PRACTICE PAPER 06

CHAPTER 06 APPLICATION OF DERIVATIVES

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**
CLASS : XII | **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The function $f(x) = x^3 + 3x$ is increasing in interval
(a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) R (d) $(0, 1)$

2. In a sphere of radius 'r', a right circular cone of height 'h' having maximum curved surface area is inscribed. The expression for the square of curved surface of cone is
(a) $2\pi^2rh(2rh + h^2)$ (b) $\pi^2hr(2rh + h^2)$ (c) $2\pi^2r(2rh^2 - h^3)$ (d) $2\pi^2r^2(2rh - h^2)$

3. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval
(a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, 2)$ (c) $(-\infty, 2] \cup [3, \infty)$ (d) $(3, \infty)$

4. A function $f : R \to R$ is defined as $f(x) = x^3 + 1$. Then the function has
(a) no minimum value (b) no maximum value
(c) both maximum and minimum values (d) neither maximum value nor minimum value

5. The point(s) on the curve $y = x^2$, at which y-coordinate is changing six times as fast as x-coordinate is/are
(*a*) (2, 4) (*b*) (3, 9) (*c*) (3, 9), (9, 3) (*d*) (6, 2)

6. The edge of a cube is increasing at the rate of 0.3 cm/s, the rate of change of its surface area when edge is 3 cm is
(a) 10.8 cm (b) 10.8 cm^2 (c) 10.8 cm^2/s (d) 10.8 cm/s

7. The function $f(x) = 4 - 3x + 3x^2 - x^3$, $x \in R$ is
(a) decreasing function (b) increasing function
(c) strictly increasing on R (d) neither increasing nor decreasing on R

8. If at x = 1, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2]. Then the value of a is
(a) 124 (b) –124 (c) 120 (d) –120

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The maximum value of the function $f(x) = x^5$, $x \in [-1, 1]$, is attained at its critical point, $x = 0$.
Reason (R): The maximum value of a function can only occur at points where derivative is zero.

10. **Assertion (A):** The function $f(x) = x^3 - 12x$ is strictly increasing in $(-\infty, -2) \cup (2, \infty)$.
Reason (R): For strictly increasing function $f'(x) > 0$.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. For what values of x is the rate of increase of $x^3 - 5x^2 + 5x + 8$ is twice the rate of increase of x?

12. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

13. A spherical balloon is being inflated by pumping in 16 cm^3/s of gas. At the instant when balloon contains 36π cm^3 of gas, how fast is its radius increasing?

14. Find the least value of a such that the function $f(x) = x^2 + ax + 1$ is strictly increasing on [1, 2].

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is (*i*) strictly increasing (*ii*) strictly decreasing

16. Find the absolute maximum value and the absolute minimum value for the function $f(x) = 4x - \frac{1}{2}x^2$, in the given interval $x \in \left[-2, \frac{9}{2}\right]$.

17. A man 1.6 m tall walks at the rate of 0.5 m/s away from a lamp post, 8 metres high. Find the rate at which his shadow is increasing and the rate with which the tip of shadow is moving away from the pole.

SECTION – D

Questions 18 carry 5 marks.

18. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank?

OR

Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. **Case-Study 1:**
The use of electric vehicles will curb air pollution in the long run.

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V:

$$V(t)=\frac{1}{5}t^3-\frac{5}{2}t^2+25t-2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

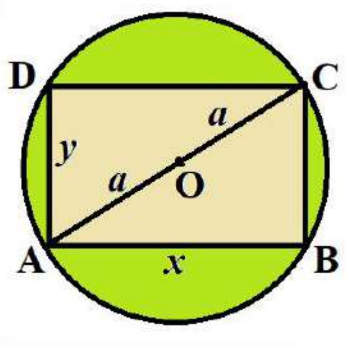
Based on the above information, answer the following questions:

(a) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.

(b) Prove that the function V(t) is an increasing function.

20. Case-Study 2:

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Find the perimeter of rectangle in terms of any one side and radius of circle.

(ii) Find critical points to maximize the perimeter of rectangle?

(iii) Check for maximum or minimum value of perimeter at critical point.

OR

(iii) If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region.

PRACTICE PAPER 06
CHAPTER 06 APPLICATION OF DERIVATIVES
(ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**
CLASS : XII | **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

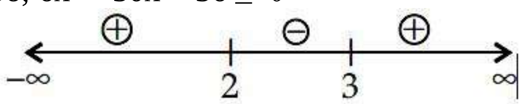
SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The function $f(x) = x^3 + 3x$ is increasing in interval
(a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) R (d) $(0, 1)$
Ans: (c) R
$f(x) = x^3 + 3x$
$\Rightarrow f'(x) = 3x^2 + 3$
For increasing $f'(x) > 0$
$3x^2 + 3 > 0$ ($x \in R \therefore x^2 > 0$)

2. In a sphere of radius 'r', a right circular cone of height 'h' having maximum curved surface area is inscribed. The expression for the square of curved surface of cone is
(a) $2\pi^2 rh(2rh + h^2)$ (b) $\pi^2 hr(2rh + h^2)$ (c) $2\pi^2 r(2rh^2 - h^3)$ (d) $2\pi^2 r^2(2rh - h^2)$
Ans: (c) $2\pi^2 r(2rh^2 - h^3)$

3. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval
(a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, 2)$ (c) $(-\infty, 2] \cup [3, \infty)$ (d) $(3, \infty)$
Ans: (c) $(-\infty, 2] \cup [3, \infty)$
Given, $f(x) = 2x^3 - 15x^2 + 36x + 6$
$\Rightarrow f'(x) = 6x^2 - 30x + 36$
It $f'(x) \geq 0$, then f(x) is increasing.
So, $6x^2 - 30x + 36 \geq 0$



$\Rightarrow 6x^2 - 30x + 36 > 0$
$\Rightarrow x^2 - 5x + 6 > 0$
$\Rightarrow (x - 3)(x - 2) > 0$
$\Rightarrow x \in (-\infty, 2] \cup [3, \infty)$

4. A function $f : R \to R$ is defined as $f(x) = x^3 + 1$. Then the function has
(a) no minimum value (b) no maximum value
(c) both maximum and minimum values (d) neither maximum value nor minimum value
Ans: (d) neither maximum value nor minimum value

5. The point(s) on the curve $y = x^2$, at which y-coordinate is changing six times as fast as x-coordinate is/are
(*a*) (2, 4) (*b*) (3, 9) (*c*) (3, 9), (9, 3) (*d*) (6, 2)

Ans: (b) (3, 9)

as $\frac{dy}{dt} = 2x \cdot \frac{dx}{dt} \Rightarrow 6 \cdot \frac{dx}{dt} = 2x \cdot \frac{dx}{dt} \Rightarrow x = 3$

From curve, $y = 9$. Point is (3, 9).

6. The edge of a cube is increasing at the rate of 0.3 cm/s, the rate of change of its surface area when edge is 3 cm is

(a) 10.8 cm (b) 10.8 cm^2 (c) 10.8 cm^2/s (d) 10.8 cm/s

Ans: (c) 10.8 cm^2/s

as $\frac{dx}{dt} = 0.3$ cm/s, x is edge of a cube.

then $S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x \cdot \frac{dx}{dt} = 12x \times 0.3$

$\Rightarrow \frac{dS}{dt} = 3.6x$

and $\left.\frac{dS}{dt}\right]_{x=3} = 3.6 \times 3 = 10.8$ cm^2/s

7. The function $f(x) = 4 - 3x + 3x^2 - x^3$, $x \in R$ is

(a) decreasing function (b) increasing function

(c) strictly increasing on R (d) neither increasing nor decreasing on R

Ans: (a) decreasing function

$f(x) = 4 - 3x + 3x^2 - x^3$

$\Rightarrow f'(x) = -3 + 6x - 3x^2 = -3(x^2 - 2x + 1)$

$= -3(x - 1)^2 \leq 0$

$\therefore$ function is decreasing on R.

8. If at x = 1, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2]. Then the value of a is

(a) 124 (b) –124 (c) 120 (d) –120

Ans: (c) 120

as $f'(x) = 4x^3 - 124x + a$, for a point of maximum $f'(1) = 0 \Rightarrow 4 - 124 + a = 0 \Rightarrow a = 120$C

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The maximum value of the function $f(x) = x^5$, $x \in [-1, 1]$, is attained at its critical point, x = 0.

Reason (R): The maximum value of a function can only occur at points where derivative is zero.

Ans. (d) Assertion (A) is false but reason (R) is true.

Assertion (A) is false because the maximum value of the function $f(x) = x^5$ on the interval $x \in [-1, 1]$ is not attained at its critical point, x = 0.

To find the maximum value, we can evaluate the function at the endpoints of the interval:

f(–1) = (–1)5 = – 1

⇒ f(1) = (1)5 = 1

So, the maximum value of the function is 1, which occurs at x = 1 and not at the critical point x = 0.

10. **Assertion (A):** The function $f(x) = x^3 - 12x$ is strictly increasing in $(-\infty, -2) \cup (2, \infty)$.

Reason (R): For strictly increasing function $f'(x) > 0$.

Ans: (a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).

$f(x) = x^3 - 12x$

$\Rightarrow f(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x - 2)(x + 2)$

$\Rightarrow f'(x) = 0 \Rightarrow x = 2, -2$

For strictly increasing function $f'(x) > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow$ Assertion A is true Also Reason (R) is true (Definition of strictly increasing function)

Clearly R is correct explanation of A

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. For what values of x is the rate of increase of $x^3 - 5x^2 + 5x + 8$ is twice the rate of increase of x?

Ans: Let $y = x^3 - 5x^2 + 5x + 8$

Also $\frac{dy}{dt} = 2\frac{dx}{dt}$

$\Rightarrow 3x^2\frac{dx}{dt} - 10x\frac{dx}{dt} + 5\frac{dx}{dt} + 0 = 2\frac{dx}{dt}$

$\Rightarrow 3x^2 - 10x + 5 - 2 = 0$

$\Rightarrow 3x^2 - 10x + 3 = 0$

$\Rightarrow 3x^2 - 9x - x + 3 = 0$

$\Rightarrow 3x(x - 3) - 1(x - 3) = 0 \Rightarrow (3x - 1)(x - 3) = 0$

$\Rightarrow 3x - 1 = 0$ or $x - 3 = 0 \Rightarrow x = \frac{1}{3}, 3$

12. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Ans:

Consider $f(x) = \tan^{-1}(\sin x + \cos x)$

$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \cdot (\cos x - \sin x) \quad ...(i)$

Sign of $f'(x)$ depends upon $(\cos x - \sin x)$

We know for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ $\cos x < \sin x$

$\Rightarrow \cos x - \sin x < 0$

$\Rightarrow f'(x) < 0$ [from (i)]

$\therefore$ f is decreasing for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

13. A spherical balloon is being inflated by pumping in 16 cm^3/s of gas. At the instant when balloon contains 36π cm^3 of gas, how fast is its radius increasing?

Ans:

$\frac{dV}{dt} = 16 \text{ cm}^3/\text{s}$, given $V = \frac{4}{3}\pi r^3 = 36\pi$

$\Rightarrow r = 3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\Rightarrow \left.\frac{dr}{dt}\right]_{r=3} = \frac{16}{4\pi \times 9} = \frac{4}{9\pi} \text{ cm/s} = 0.14 \text{ cm/s}$

$\left[\because \frac{dV}{dt} = 16 \text{ cm}^3/\text{s}\right]$

14. Find the least value of a such that the function $f(x) = x^2 + ax + 1$ is strictly increasing on [1, 2].

Ans: $f(x) = x^2 + ax + 1$

Differentiating both sides w.r.t. x, we get

$f'(x) = 2x + a$,

$1 < x < 2 \Rightarrow 2 < 2x < 4$

$\Rightarrow 2 + a < 2x + a < 4 + a$

or $4 + a > 2x + a > 2 + a$

For increasing $f'(x) > 0$

$\Rightarrow$ for least value $2 + a = 0 \Rightarrow a = -2$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is (*i*) strictly increasing (*ii*) strictly decreasing

Ans: Consider $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Differentiating both sides, w.r.t. x, we get

$f'(x) = 12x^3 - 12x^2 - 24x$

$= 12x(x^2 - x - 2)$

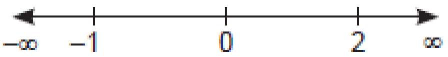
$= 12x(x - 2)(x + 1)$ ...(*i*)

For critical (stationary) points,

$f'(x) = 0$

$\Rightarrow 12x(x - 2)(x + 1) = 0$

$\Rightarrow x = 0, 2, -1$



From (*i*)

	$x < -1$	$-1 < x < 0$	$0 < x < 2$	$x > 2$
$12x$	–	–	+	+
$x - 2$	–	–	–	+
$x + 1$	–	+	+	+
$f'(x)$	–	+	–	+
	↓	↑	↓	↑

Hence, (*i*) function strictly increasing for $(-1, 0) \cup (2, \infty)$

(*ii*) function strictly decreasing for $(-\infty, -1) \cup (0, 2)$

16. Find the absolute maximum value and the absolute minimum value for the function $f(x) = 4x - \frac{1}{2}x^2$, in the given interval $x \in \left[-2, \frac{9}{2}\right]$.

Ans: $f(x) = 4x - \frac{1}{2}x^2$, $x \in \left[-2, \frac{9}{2}\right]$

$\Rightarrow f'(x) = 4 - x$

For a point of absolute maximum or minimum $f'(x) = 0$

$\Rightarrow 4 - x = 0 \Rightarrow x = 4$

$f(-2) = -8 - 2 = -10$

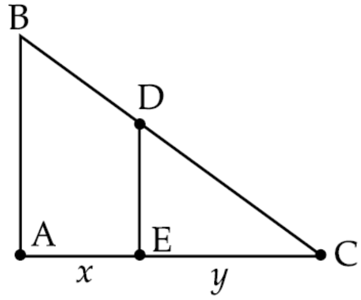
$f(4) = 16 - 8 = 8$

$f\left(\frac{9}{2}\right) = 18 - \frac{81}{8} = 7.87$

Absolute maximum value is 8 at $x = 4$ and absolute minimum value is –10 at $x = -2$.

17. A man 1.6 m tall walks at the rate of 0.5 m/s away from a lamp post, 8 metres high. Find the rate at which his shadow is increasing and the rate with which the tip of shadow is moving away from the pole.

Ans: Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height of 1.6 m be at a distance of x m from AB and the length of his shadow EC by y m.



Using similarity of triangles, we have $\frac{4}{1.6}=\frac{x+y}{y}\Rightarrow 3y=2x$

Differentiating both sides w.r.t. t we get,

$3\frac{dy}{dt}=2\frac{dx}{dt}\Rightarrow\frac{dy}{dt}=\frac{2}{3}\times\frac{dx}{dt}\Rightarrow\frac{dy}{dt}=0.2m/s$

At any time t seconds, the tip of his shadow is at a distance of (x + y) m from AB.

The rate at which the tip of his shadow moving = $\left(\frac{dx}{dt}+\frac{dy}{dt}\right)=0.5m/s$

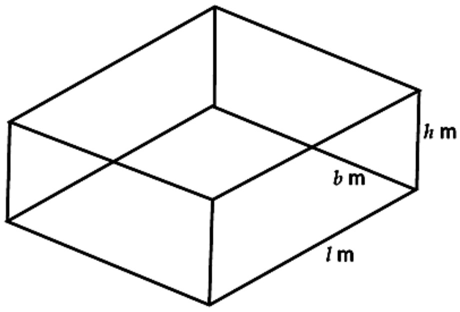
The rate at which his shadow is lengthening = $\frac{dy}{dt}=0.2m/s$

SECTION – D

Questions 18 carry 5 marks.

18. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank?

Ans: Let l, b and h metre be the length, breadth and height of the tank respectively.



Given h = 2m and volume of tank = $l \times b \times h$

$\Rightarrow 8 = l \times b \times 2 \Rightarrow lb = 4 \Rightarrow b = 4/l$

Now, area of the base, $lb = 4\ m^2$

and, area of four walls, A = 2 (l + b)× h $=2\left(l+\frac{4}{l}\right)\times 2\Rightarrow A=4\left(l+\frac{4}{l}\right)$

For minimum cost, $\frac{dA}{dl}=0\Rightarrow 4\left(1-\frac{4}{l^2}\right)=0\Rightarrow\frac{4}{l^2}=1\Rightarrow l^2=4$

∴ l = 2m (∵ length cannot be negative) and b = 4/l = 4/2 = 2 m

Now, $\frac{d^2A}{dl^2}=\frac{32}{l^2}=\frac{32}{4}=8>0$

∴ Area will be minimum when l = 2m, b = 2m, h = 2m

∴ Cost of building of tank = 70 × (l × b) = 70 × 2 ×2 = Rs. 280

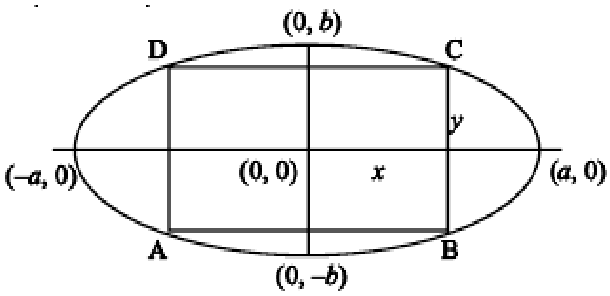
and cost of building the walls = 45 × 2h (l + b) = 90 × 2 (2 + 2) = Rs. 720

∴ Total cost for building the tank = 280 + 720 = Rs. 1,000

OR

Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Ans: Let ABCD be a rectangle having area A inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$…..(i)



The area A of the rectangle is 4xy i.e. A = 4xy which gives $A^2 = 16x^2y^2 = s$ (say)

Therefore, $s = 16x^2\left(1-\frac{x^2}{a^2}\right).b^2 = \frac{16b^2}{a^2}(a^2x^2 - x^4) \Rightarrow \frac{ds}{dx} = \frac{16b^2}{a^2}.[2a^2x - 4x^3]$.

Again, $\frac{ds}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}$ and $y = \frac{b}{\sqrt{2}}$

Now, $\frac{d^2s}{dx^2} = \frac{16b^2}{a^2}[2a^2 - 12x^2]$

At $x = \frac{a}{\sqrt{2}}$, $\frac{d^2s}{dx^2} = \frac{16b^2}{a^2}[2a^2 - 6a^2] = \frac{16b^2}{a^2}(-4a^2) < 0$

Thus at $x = \frac{a}{\sqrt{2}}, y = \frac{b}{\sqrt{2}}$, s is maximum and hence the area A is maximum.

Maximum area $= 4.x.y = 4.\frac{a}{\sqrt{2}}.\frac{b}{\sqrt{2}} = 2ab$ sq units.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1:

The use of electric vehicles will curb air pollution in the long run.

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V:

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.
Based on the above information, answer the following questions:
(a) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.
(b) Prove that the function V(t) is an increasing function.

Ans: (a) $V(t)=\frac{1}{5}t^3-\frac{5}{2}t^2+25t-2$

∵ For 2000, t = 0
∴ V(0) = 0 – 0 + 0 – 2
Since, number of vehicles cannot be negative.
Therefore, the given function cannot be used to estimate number of vehicles in the year 2000.

(b) $V(t)=\frac{1}{5}t^3-\frac{5}{2}t^2+25t-2$

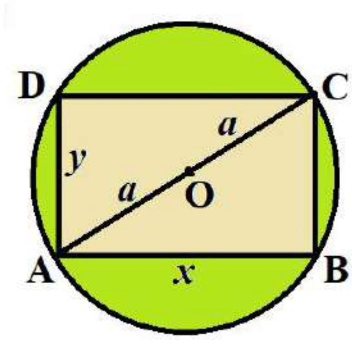
$$V'(t) = \frac{3}{5}t^2-5t+25 = \frac{3}{5}\left[t^2-\frac{25}{3}t+\frac{125}{3}\right]$$

$$= \frac{3}{5}\left[\left(t-\frac{25}{6}\right)^2-\frac{625}{36}+\frac{125}{3}\right] = \frac{3}{5}\left[\left(t-\frac{25}{6}\right)^2+\frac{875}{36}\right]$$

$V'(t) > 0$ for any value of t.
∴ The given function *V(t)* is an increasing function.

20. Case-Study 2:

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Find the perimeter of rectangle in terms of any one side and radius of circle.
(ii) Find critical points to maximize the perimeter of rectangle?
(iii) Check for maximum or minimum value of perimeter at critical point.

OR

(iii) If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region.

Ans: (i) Let 'y' be the breadth and 'x' be the length of rectangle and 'a' is radius of given circle.
From figure, $4a^2 = x^2 + y^2 \Rightarrow y^2 = 4a^2 - x^2$

$\Rightarrow y=\sqrt{4a^2-x^2}$

Perimeter (P) = 2x + 2y = $2\left(x+\sqrt{4a^2-x^2}\right)$

(ii) We know that P = 2x + 2y = $2\left(x+\sqrt{4a^2-x^2}\right)$

Critical points to maximize perimeter, $\frac{dP}{dx}=0$

$$\Rightarrow \frac{dP}{dx}=2\left[1+\frac{1}{2\sqrt{4a^2-x^2}}(-2x)\right]=0 \Rightarrow 2\left(\frac{\sqrt{4a^2-x^2}-x}{\sqrt{4a^2-x^2}}\right)=0 \Rightarrow \sqrt{4a^2-x^2}-x=0$$

$$\Rightarrow \sqrt{4a^2 - x^2} = x \Rightarrow 4a^2 - x^2 = x^2 \Rightarrow 2x^2 = 4a^2 \Rightarrow x^2 = 2a^2 \Rightarrow x = \pm\sqrt{2}a$$

But $x = -\sqrt{2}a$ is not possible as length is never negative

$\therefore x = \sqrt{2}a \Rightarrow y = \sqrt{2}a$

Hence, critical point is $(\sqrt{2}a, \sqrt{2}a)$

(iii) We have, $\frac{dP}{dx} = 2\left[1 - \frac{x}{\sqrt{4a^2 - x^2}}\right]$

$$\Rightarrow \frac{d^2P}{dx^2} = -2\left[\frac{\sqrt{4a^2 - x^2} - (x)\frac{-2x}{2\sqrt{4a^2 - x^2}}}{\sqrt{4a^2 - x^2}}\right] = -2\left[\frac{\sqrt{4a^2 - x^2} + \frac{x^2}{\sqrt{4a^2 - x^2}}}{\sqrt{4a^2 - x^2}}\right]$$

$$\Rightarrow \frac{d^2P}{dx^2} = -2\left[\frac{4a^2 - x^2 + x^2}{(4a^2 - x^2)^{3/2}}\right] = -2\left[\frac{4a^2}{(4a^2 - x^2)^{3/2}}\right]$$

$$\Rightarrow \left[\frac{d^2P}{dx^2}\right]_{x = a\sqrt{2}} = -2\left[\frac{4a^2}{(4a^2 - 2a^2)^{3/2}}\right] = \frac{-2}{2\sqrt{2}a} < 0$$

Hence, Perimeter is maximum at a critical point.

OR

From the above results know that x = y = $\sqrt{2}a$

a = radius

Here, x = y = $10\sqrt{2}$

Perimeter = P = 4 x side = $40\sqrt{2}$ cm