PRACTICE PAPER 02
CHAPTER 02 INVERSE TRIGONOMETRIC FUNCTIONS

SUBJECT: MATHEMATICS — **MAX. MARKS : 40**
CLASS : XII — **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

2. Principal value of the expression $\cos^{-1}[\cos(-680°)]$ is

(a) $\frac{2\pi}{9}$ (b) $-\frac{2\pi}{9}$ (c) $\frac{34\pi}{9}$ (d) $\frac{\pi}{9}$

3. If $\sec^{-1}x + \sec^{-1}y = \frac{\pi}{2}$, the value of $\operatorname{cosec}^{-1}x + \operatorname{cosec}^{-1}y$ is

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) $-\pi$

4. The domain of $y = \cos^{-1}(x^2 - 4)$ is

(*a*) [3, 5] (*b*) [0, π]
(*c*) $[-\sqrt{5}, -\sqrt{3}] \cap [-\sqrt{5}, \sqrt{3}]$ (*d*) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

5. The value of $\cos^{-1}\left(\frac{1}{2}\right) + 3\sin^{-1}\left(\frac{1}{2}\right)$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

6. $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$ is equal to

(a) –1 (b) 1 (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{4}$

7. The domain of the function $y = \sin^{-1}(-x^2)$ is

(*a*) [0, 1] (*b*) (0, 1) (*c*) [–1, 1] (*d*) ϕ

8. The value of $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is

(a) $-\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** Domain of y = cos–1 (x) is [–1, 1].

Reason (R): The range of the principal value branch of y = $\cos^{-1}$ (x) is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

10. **Assertion (A):** All trigonometric functions have their inverses over their respective domains.
Reason (R): The inverse of $\tan^{-1}$x exists for some x ∈ R

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Write the domain and range (principle value branch) of the following functions: f (x) = $\tan^{-1}$ x

OR

Draw the graph of $\cos^{-1}$ x, where x ∈ [–1, 0]. Also write its range.

12. Find value of k if $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}$

13. Simplify: $\cos^{-1}x+\cos^{-1}x\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right],\frac{1}{2}\le x\le 1$

14. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(1)$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Prove that $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

16. If a = $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)$ and b = $\tan^{-1}(\sqrt{3})-\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ then find the value of a + b.

17. Prove that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$

SECTION – D

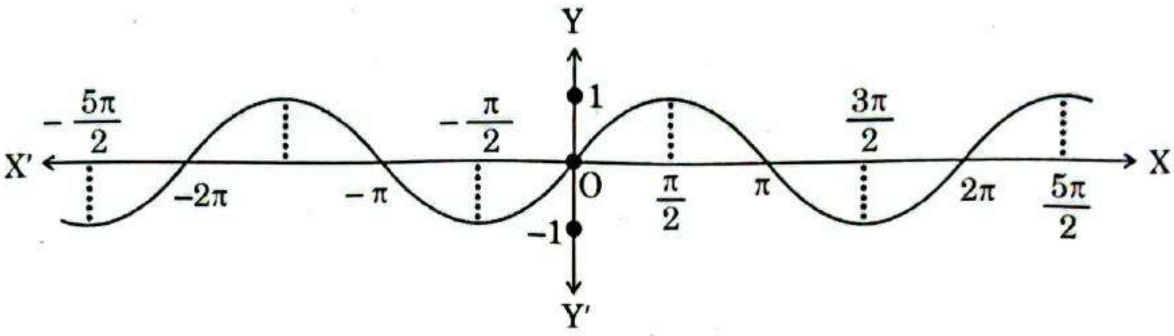
Questions 18 carry 5 marks.

18. Prove that $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. If a function f : X → Y defined as f (x) = y is one-one and onto, then we can define a unique function g : Y → X such that g(y) = x , where x ∈ X and y = f (x), y ∈ Y . Function g is called the inverse of f.
The domain of sine function is R and function sine: R → R is neither one-one nor onto. The following graph shows the sine function.



y = sin x

Let sine function be defined from set A to [–1, 1] such that inverse of sine function exists, i.e., $\sin^{-1}x$ is defined from [–1, 1] to A.

On the basis of above information, answer the following questions.

(i) If A is the interval other than principal value branch, give an example of one such interval. (1)

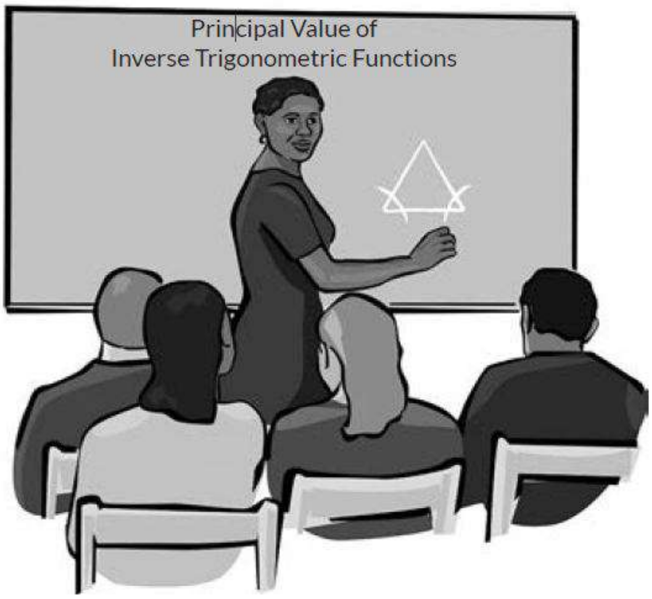
(ii) If $\sin^{-1}x$ is defined from [–1, 1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right)-\sin^{-1}(1)$ (1)

(iii) Draw the graph of $\sin^{-1}x$ from [–1, 1] to its principal value branch. (2)

OR

(iii) Find the domain and range of $f(x) = 2\sin^{-1}(1-x)$. (2)

20. A math teacher explained to the students about topic "Principal Value of Inverse Trigonometric Functions". He told that the value of an inverse trigonometric functions which lies in the range of principal branch is called the principal value of that inverse.



Based on the given information, answer the following questions:

(i) Evaluate : $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ (1)

(ii) Find the principal values of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ (1)

(iii) Prove that $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in(0,1)$ (2)

OR

(iii) Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]+\tan^{-1}1$

PRACTICE PAPER 02

CHAPTER 02 INVERSE TRIGONOMETRIC FUNCTIONS

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : XII **DURATION : 1½ hrs**

General Instructions:

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(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Ans: (a) 1

$$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\sin\frac{\pi}{2}=1$$

2. Principal value of the expression $\cos^{-1}[\cos(-680°)]$ is

(a) $\frac{2\pi}{9}$ (b) $-\frac{2\pi}{9}$ (c) $\frac{34\pi}{9}$ (d) $\frac{\pi}{9}$

Ans: (a) $\frac{2\pi}{9}$

$\cos(-680°) = \cos 680° = \cos(720° - 40°) = \cos 40°$

$\therefore \cos^{-1}[\cos(-680°)] = \cos^{-1}(\cos 40°) = 40° = \frac{2\pi}{9}$

3. If $\sec^{-1}x + \sec^{-1}y = \frac{\pi}{2}$, the value of $\text{cosec}^{-1}x + \text{cosec}^{-1}y$ is

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) $-\pi$

Ans: (b) $\frac{\pi}{2}$

$\sec^{-1}x + \sec^{-1}y = \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} - \text{cosec}^{-1}x + \frac{\pi}{2} - \text{cosec}^{-1}y = \frac{\pi}{2}$

$\Rightarrow \text{cosec}^{-1}x + \text{cosec}^{-1}y = \frac{\pi}{2}$.

4. The domain of $y = \cos^{-1}(x^2 - 4)$ is

(*a*) [3, 5] (*b*) [0, π]

(*c*) $[-\sqrt{5}, -\sqrt{3}] \cap [-\sqrt{5}, \sqrt{3}]$ (*d*) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Ans: (*d*), as $y = \cos^{-1}(x^2 - 4) \Rightarrow \cos y = x^2 - 4$
Since $-1 \le \cos y \le 1$
i.e. $-1 \le x^2 - 4 \le 1 \Rightarrow 3 \le x^2 \le 5$
$\Rightarrow \sqrt{3} \le |x| \le \sqrt{5}$
$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

5. The value of $\cos^{-1}\left(\frac{1}{2}\right) + 3\sin^{-1}\left(\frac{1}{2}\right)$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

Ans: (d) $\frac{5\pi}{6}$

$\cos^{-1}\left(\frac{1}{2}\right) + 3\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 3 \times \frac{\pi}{6} = \frac{5\pi}{6}$

6. $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$ is equal to

(a) –1 (b) 1 (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{4}$

Ans: (d) $-\frac{\pi}{4}$

$\sin\left(-\frac{\pi}{2}\right) = -1$, and $\tan^{-1}(-1) = -\frac{\pi}{4}$

7. The domain of the function $y = \sin^{-1}(-x^2)$ is
(*a*) [0, 1] (*b*) (0, 1) (*c*) [–1, 1] (*d*) ϕ
Ans: (*c*) [–1, 1]
$-1 \le -x^2 \le 1 \Rightarrow 1 \ge x^2 \ge -1 \Rightarrow 0 \le x^2 \le 1 \Rightarrow |x| \le 1 \Rightarrow -1 \le x \le 1.$

8. The value of $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is

(a) $-\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$

Ans: (b) $-\frac{\pi}{10}$

We have, $\sin^{-1}\left(\cos\frac{13\pi}{5}\right) = \sin^{-1}\left[\cos\left(2\pi + \frac{3\pi}{5}\right)\right]$

$= \sin^{-1}\left[\cos\frac{3\pi}{5}\right] = \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$

$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right) = -\sin^{-1}\left(\sin\frac{\pi}{10}\right) = -\frac{\pi}{10}$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** Domain of y = cos–1 (x) is [–1, 1].

Reason (R): The range of the principal value branch of y = $\cos^{-1}$ (x) is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

Ans: (c) A is true but R is false.
Range of $\cos^{-1}$ x is [0, π]

10. **Assertion (A):** All trigonometric functions have their inverses over their respective domains.
Reason (R): The inverse of $\tan^{-1}$x exists for some x ∈ R
Ans: (d) A is false but R is true.
All trigonometric functions are periodic and hence not invertible over their respective domains but all trigonometric functions have inverse over their restricted domains.

Inverse of $\tan^{-1}$x is tanx which is defined for $x\in R-(2n+1)\frac{\pi}{2}, n\in Z$

Hence, Assertion is false and reason is true.

SECTION – B

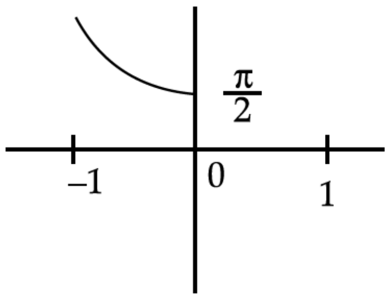
Questions 11 to 14 carry 2 marks each.

11. Write the domain and range (principle value branch) of the following functions: f (x) = $\tan^{-1}$ x
Ans: f(x) = $\tan^{-1}$x
Domain = Real number

Range = $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

OR

Draw the graph of $\cos^{-1}$ x, where x ∈ [–1, 0]. Also write its range.
Ans: Range of $\cos^{-1}$ x is [0, π]



12. Find value of k if $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}$

Ans: Given that $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}$

$\Rightarrow k\tan\left(2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right)=\sin\frac{\pi}{3}$

$\Rightarrow k\tan\frac{\pi}{3}=\frac{\sqrt{3}}{2}\Rightarrow k\sqrt{3}=\frac{\sqrt{3}}{2}\Rightarrow k=\frac{1}{2}$

13. Simplify: $\cos^{-1}x+\cos^{-1}x\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right], \frac{1}{2}\le x\le 1$

Ans: Let $\cos^{-1}x=\alpha\Rightarrow x=\cos\alpha$

$$\cos^{-1}x+\cos^{-1}x\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right],\frac{1}{2}\le x\le 1$$

$$=\alpha+\cos^{-1}\left[\frac{\cos\alpha}{2}+\frac{\sqrt{3-3\cos^2\alpha}}{2}\right]$$

$$=\alpha+\cos^{-1}\left[\frac{\cos\alpha}{2}+\frac{\sqrt{3}.\sqrt{1-\cos^2\alpha}}{2}\right]=\alpha+\cos^{-1}\left[\frac{\cos\alpha}{2}+\frac{\sqrt{3}}{2}\sin\alpha\right]$$

$$=\alpha+\cos^{-1}\left[\cos\frac{\pi}{3}\cos\alpha+\sin\frac{\pi}{3}\sin\alpha\right]=\alpha+\cos^{-1}\left[\cos\left(\frac{\pi}{3}-\alpha\right)\right]=\frac{\pi}{3}$$

14. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(1)$

Ans:

$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(1)$$

$$=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{4}\right)\right)+\pi+\tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$=\sin^{-1}\sin\left(\frac{\pi}{4}\right)+\pi+\frac{\pi}{4}=\frac{\pi}{4}+\pi+\frac{\pi}{4}=\frac{3\pi}{2}$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Prove that $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

Ans:

$$LHS=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)=\frac{9}{4}\cos^{-1}\frac{1}{3}$$

$$Let\ \cos^{-1}\frac{1}{3}=x\Rightarrow\cos x=\frac{1}{3}\Rightarrow\sin x=\sqrt{1-\cos^2 x}$$

$$\Rightarrow\sin x=\sqrt{1-\left(\frac{1}{3}\right)^2}=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$$

$$\Rightarrow x=\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\Rightarrow\cos^{-1}\frac{1}{3}=\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\therefore\frac{9}{4}\cos^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)=RHS$$

16. If a = $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)$ and b = $\tan^{-1}(\sqrt{3})-\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ then find the value of a + b.

Ans: a = $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)$

$$=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\cos^{-1}\left(-\frac{1}{2}\right)=\frac{\pi}{4}+\frac{2\pi}{3}=\frac{11\pi}{12}$$

$$b = \tan^{-1}(\sqrt{3}) - \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3} - \frac{2\pi}{3} = \frac{-\pi}{3}$$

Now a + b $= \frac{11\pi}{12} - \frac{\pi}{3} = \frac{11\pi - 4\pi}{12} = \frac{7\pi}{12}$

17. Prove that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

Ans:

$$Let \ \sin^{-1}\frac{3}{4} = \alpha \Rightarrow \sin\alpha = \frac{3}{4}$$

$$\Rightarrow \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{3}{4} \qquad \left[\because \sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}\right]$$

$$\Rightarrow 3 + 3\tan^2\frac{\alpha}{2} = 8\tan\frac{\alpha}{2} \Rightarrow 3\tan^2\frac{\alpha}{2} - 8\tan\frac{\alpha}{2} + 3 = 0$$

$$\Rightarrow \tan\frac{\alpha}{2} = \frac{8 \pm \sqrt{64-36}}{6} = \frac{8 \pm \sqrt{28}}{6}$$

$$\Rightarrow \tan\frac{\alpha}{2} = \frac{8 \pm 2\sqrt{7}}{6} \Rightarrow \tan\frac{\alpha}{2} = \frac{4 \pm \sqrt{7}}{3}$$

$$\Rightarrow \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$$

SECTION – D

Questions 18 carry 5 marks.

18. Prove that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Ans:

$$LHS = \left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7}\right) + \left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}.\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}.\frac{1}{8}}\right) \qquad \left(\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right)$$

$$= \tan^{-1}\left(\frac{\frac{7+5}{35}}{1-\frac{1}{35}}\right) + \tan^{-1}\left(\frac{\frac{8+3}{24}}{1-\frac{1}{24}}\right) = \tan^{-1}\left(\frac{\frac{12}{35}}{\frac{30}{35}}\right) + \tan^{-1}\left(\frac{\frac{11}{24}}{\frac{23}{24}}\right)$$

$$= \tan^{-1}\left(\frac{12}{34}\right) + \tan^{-1}\left(\frac{11}{23}\right) = \tan^{-1}\left(\frac{6}{17}\right) + \tan^{-1}\left(\frac{11}{23}\right)$$

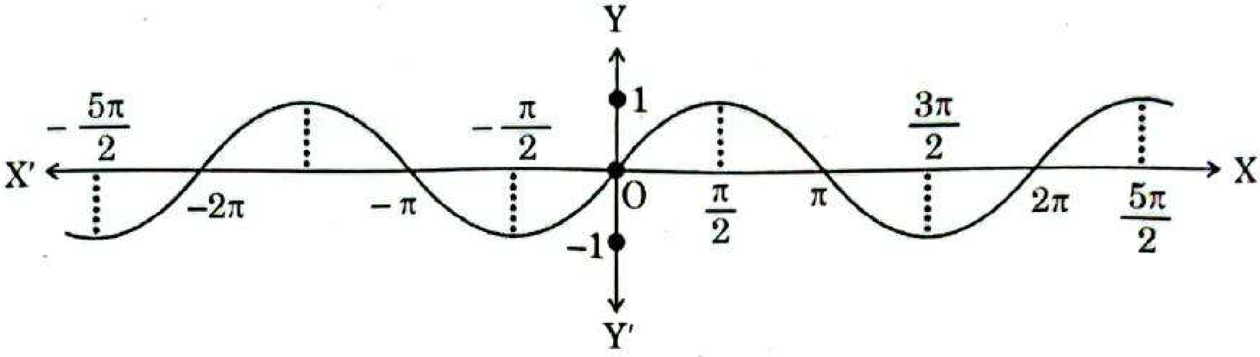
$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}.\frac{11}{23}}\right) = \tan^{-1}\left(\frac{\frac{138+187}{391}}{1-\frac{66}{391}}\right) = \tan^{-1}\left(\frac{\frac{325}{391}}{\frac{325}{391}}\right) = \tan^{-1}(1) = \frac{\pi}{4} = RHS$$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. If a function $f : X \rightarrow Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \rightarrow X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of f.

The domain of sine function is R and function sine: $R \rightarrow R$ is neither one-one nor onto. The following graph shows the sine function.



y = sin x

Let sine function be defined from set A to [–1, 1] such that inverse of sine function exists, i.e., $\sin^{-1}x$ is defined from [–1, 1] to A.

On the basis of above information, answer the following questions.

(i) If A is the interval other than principal value branch, give an example of one such interval.

(ii) If $\sin^{-1}x$ is defined from [–1, 1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right)-\sin^{-1}(1)$

(iii) Draw the graph of $\sin^{-1}x$ from [–1, 1] to its principal value branch.

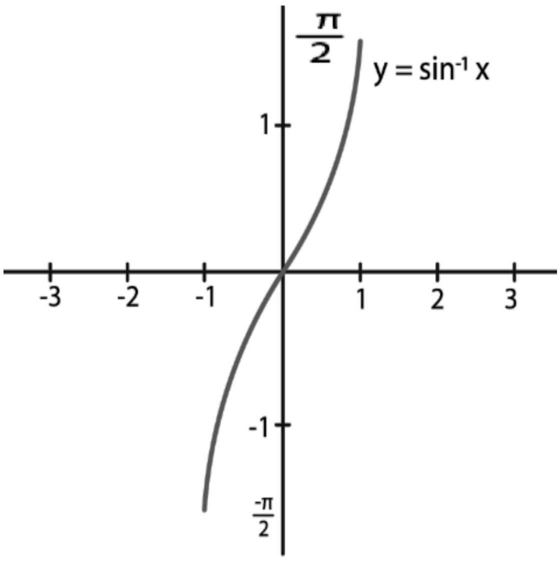
OR

(iii) Find the domain and range of $f(x) = 2\sin^{-1}(1-x)$.

Ans: (i) $A = \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ or any other correct interval

(ii) $\sin^{-1}\left(-\frac{1}{2}\right)-\sin^{-1}(1) = -\frac{\pi}{6}-\frac{\pi}{2} = -\frac{2\pi}{3}$

(iii) Graph is shown below.



OR

(iii)
$f(x)=2\sin^{-1}(1-x)$ is defined when $-1\le(1-x)\le 1$

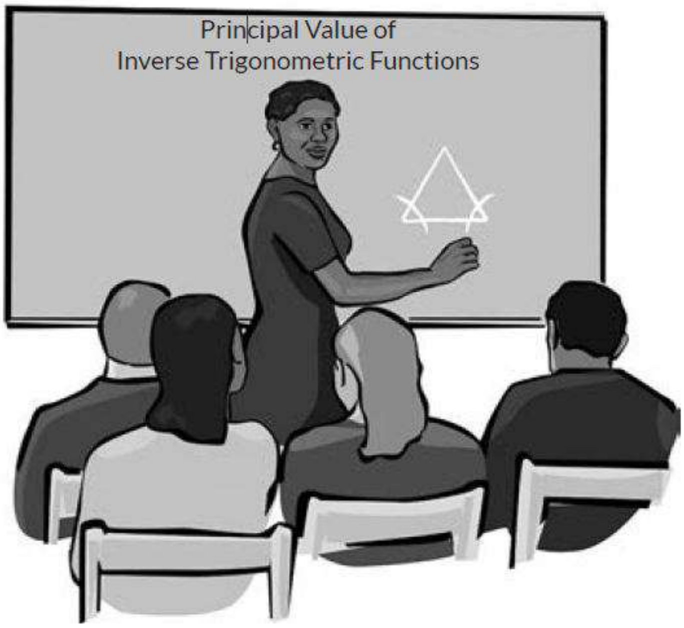
$\Rightarrow -2\le(-x)\le 0 \Rightarrow 0\le x\le 2$

Therefore, domain of given function is $x\in[0,2]$.

Also for $x\in[0,2]$, $-\frac{\pi}{2}\le\sin^{-1}(1-x)\le\frac{\pi}{2}$ i.e., $-\pi\le 2\sin^{-1}(1-x)\le\pi$ i.e., $-\pi\le f(x)\le\pi$.

Hence, the range of the function is $[-\pi, \pi]$.

20. A math teacher explained to the students about topic "Principal Value of Inverse Trigonometric Functions". He told that the value of an inverse trigonometric functions which lies in the range of principal branch is called the principal value of that inverse.



Based on the given information, answer the following questions:

(i) Evaluate : $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ (1)

Ans: $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]=\sin\left[\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right]$

$=\sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]=\sin\frac{\pi}{2}=1$

(ii) Find the principal values of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ (1)

Ans: $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)=\tan^{-1}\left(\tan\left(\pi-\frac{\pi}{4}\right)\right)$

$=\tan^{-1}\left(-\tan\frac{\pi}{4}\right)=\tan^{-1}(-1)=-\frac{\pi}{4}$

(iii) Prove that $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in(0,1)$ (2)

Ans: $LHS=\tan^{-1}\sqrt{x}=\frac{1}{2}\left(2\tan^{-1}\sqrt{x}\right)$

$$=\frac{1}{2}\cos^{-1}\left(\frac{1-\left(\sqrt{x}\right)^2}{1+\left(\sqrt{x}\right)^2}\right) \qquad \left[\because 2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)=RHS$$

OR

(iii) Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]+\tan^{-1}1$

Ans: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]+\tan^{-1}1$

$$=\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]+\tan^{-1}1=\tan^{-1}\left[2\cos\left(\frac{\pi}{3}\right)\right]+\tan^{-1}1$$

$$=\tan^{-1}\left[2\times\frac{1}{2}\right]+\tan^{-1}1=\tan^{-1}1+\tan^{-1}1=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$$