PRACTICE PAPER 04
CHAPTER 04 DETERMINANTS

SUBJECT: MATHEMATICS — **MAX. MARKS : 40**
CLASS : XII — **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. If A and B are invertible square matrices of the same order, then which of the following is not correct?
(a) adj $A = |A|. A^{-1}$ (b) det $(A)^{-1} = [\det (A)]^{-1}$
(c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A + B)^{-1} = B^{-1} + A^{-1}$

2. If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is:
(a) 1 (b) –1 (c) 2 (d) 0

3. If A is a square matrix of order 3 and |A| = 5, then |adj A| =
(a) 5 (b) 25 (c) 125 (d) 1/5

4. If A is a square matrix of order 3, |A'| = – 3, then |AA'| =
(a) 9 (b) –9 (c) 3 (d) – 3

5. If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1}$ =
(a) $A^{-1}B$ (b) $A^{-1}B^{-1}$ (c) BA^{-1} (d) AB

6. If A = $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix}$ and $|A|^3 = 125$, then a is
(a) ±3 (b) 5 (c) ±2 (d) 4

7. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) or 'x' is/are
(a) 3 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\sqrt{3}, -\sqrt{3}$

8. Three points P(2x, x + 3), Q(0, x) and R(x + 3, x + 6) are collinear, then x is equal to:
(a) 0 (b) 2 (c) 3 (d) 1

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The system of equations $2x - y = -2$; $3x + 4y = 3$ has unique solution and $x = -5/11$ and $y = 12/11$.
Reason (R): The system of equations AX = B has a unique solution, if $|A| \neq 0$.

10. **Assertion (A):** Minor of an element of a determinant of order n ($n \geq 2$) is a determinant of order n.
Reason (R): If A is an invertible matrix of order 2, then det (A^{-1}) is equal to $1/|A|$.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If area of triangle is 35 sq units with vertices (2, – 6), (5, 4) and (*k*, 4). Then find the value of *k*.

12. Let $A = \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. If AB = C, then find A^2.

13. Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

14. If points (2, 0), (0, 5) and (*x*, *y*) are collinear, then show that $\frac{x}{2} + \frac{y}{5} = 1$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

16. Show that $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $x^2 + 4x - 42 = O$. Hence find A^{-1}.

17. If A, B are square matrices of the same order, then prove that adj (AB) = (adj B) (adj A).

SECTION – D

Questions 18 carry 5 marks.

18. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

OR

Solve the system of equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \text{ and}$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹160 from Stationary Shop. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹190. Also, Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹250.

Based on the above information, answer the following questions:

(i) Convert the given above situation into a matrix equation of the form AX = B.

(ii) Find |A|.

(iii) Find A^{-1}.

OR

Determine $P = A^2 - 5A$.

20. The Palace of Peace and Reconciliation, also known as the Pyramid of peace and Accord is a 62-metre high Pyramid in Mursultan, the capital of Kazakhstan, that serves as a non-denominational national spiritual centre and an event venue. It is designed by faster and partners with a stained glass apex. It has 25 smaller equilateral triangles as shown in the figure.

(i) If the vertices of one triangle are (0, 0), $(3, \sqrt{3})$ and $(3, -\sqrt{3})$ then find the area.

(ii) Find the area of face of the Pyramid.

(iii) Find the length of a altitude of a smaller equilateral triangle.

OR

Using determinants, find the equation of the line joining the points A (1, 2) and B (3, 6).

PRACTICE PAPER 04 CHAPTER 04 DETERMINANTS
(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XII **DURATION : 1½ hrs**

General Instructions:
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SECTION – A
Questions 1 to 10 carry 1 mark each.

1. If A and B are invertible square matrices of the same order, then which of the following is not correct?
(a) adj $A = |A|.\ A^{-1}$ (b) det $(A)^{-1} = [\det (A)]^{-1}$
(c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A + B)^{-1} = B^{-1} + A^{-1}$
Ans: (d) $(A + B)^{-1} = B^{-1} + A^{-1}$

2. If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is:
(a) 1 (b) –1 (c) 2 (d) 0
Ans: (d) 0
$|A| = |kA|$
$|A| = k^n|A|$ where n is the order of matrix
$\Rightarrow 1 = k^n$
$\Rightarrow k^2 = 1$
$\Rightarrow k = \pm 1$
Sum of all values of $k = +1 - 1 = 0$

3. If A is a square matrix of order 3 and |A| = 5, then |adj A| =
(a) 5 (b) 25 (c) 125 (d) 1/5
Ans: (b) 25
$|adj\ A| = |A|^{n-1}$
$\Rightarrow\ |adj\ A| = 25$

4. If A is a square matrix of order 3, |A'| = – 3, then |AA'| =
(a) 9 (b) –9 (c) 3 (d) – 3
Ans: (a) 9
$|AA'| = |A||A'| = (-3)(-3) = 9$

5. If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1}$ =
(a) $A^{-1}B$ (b) $A^{-1}B^{-1}$ (c) BA^{-1} (d) AB
Ans: (c) BA^{-1}
$(AB^{-1})^{-1} = (B^{-1})^{-1} A^{-1} = BA^{-1}$

6. If A = $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix}$ and $|A|^3 = 125$, then a is

(a) ±3 (b) 5 (c) ±2 (d) 4

Ans: (a) ±3

$|A|^3 = 125 \Rightarrow |A| = 5$

$\therefore \quad \begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} = 5$

$\Rightarrow \; a^2 - 4 = 5$

$\Rightarrow \; a^2 = 9 \Rightarrow a = \pm 3.$

7. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) or 'x' is/are

(a) 3 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\sqrt{3}, -\sqrt{3}$

Ans: (d) $\sqrt{3}, -\sqrt{3}$

$2 - 20 = 2x^2 - 24$

$\Rightarrow \; 2x^2 = 6 \quad \Rightarrow \; x^2 = 3 \quad \Rightarrow \; x = \pm\sqrt{3}$

8. Three points P(2x, x + 3), Q(0, x) and R(x + 3, x + 6) are collinear, then x is equal to:

(a) 0 (b) 2 (c) 3 (d) 1

Ans: (d) 1

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

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(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): The system of equations 2x – y = – 2; 3x + 4y = 3 has unique solution and x = −5/11 and y = 12/11.

Reason (R): The system of equations AX = B has a unique solution, if |A| ≠ 0.

Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. Assertion (A): Minor of an element of a determinant of order n (n ≥ 2) is a determinant of order n.

Reason (R): If A is an invertible matrix of order 2, then det (A^{-1}) is equal to 1/| A |.

Ans: (d) Assertion (A) is false but reason (R) is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If area of triangle is 35 sq units with vertices (2, – 6), (5, 4) and (k, 4). Then find the value of k.

Ans: We have Area of triangle $= \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = 35$

$\Rightarrow |2\,(4 - 4) + 6(5 - k) + 1(20 - 4k)| = 70$

$\Rightarrow 2\,(4 - 4) + 6\,(5 - k) + 1(20 - 4k) = \pm\, 70$

$\Rightarrow 30 - 6k + 20 - 4k = \pm\, 70$

On taking positive sign, $-\,10k + 50 = 70$

$\Rightarrow -\,10k = 20 \Rightarrow \; k = -\,2$

On taking negative sign, $-\,10k + 50 = -\,70$

$\Rightarrow -\,10k = -\,120 \Rightarrow \; k = 12$

$\therefore\ k = 12, -2$

12. Let A = $\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$, B = $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and C = $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$. If AB = C, then find A^2.

Ans:

Here, $\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \Rightarrow \quad \begin{matrix} 2x+y=3 \\ 3x+y=2 \end{matrix}$

On solving above equations, we get $x = -1$ and $y = 5$.

$\therefore\ A = \begin{bmatrix} -1+5 & 5 \\ 2(-1) & -1-5 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$

Thus, $A^2 = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix} = \begin{bmatrix} 6 & -10 \\ 4 & -26 \end{bmatrix}$

13. Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Ans: Given that $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Cofactors of the elements of second row

$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9-16) = 7$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = (15-8) = 7$

and $A_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10-3) = -7$

Now, expansion of Δ using cofactors of elements of second row is given by

$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 2 \times 7 + 0 \times 7 + 1(-7) = 14 - 7 = 7$

14. If points (2, 0), (0, 5) and (x, y) are collinear, then show that $\frac{x}{2} + \frac{y}{5} = 1$.

Ans: Since the given points are collinear then $\frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ 0 & 5 & 1 \\ x & y & 1 \end{vmatrix} = 0$

$\Rightarrow 2(5 - y) + 1(-5x) = 0$

$\Rightarrow 5x + 2y = 10 \Rightarrow \frac{x}{2} + \frac{y}{5} = 1$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

Ans: Given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Now, $A^2 - 5A + 7I = O$

$$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-2 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$\therefore$ $A^2 - 5A + 7I = O$

$$\because |A| = \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} = 6 + 1 = 7 \neq 0$$

$\therefore$ A^{-1} exists.

Now, $A.A - 5A = -7I$

Multiplying by A^{-1} on both sides, we get

$A.A(A^{-1}) - 5A(A^{-1}) = -7I(A^{-1})$

$\Rightarrow AI - 5I = -7A^{-1}$ (using $AA^{-1} = I$ and $IA^{-1} = A^{-1}$)

$$A^{-1} = -\frac{1}{7}(A - 5I) = \frac{1}{7}(5I - A) = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right) = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

16. Show that $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $x^2 + 4x - 42 = O$. Hence find A^{-1}.

Ans: $A^2 + 4A - 42I =$

$$\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + 4\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} - 42\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 64+10 & -40+20 \\ -16+8 & 10+16 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix}$$

$$= \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix}$$

$$= \begin{bmatrix} 74-32-42 & -20+20-0 \\ -8+8-0 & 26+16-42 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A^2 + 4A - 42I = O$

$\Rightarrow A^{-1}(A^2) + 4A^{-1}A - 42A^{-1}I = A^{-1}O$ (Pre-multiplying by A^{-1})

$\Rightarrow (A^{-1}A)A + 4I - 42A^{-1} = O \Rightarrow 42A^{-1} = IA + 4I$

$\Rightarrow 42A^{-1} = A + 4I$

$\Rightarrow A^{-1} =$

$$\frac{1}{42}\left\{\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right\}$$

$\Rightarrow A^{-1} =$

$$\frac{1}{42}\begin{bmatrix} -8+4 & 5+0 \\ 2+0 & 4+4 \end{bmatrix} = \frac{1}{42}\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$$

17. If A, B are square matrices of the same order, then prove that adj (AB) = (adj B) (adj A).

Ans: We know that (AB)*adj* (AB) =|AB|= *adj* (AB)(AB)

(AB)(*adj* B *adj* A) = A · B *adj* B · *adj* · A

= A (B *adj* B) *adj* A = A(|B|I) *adj* A [Q B *adj* B = |B|I]

=|B|(A · *adj* A) =|B||A|I

=|A||B|I =|AB|I

From (i) and (ii), we get

AB(*adj* AB) =AB(*adj* B *adj* A)

On multiplying both sides by $(AB)^{-1}$, we get

$(AB)^{-1}$[(AB) *adj* AB] = $(AB)^{-1}$ [(AB) *adj* B · *adj* A]

⇒ *adj* AB = *adj* B · *adj* A

SECTION – D

Questions 18 carry 5 marks.

18. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

Ans: Let the prices (per kg) of onion, wheat and rice be Rs. x, Rs. y and Rs. z, respectively then

$4x + 3y + 2z = 60$, $2x + 4y + 6z = 90$, $6x + 2y + 3z = 70$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ and \ B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 4(12 - 12) - 3(6 - 36) + 2(4 - 24)$$

$= 0 + 90 - 40 = 50 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists. Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are,

$A_{11} = 12 - 12 = 0,$

$A_{12} = -(6 - 36) = 30,$

$A_{13} = 4 - 24 = -20,$

$A_{21} = -(9 - 4) = -5,$

$A_{22} = 12 - 12 = 0,$

$A_{23} = -(8 - 18) = 10,$

$A_{31} = (18 - 8) = 10,$

$A_{32} = -(24 - 4) = -20,$

$A_{33} = 16 - 6 = 10$

$$adj(A) = \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}^T = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0-450+700 \\ 1800+0-1400 \\ -1200+900+700 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore$ $x = 5$, $y = 8$ and $z = 8$.

Hence, price of onion per kg is Rs. 5, price of wheat per kg is Rs. 8 and that of rice per kg is Rs. 8.

OR

Solve the system of equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Ans: Let $\frac{1}{x} = p$, $\frac{1}{y} = q$ *and* $\frac{1}{z} = r$, then the given equations become

$2p + 3q + 10r = 4$, $4p - 6q + 5r = 1$, $6p + 9q - 20r = 2$

This system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 2(120-45) - 3(-80-30) + 10(36+36)$$

$= 150 + 330 + 720 = 1200 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Therefore, the above system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are

$A_{11} = 120 - 45 = 75$,

$A_{12} = -(-80 - 30) = 110$,

$A_{13} = (36 + 36) = 72$,

$A_{21} = -(-60 - 90) = 150$,

$A_{22} = (-40 - 60) = -100$,

$A_{23} = -(18 - 18) = 0$,

$A_{31} = 15 + 60 = 75$,

$A_{32} = -(10 - 40) = 30$,

$A_{33} = -12 - 12 = -24$

$$adj(A) = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\Rightarrow p = \frac{1}{2}, q = \frac{1}{3}, r = \frac{1}{5}$

$\Rightarrow \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$

$\Rightarrow x = 2, y = 3$ and $z = 5$.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ` 160. From the same shop. Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ` 190. Also, Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ` 250.

Based on the above information, answer the following questions:

(i) Convert the given above situation into a matrix equation of the form AX = B.

(ii) Find |A|.

(iii) Find A^{-1}.

OR

Determine $P = A^2 - 5A$.

Ans:

	Pen	*Bags*	*Instrument*
Gautam	5	3	1
Vikram	2	1	3
Ankur	1	2	4

(i) $\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$

where x = cost of Pen

y = cost of Bag

z = cost of Instrument

(ii) $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = 5(4-6) - 3(8-3) + 1(4-1)$$
$$= -10 - 15 + 3$$
$$= -22$$

(iii) $C_{11} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = -2,\ C_{12} = -\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = -5,\ C_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3,$

$$C_{21} = -\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = -10,\ C_{22} = \begin{vmatrix} 5 & 1 \\ 1 & 4 \end{vmatrix} = 19,\ C_{23} = -\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -7,$$

$$C_{31} = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 8,\ C_{32} = -\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} = -13,\ C_{33} = \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} = -1,$$

$$\text{Adj } A = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{Adj\ A}{|A|} = -\frac{1}{22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix} = \frac{1}{22}\begin{bmatrix} 2 & 10 & -8 \\ 5 & -19 & 13 \\ -3 & 7 & 1 \end{bmatrix}$$

OR

$P = A^2 - 5A$

$$= \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} - 5\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 25+6+1 & 15+3+2 & 5+9+4 \\ 10+2+3 & 6+1+6 & 2+3+12 \\ 5+4+4 & 3+2+8 & 1+6+16 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

20. The Palace of Peace and Reconciliation, also known as the Pyramid of peace and Accord is a 62-metre high Pyramid in Mursultan, the capital of Kazakhstan, that serves as a non-denominational national spiritual centre and an event venue. It is designed by faster and partners with a stained glass apex. It has 25 smaller equilateral triangles as shown in the figure.

(i) If the vertices of one triangle are (0, 0), $(3,\sqrt{3})$ and $(3,-\sqrt{3})$ then find the area.
(ii) Find the area of face of the Pyramid.
(iii) Find the length of a altitude of a smaller equilateral triangle.

OR

Using determinants, find the equation of the line joining the points A (1, 2) and B (3, 6).

Ans: (i) Required Area

$$= \left|\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & \sqrt{3} & 1 \\ 3 & -\sqrt{3} & 1 \end{vmatrix}\right|$$

$$= \left|\frac{1}{2}\left[1\left(-3\sqrt{3} - 3\sqrt{3}\right)\right]\right| = \frac{6\sqrt{3}}{2} = 3\sqrt{3} \text{ sq units}$$

(ii) Since, a face of the Pyramid consists of 25 smaller equilateral triangles.

∴ Area of a face of the Pyramid = $25 \times 3\sqrt{3} = 75\sqrt{3}$ sq. units

(iii) Area of equilateral triangle = $\frac{\sqrt{3}}{4}$ $(\text{side})^2$

$\therefore 3\sqrt{3} = \frac{\sqrt{3}}{4}(\text{side})^2$

[As calculated above area of equilateral triangle is $2\sqrt{3}$ sq. units]

$\Rightarrow (\text{side})^2 = 12$

$\Rightarrow$ side = $2\sqrt{3}$ units

Let h be the length of the altitude of a smaller equilateral triangle.

Then, $\frac{1}{2}$ × base × height = $3\sqrt{3}$

$\Rightarrow \frac{1}{2}$ × side × height = $3\sqrt{3}$

$\Rightarrow$ height = $= \frac{3\sqrt{3} \times 2}{2\sqrt{3}}$ = 3 units

OR

Let the third point on the line be (x, y).

The area of triangle with vertices (x, y), (1, 2), (3, 6) $= \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix}$

Since the three points are collinear, the area formed will be zero.

$$\Rightarrow \begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

$\Rightarrow x(2 - 6) - y(1 - 3) + 1(6 - 6) = 0$

$\Rightarrow -4x + 2y = 0$

$\Rightarrow 2x - y = 0$

Hence, the equation of line joining (1, 2) and (3, 6) is $2x - y = 0$.