$.

The area of the given triangle is $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

Now, $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 2\\ 1 & 2 & 0\end{vmatrix}=-4\hat{i}+2\hat{j}-\hat{k}$

Therefore, $|\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{16+4+1}=\sqrt{21}$

Thus, the required area is $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{1}{2}\sqrt{21}$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find a unit vector perpendicular to each of the vectors $\vec{a}+2\vec{b}$ and $2\vec{a}+\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

Ans. Given that $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$

$\therefore \vec{a}+2\vec{b}=3\hat{i}+2\hat{j}+2\hat{k}+2(\hat{i}+2\hat{j}-2\hat{k})$

$=3\hat{i}+2\hat{j}+2\hat{k}+2\hat{i}+4\hat{j}-4\hat{k}=5\hat{i}+6\hat{j}-2\hat{k}$

and $2\vec{a}+\vec{b}=2(3\hat{i}+2\hat{j}+2\hat{k})+\hat{i}+2\hat{j}-2\hat{k}$

$=6\hat{i}+4\hat{j}+4\hat{k}+\hat{i}+2\hat{j}-2\hat{k}=7\hat{i}+6\hat{j}+2\hat{k}$

Now, perpendicular vector of $\vec{a}+2\vec{b}$ and $2\vec{a}+\vec{b}$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & 6 & -2\\ 7 & 6 & 2\end{vmatrix}=(12+12)\hat{i}-(10+14)\hat{j}+(30-42)\hat{k}$

$= 24\hat{i} - 24\hat{j} - 12\hat{k} = 12(2\hat{i} - 2\hat{j} - \hat{k})$

$\therefore$ Required unit vector = $\pm \dfrac{12(2\hat{i} - 2\hat{j} - \hat{k})}{12\sqrt{4+4+1}} = \pm \dfrac{2\hat{i} - 2\hat{j} - \hat{k}}{\sqrt{9}}$

$= \pm \dfrac{2\hat{i} - 2\hat{j} - \hat{k}}{3} = \pm\left(\dfrac{2}{3}\hat{i} - \dfrac{2}{3}\hat{j} - \dfrac{1}{3}\hat{k}\right)$

16. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

Ans: Given that $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

$\therefore\ \vec{a} + \vec{b} = \hat{i} - \hat{j} + 7\hat{k} + 5\hat{i} - \hat{j} + \lambda\hat{k} = 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$

and $\vec{a} - \vec{b} = \hat{i} - \hat{j} + 7\hat{k} - 5\hat{i} + \hat{j} - \lambda\hat{k} = -4\hat{i} + (7-\lambda)\hat{k}$

Now, $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors

$\Rightarrow (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$

$\Rightarrow (6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}).(-4\hat{i} + (7-\lambda)\hat{k} = 0$

$\Rightarrow -24 + 0 + (7+\lambda)(7-\lambda) = 0$

$\Rightarrow -24 + 49 - \lambda^2 = 0 \Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5$

17. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

Ans: Let A= $2\hat{i} - \hat{j} + \hat{k}$ B = $\hat{i} - 3\hat{j} - 5\hat{k}$ and C = $3\hat{i} - 4\hat{j} - 4\hat{k}$

$\overrightarrow{AB} = (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = -\hat{i} - 2\hat{j} - 6\hat{k}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{1+4+36} = \sqrt{41}$

$\overrightarrow{BC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k}) = 2\hat{i} - \hat{j} + \hat{k}$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{4+1+1} = \sqrt{6}$

and $\overrightarrow{AC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = \hat{i} - 3\hat{j} - 5\hat{k}$

$\Rightarrow |\overrightarrow{AC}| = \sqrt{1+9+25} = \sqrt{35}$

$\therefore |\overrightarrow{AB}|^2 = |\overrightarrow{AC}|^2 + |\overrightarrow{BC}|^2$

Hence, ABC is a right angled triangle.

SECTION – D

Questions 18 carry 5 marks.

18. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Ans: Given that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors.

$\therefore\ \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

It is also given that $|\vec{a}| = |\vec{b}| = |\vec{c}|$

Let vector $\vec{a} + \vec{b} + \vec{c}$ be inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$ at angles α, β and γ respectively.

$\cos\alpha = \dfrac{(\vec{a} + \vec{b} + \vec{c}).\vec{a}}{|\vec{a} + \vec{b} + \vec{c}||\vec{a}|} = \dfrac{\vec{a}.\vec{a} + \vec{b}.\vec{a} + \vec{c}.\vec{a}}{|\vec{a} + \vec{b} + \vec{c}||\vec{a}|} = \dfrac{|\vec{a}|^2 + 0 + 0}{|\vec{a} + \vec{b} + \vec{c}||\vec{a}|}$

$= \dfrac{|\vec{a}|^2}{|\vec{a} + \vec{b} + \vec{c}||\vec{a}|} = \dfrac{|\vec{a}|}{|\vec{a} + \vec{b} + \vec{c}|}$

$$\cos\beta = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|} = \frac{\vec{a}.\vec{b}+\vec{b}.\vec{b}+\vec{c}.\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} = \frac{0+|\vec{b}|^2+0}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$= \frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} = \frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\gamma = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|} = \frac{\vec{a}.\vec{c}+\vec{b}.\vec{c}+\vec{c}.\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} = \frac{0+0+|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$= \frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} = \frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

Now as $|\vec{a}|=|\vec{b}|=|\vec{c}|$, therefore, cosα = cosβ = cosγ

$\therefore\ \alpha = \beta = \gamma$

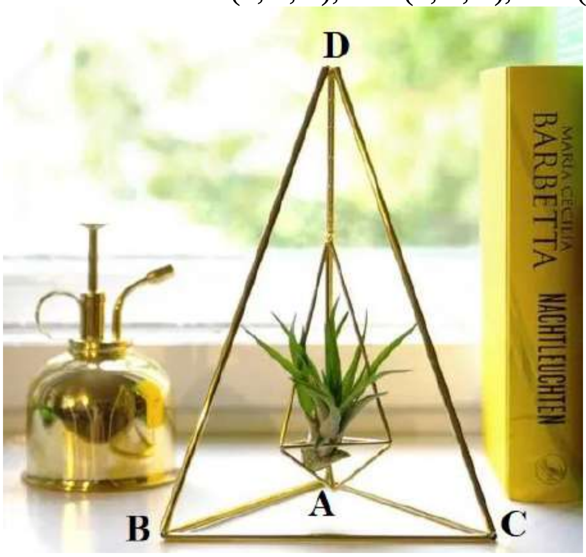
Hence, the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

Aditi purchased an air plant holder which is in the shape of a tetrahedron. Let A, B, C and D are the coordinates of the air plant holder where A = (1, 1, 1), B = (2, 1, 3), C = (3, 2, 2) and D = (3, 3, 4).



Based on the above information, answer the following questions.

(i) Find the position vector $\overrightarrow{AB}$. (1)

(ii) Find the position vector $\overrightarrow{AC}$. (1)

(iii) Find the unit vector along $\overrightarrow{AD}$ vector. (2)

OR

(iii) Find the area of ΔABC). (2)

Ans. (i) Position vector $\overrightarrow{AB}$

$= (2-1)\hat{i} + (1-1)\hat{j} + (3-1)\hat{k} = \hat{i} + 2\hat{k}$

(ii) Position vector $\overrightarrow{AC}$

$= (3-1)\hat{i} + (2-1)\hat{j} + (2-1)\hat{k} = 2\hat{i} + \hat{j} + \hat{k}$

(iii) Unit vector along $\overrightarrow{AD} = \dfrac{\overrightarrow{AD}}{|\overrightarrow{AD}|}$

$$= \frac{2\hat{i}+2\hat{j}+3k}{\sqrt{2^2+2^2+3^2}} = \frac{2\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{4+4+9}} = \frac{1}{\sqrt{17}}(2\hat{i}+2\hat{j}+3\hat{k})$$

OR

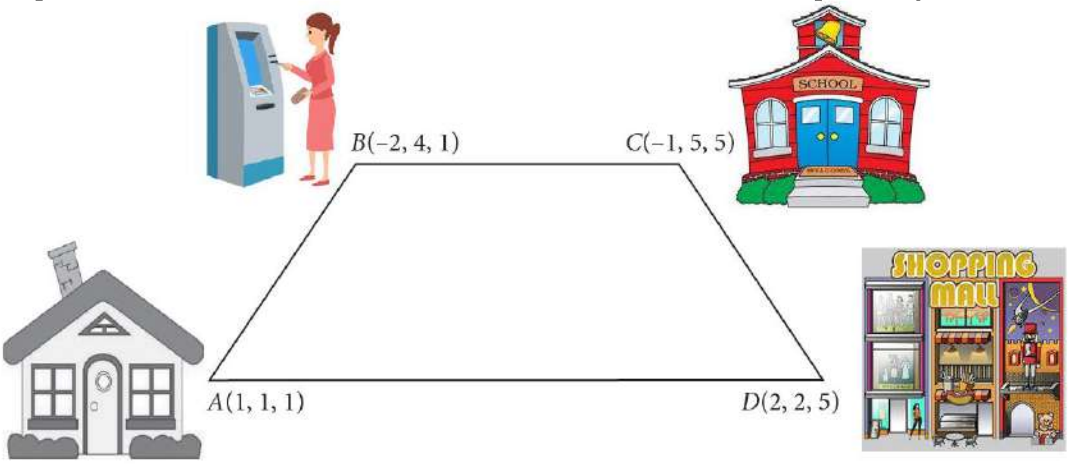
(iii) Area of ΔABC = $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 2\\ 2 & 1 & 1\end{vmatrix}=\hat{i}(0-2)-\hat{j}(1-4)+\hat{k}(1-0)$$
$$=-2\hat{i}+3\hat{j}+\hat{k}$$

$$\Rightarrow |\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{(-2)^2+3^2+1^2}=\sqrt{4+9+1}=\sqrt{14}$$

20. Case-Study 2: Read the following passage and answer the questions given below.

Ritika starts walking from his house to shopping mall. Instead of going to the mall directly, she first goes to a ATM, from there to her daughter's school and then reaches the mall. In the diagram, A, B, C and D represent the coordinates of House, ATM, School and Mall respectively.



(i) Find the position vector $\overrightarrow{AB}$. (1)

(ii) Find the position vector $\overrightarrow{BC}$. (1)

(iii) Find the unit vector along $\overrightarrow{AD}$ vector. (2)

Ans. (i) Position vector $\overrightarrow{AB}$

$=(-2-1)\hat{i}+(4-1)\hat{j}+(1-1)\hat{k}=-3\hat{i}+3\hat{j}$

(ii) Position vector $\overrightarrow{BC}$

$=(-1+2)\hat{i}+(5-4)\hat{j}+(5-1)\hat{k}=\hat{i}+\hat{j}+4\hat{k}$

(iii) Unit vector along $\overrightarrow{AD}=\frac{\overrightarrow{AD}}{|\overrightarrow{AD}|}$

$$=\frac{\hat{i}+\hat{j}+4\hat{k}}{\sqrt{1^2+1^2+4^2}}=\frac{\hat{i}+\hat{j}+4\hat{k}}{\sqrt{1+1+16}}=\frac{1}{\sqrt{18}}(\hat{i}+\hat{j}+4\hat{k})=\frac{1}{3\sqrt{2}}(\hat{i}+\hat{j}+4\hat{k})$$