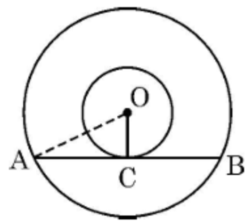
Set 430/6/3

SOLUTIONS
MATHEMATICS (BASIC)

SECTION A

*This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each.* $20\times1=20$

1. In two concentric circles centred at O, a chord AB of the larger circle touches the smaller circle at C. If OA = 3.5 cm, OC = 2.1 cm, then AB is equal to



(A) 5.6 cm (B) 2.8 cm
(C) 3.5 cm (D) 4.2 cm

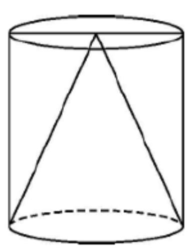
Ans: (A) 5.6 cm	1

2. Three coins are tossed together. The probability that at least one head comes up, is

(A) $\frac{3}{8}$ (B) $\frac{7}{8}$
(C) $\frac{1}{8}$ (D) $\frac{3}{4}$

Ans: (B) $\frac{7}{8}$	1

3. The volume of air in a hollow cylinder is 450 cm^3. A cone of same height and radius as that of cylinder is kept inside it. The volume of empty space in the cylinder is



(A) 225 cm^3 (B) 150 cm^3
(C) 250 cm^3 (D) 300 cm^3

Ans: (D) 300 cm^3	1

4. If the length of the shadow of a tower is $\sqrt{3}$ times its height, then the angle of elevation of the sun is

(A) 45° (B) 30°

(C) 60° (D) 0°

Ans: (B) 30^0	1

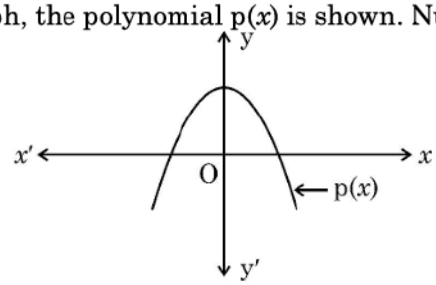
5. 22^{nd} term of the A.P. : $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}$, ... is

(A) $\frac{45}{2}$ (B) -9

(C) $\frac{-39}{2}$ (D) -21

Ans: (C) $\frac{-39}{2}$	1

6. In the given graph, the polynomial p(x) is shown. Number of zeroes of p(x) is



(A) 3 (B) 2

(C) 1 (D) 4

Ans: (B) 2	1

7. If probability of happening of an event is 57%, then probability of non-happening of the event is

(A) 0.43 (B) 0.57

(C) 53% (D) $\frac{1}{57}$

Ans: (A) 0.43	1

8. OAB is sector of a circle with centre O and radius 7 cm. If length of arc $\widehat{AB} = \frac{22}{3}$ cm, then $\angle AOB$ is equal to

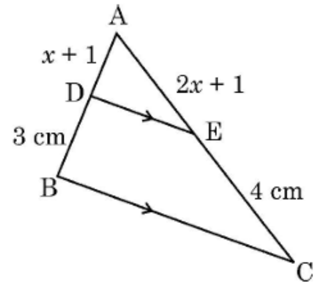
(A) $\left(\frac{120}{7}\right)^\circ$ (B) 45°

(C) 60° (D) 30°

Ans: (C) 60°	1

9. If the sum of first n terms of an A.P. is given by $S_n = \frac{n}{2}(3n + 1)$, then the first term of the A.P. is (A) 2 (B) $\frac{3}{2}$ (C) 4 (D) $\frac{5}{2}$	
Ans: (A) 2	1
10. To calculate mean of a grouped data, Rahul used assumed mean method. He used $d = (x - A)$, where A is assumed mean. Then $\bar{x}$ is equal to (A) $A + \bar{d}$ (B) $A + h\bar{d}$ (C) $h(A + \bar{d})$ (D) $A - h\bar{d}$	
Ans: (A) $A + \bar{d}$	1
11. The point (3, – 5) lies on the line $mx - y = 11$. The value of m is (A) 3 (B) – 2 (C) 8 (D) 2	
Ans: (D) 2	1
12. If $\sqrt{3} \sin \theta = \cos \theta$, then value of θ is (A) $\sqrt{3}$ (B) 60° (C) $\frac{1}{\sqrt{3}}$ (D) 30°	
Ans: (D) 30°	1
13. ABCD is a rectangle with its vertices at (2, –2), (8, 4), (4, 8) and (– 2, 2) taken in order. Length of its diagonal is (A) $4\sqrt{2}$ (B) $6\sqrt{2}$ (C) $4\sqrt{26}$ (D) $2\sqrt{26}$	
Ans: (D) $2\sqrt{26}$	1
14. Two dice are rolled together. The probability of getting a sum more than 9 is (A) $\frac{5}{6}$ (B) $\frac{5}{18}$ (C) $\frac{1}{6}$ (D) $\frac{1}{2}$	
Ans: (C) $\frac{1}{6}$	1

15. In ΔABC, $DE \| BC$. If $AE = (2x + 1)$ cm, $EC = 4$ cm, $AD = (x + 1)$ cm and $DB = 3$ cm, then value of x is



(A) 1 (B) $\frac{1}{2}$

(C) -1 (D) $\frac{1}{3}$

Ans: (B) $\frac{1}{2}$	1

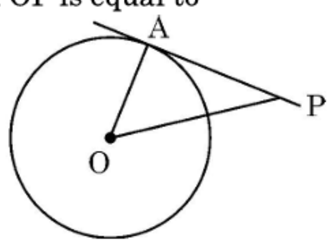
16. The value of k for which the system of equations $3x - 7y = 1$ and $kx + 14y = 6$ is inconsistent, is

(A) -6 (B) $\frac{2}{3}$

(C) 6 (D) $\frac{-3}{2}$

Ans: (A) -6	1

17. In the given figure, PA is tangent to a circle with centre O. If $\angle APO = 30°$ and $OA = 2.5$ cm, then OP is equal to

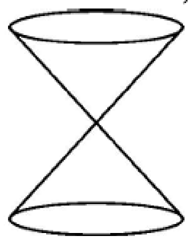


(A) 2.5 cm (B) 5 cm

(C) $\frac{5}{\sqrt{3}}$ cm (D) 2 cm

Ans: (B) 5 cm	1

18. Two identical cones are joined as shown in the figure. If radius of base is 4 cm and slant height of the cone is 6 cm, then height of the solid is



(A) 8 cm (B) $4\sqrt{5}$ cm
(C) $2\sqrt{5}$ cm (D) 12 cm

Ans: (B) $4\sqrt{5}$ cm	1

(Assertion – Reason based questions)

Directions : In question numbers **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option :

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not correct explanation for Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** $(a + \sqrt{b}) \cdot (a - \sqrt{b})$ is a rational number, where a and b are positive integers.

Reason (R) : Product of two irrationals is always rational.

Ans: (C) Assertion (A) is true, but Reason (R) is false.	1

20. **Assertion (A) :** $\Delta ABC \sim \Delta PQR$ such that $\angle A = 65°$, $\angle C = 60°$. Hence $\angle Q = 55°$.

Reason (R) : Sum of all angles of a triangle is 180°.

Ans: (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).	1

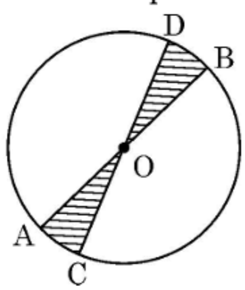
SECTION B

*This section has **5** Very Short Answer (VSA) type questions of **2** marks each.* *5×2=10*

21. A box contains 120 discs, which are numbered from 1 to 120. If one disc is drawn at random from the box, find the probability that (i) it bears a 2– digit number (ii) the number is a perfect square.	
Solution: (i) P(2 - digit number) $=\frac{90}{120}$ or $\frac{3}{4}$ (ii) P(the number is a perfect square) = $\frac{10}{120}$ or $\frac{1}{12}$	1 1
22. (a) Evaluate : $\frac{\cos 45^\circ}{\tan 30^\circ + \sin 60^\circ}$ **OR** (b) Verify that $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$, for A = 30°.	
Solution: (a) $\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{3}}+\frac{\sqrt{3}}{2}}$ $= \frac{2\sqrt{3}}{5\sqrt{2}}$ or $\frac{\sqrt{6}}{5}$ **OR** (b) LHS = $\sin 60^\circ = \frac{\sqrt{3}}{2}$ RHS = $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$ $= \frac{\sqrt{3}}{2}$ = LHS	1½ ½ ½ 1 ½
23. (a) Solve the quadratic equation $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$ using quadratic formula. **OR** (b) Find the nature of roots of the equation $4x^2 - 4a^2x + a^4 - b^4 = 0$, $b \neq 0$	

Solution: (a) Discriminant = 16 $x = \frac{-10 \pm \sqrt{16}}{2\sqrt{3}}$ $x = -\frac{3}{\sqrt{3}}, -\frac{7}{\sqrt{3}}$ or $x = -\sqrt{3}, -\frac{7}{3}\sqrt{3}$ **OR** (b) Discriminant = $(-4a^2)^2 - 4 \times 4 \times (a^4 - b^4) = 16b^4$ Since, Discriminant > 0 Therefore, the given equation has real and distinct roots	1 1 1 1
24. Using prime factorisation, find the HCF of 180, 140 and 210.	
Solution: $180 = 2^2 \times 3^2 \times 5$, $140 = 2^2 \times 5 \times 7$, $210 = 2 \times 3 \times 5 \times 7$ HCF (180, 140, 210) = 2 × 5 = 10	1½ ½
25. The perimeters of two similar triangles are 22 cm and 33 cm respectively. If one side of first triangle is 9 cm, then find the length of corresponding side of the second triangle.	
Solution: Let the length of corresponding side of the second triangle be x cm $\frac{9}{x} = \frac{22}{33}$ $\Rightarrow x = 13 \cdot 5$ ∴ The length of corresponding side of the second triangle = 13·5 cm	1 1
Section – C **(Short Answer Type Questions)** **6 × 3 = 18** Q. Nos. **26** to **31** are Short Answer type questions of **3** marks each. **26. Given that $\sqrt{5}$ is an irrational number, prove that $2 + 3\sqrt{5}$ is an irrational number.**	
Solution: Let $2 + 3\sqrt{5} = a$, where 'a' is a rational number $\Rightarrow \sqrt{5} = \frac{a-2}{3}$ Here L.H.S. is an irrational number but R.H.S. is a rational number So, our assumption is wrong Hence, $2 + 3\sqrt{5}$ is an irrational number	½ 1 1 ½

27. (a) Find the A.P. whose third term is 16 and seventh term exceeds the fifth term by 12. Also, find the sum of first 29 terms of the A.P. **OR** (b) Find the sum of first 20 terms of an A.P. whose n^{th} term is given by $a_n = 5 + 2n$. Can 52 be a term of this A.P. ?	
Solution: (a) $a + 2d = 16$ … (i)	½
$a + 6d = 12 + a + 4d$ …(ii)	½
Solving (i) and (ii) to get $d = 6$, $a = 4$	½
∴ A.P. is 4, 10, 16, ...	½
$S_{29} = \frac{29}{2}[8 + 28 \times 6]$	½
$= 2552$	½
OR (b) $a_n = 5 + 2n$	
getting $a = 7$ and $d = 2$	1
$S_{20} = \frac{20}{2}[14 + 19 \times 2]$ $= 520$	1
$52 = 7 + (n - 1) \times 2$	½
$\Rightarrow n = \frac{47}{2}$, which is not a natural number	
Therefore, 52 cannot be a term of this A.P.	½
28. Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\,\text{cosec}\,\theta$.	
Solution: LHS $= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$	1
$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{(1+\cos\theta)\sin\theta}$	½
$= \frac{2 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$	1
$= \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta = \text{RHS}$	½

29. Find length and breadth of a rectangular park whose perimeter is 100 m and area is 600 m^2.	
Solution: Let length and breadth of the park be a metres and b metres respectively. A.T.Q. $2(a + b) = 100$ …(i) and $ab = 600$ …(ii) using (i) & (ii) we get $a^2 - 50a + 600 = 0$ $\Rightarrow a = 30$ or 20 and $b = 20$ or 30 $\therefore$ length = 30 m, breadth = 20 m or length = 20 m, breadth = 30 m	½ ½ 1 ½ ½
30. AB and CD are diameters of a circle with centre O and radius 7 cm. If $\angle BOD = 30°$, then find the area and perimeter of the shaded region.  	
Solution: Area of the shaded region $= 2\left(\frac{30}{360} \times \frac{22}{7} \times 7 \times 7\right)$ $= \frac{77}{3}$ sq. cm or 25.67 sq. cm Perimeter of the shaded region $= 2\left(14 + \frac{30}{360} \times 2 \times \frac{22}{7} \times 7\right)$ $= \frac{106}{3}$ cm or 35.33 cm	1 ½ 1 ½
31. (a) α, β are zeroes of the polynomial $3x^2 - 8x + k$. Find the value of k, if $\alpha^2 + \beta^2 = \frac{40}{9}$. **OR** (b) Find the zeroes of the polynomial $2x^2 + 7x + 5$ and verify the relationship between its zeroes and co-efficients.	

Solution:	
(a) $p(x) = 3x^2 - 8x + k$ $\alpha + \beta = \frac{8}{3}$, $\alpha\beta = \frac{k}{3}$	1
$\alpha^2 + \beta^2 = \frac{40}{9} \Rightarrow \left(\frac{8}{3}\right)^2 - \frac{2k}{3} = \frac{40}{9}$	1
$\Rightarrow k = 4$	1
OR	
(b) $p(x) = 2x^2 + 7x + 5$ $= (x + 1)(2x + 5)$	
Zeroes of p(x) are -1 and $\frac{-5}{2}$	1
Sum of zeroes $= -1 - \frac{5}{2} = \frac{-7}{2} = -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$	1
Product of zeroes $= (-1)\left(\frac{-5}{2}\right) = \frac{5}{2} = \frac{\text{constant term}}{\text{coefficient of } x^2}$	1

Section – D

(Long Answer Type Questions) **4 × 5 = 20**

Q. Nos. **32** to **35** are Long Answer type questions of **5** marks each.

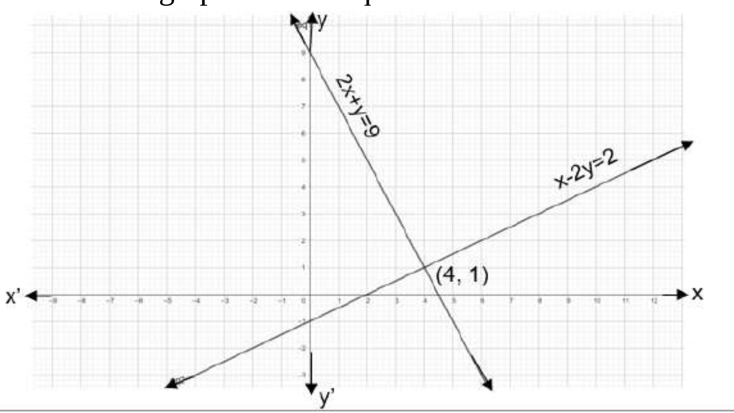
32. Find 'mean' and 'mode' marks of the following data :

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30
Number of students	2	3	8	15	14	8

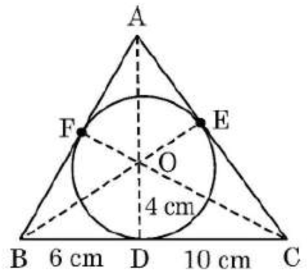
Solution:

Class	x_i	f_i	$u_i = \frac{x_i - 17.5}{5}$	f_iu_i
0 – 5	2.5	2	– 3	- 6
5 – 10	7.5	3	– 2	- 6
10 – 15	12.5	8	– 1	- 8
15 – 20	17.5	15	0	0
20 – 25	22.5	14	1	14
25 - 30	27.5	8	2	16
		$\sum f_i = 50$		$\sum f_iu_i = 10$

Correct table 1½

Mean = $17{\cdot}5 + 5 \times \frac{10}{50} = 18.5$ Modal class is 15 – 20 Mode = $15 + 5 \times \frac{15-8}{30-8-14}$ $= 19.38$	1½ 1½ ½
33. (a) Solve the following pair of linear equations by graphical method : $2x + y = 9$ and $x - 2y = 2$ **OR** (b) Nidhi received simple interest of ₹ 1,200 when invested ₹ x at 6% p.a. and ₹ y at 5% p.a. for 1 year. Had she invested ₹ x at 3% p.a. and ₹ y at 8% p.a. for that year, she would have received simple interest of ₹ 1,260. Find the values of x and y.	
Solution: (a) Correct graph of each equation  Solution x = 4, y = 1 or (4, 1) **OR** (b) A.T.Q. $\frac{6}{100}x + \frac{5}{100}y = 1200 \Rightarrow 6x + 5y = 120000$ …(i) $\frac{3}{100}x + \frac{8}{100}y = 1260 \Rightarrow 3x + 8y = 126000$ …(ii) Solving (i) and (ii) we get, x = 10000 and y = 12000	2 + 2 1 1½ 1½ 1 + 1

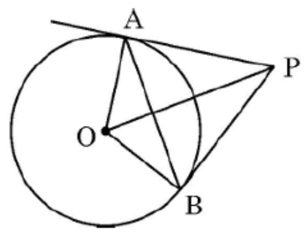
34. (a) The given figure shows a circle with centre O and radius 4 cm circumscribed by ΔABC. BC touches the circle at D such that BD = 6 cm, DC = 10 cm. Find the length of AE.



OR

(b) PA and PB are tangents drawn to a circle with centre O.

If $\angle AOB = 120°$ and OA = 10 cm, then



(i) Find $\angle OPA$. **1**

(ii) Find the perimeter of ΔOAP. **3**

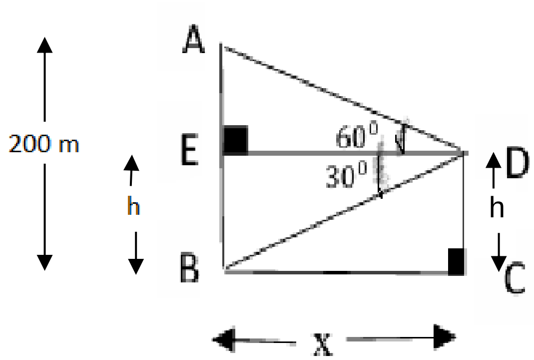
(iii) Find the length of chord AB. **1**

Solution	Marks
Solution:	
(a) Let AE = x $\Rightarrow$ AF = x and CE = 10 cm, BF = 6 cm	1
(Lengths of tangents drawn from an external point to a circle are equal)	
$s = \dfrac{16 + 10 + x + 6 + x}{2} = 16 + x$	½
$\therefore$ Area of Δ ABC = $\sqrt{(16 + x)(x)(6)(10)}$ …(i)	1
Also, area of Δ ABC = $\frac{1}{2}[16 \times 4 + (10 + x)4 + (6 + x)4]$ …(ii)	1
Equating (i) & (ii), we get $x = \frac{64}{11}$ or 5·8	1½
x = – 16 (Rejected)	
Length of AE = $\frac{64}{11}$ cm or 5.8 cm	
OR	
(b) (i) $\angle$ OPA = 30°	1
(ii) In Δ OAP, $\sin 30° = \frac{10}{OP} \Rightarrow OP = 20$ cm	1
$\tan 30° = \frac{10}{PA} \Rightarrow PA = 10\sqrt{3}$ cm	1
$\therefore$ Perimeter of Δ OPA = $(30 + 10\sqrt{3})$ cm	1

(iii) PA = PB and $\angle$ APB = 60° Δ APB is an equilateral triangle $\therefore$ PA = AB = $10\sqrt{3}$ cm	½ ½

35. A drone is flying at a height of h metres. At an instant it observes the angle of elevation of top of an industrial turbine as 60° and angle of depression of foot of the turbine as 30°. If height of turbine is 200 metres, find the value of h and the distance of drone from the turbine.

(Use $\sqrt{3} = 1.73$)

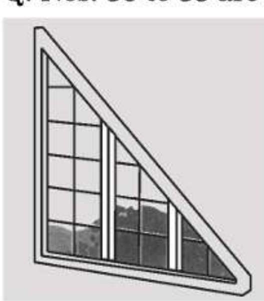
Solution: 	Correct figure 1
Let the drone be flying at the height CD = h metres, distance of drone from the turbine be x metres and height of industrial turbine is AB = 200 metres	
In Δ DEB, $\tan 30° = \frac{h}{x} \Rightarrow x = h\sqrt{3}$ (i)	1½
In Δ DEA, $\tan 60° = \frac{200-h}{x} \Rightarrow x\sqrt{3} = 200 - h$ (ii)	1½
Solving (i) & (ii) we get $x = 50\sqrt{3} = 86.5$, h = 50 $\therefore$ h = 50 m and the distance of drone from the turbine is 86.5 m	½ + ½

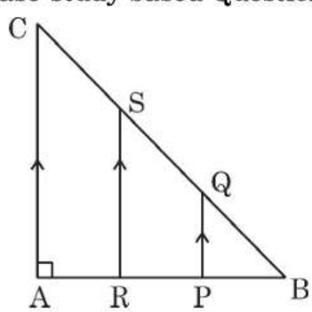
Section – E

(Case-study based Questions) **3 × 4 = 12**

Q. Nos. **36** to **38** are Case-study based Questions of **4** marks each.

36.





A triangular window of a building is shown above. Its diagram represents a ΔABC with $\angle A = 90°$ and AB = AC. Points P and R trisect AB and PQ || RS || AC.

Based on the above, answer the following questions :

(i) Show that $\Delta BPQ \sim \Delta BAC$. **1**

(ii) Prove that $PQ = \frac{1}{3} AC$. **1**

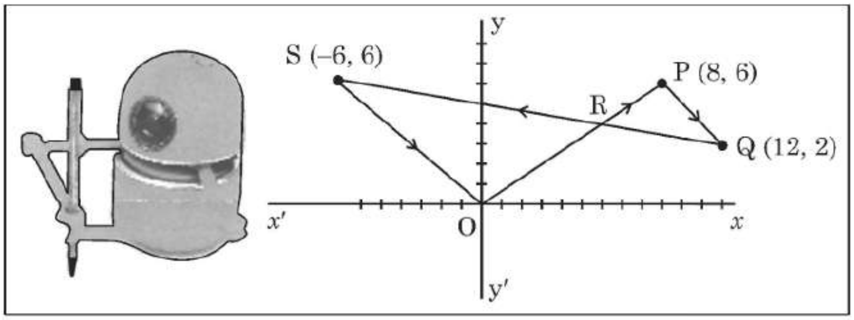
(iii) (a) If AB = 3 m, find length BQ and BS. Verify that $BQ = \frac{1}{2} BS$. **2**

OR

(iii) (b) Prove that $BR^2 + RS^2 = \frac{4}{9} BC^2$.

Solution:	
(i) In Δ BAC and Δ BPQ, PQ ∥ AC	
$\therefore \angle BQP = \angle BCA$ and $\angle B$ is common	
$\therefore \Delta BPQ \sim \Delta BAC$ (By AA similarity criterion)	1
(ii) Since, $\Delta BPQ \sim \Delta BAC \Rightarrow \frac{PQ}{AC} = \frac{BP}{BA} = \frac{1}{3}$	½
$\Rightarrow \frac{PQ}{AC} = \frac{1}{3} \Rightarrow PQ = \frac{1}{3} AC$	½
(iii) (a) $\frac{BP}{BA} = \frac{PQ}{AC}$ (corresponding sides of similar triangles)	
$\Rightarrow$ BP = PQ (as BA = AC)	
$\therefore PQ = \frac{1}{3} \times 3 = 1$ m	
Hence, $BQ = \sqrt{2}$ m	1
getting $BS = 2\sqrt{2}$ m	½
$\Rightarrow \frac{1}{2}BS = BQ$ (Hence verified)	½
OR	
(b) $BR^2 + RS^2 = \left(\frac{2}{3}AB\right)^2 + \left(\frac{2}{3}AC\right)^2$	½ + ½
$= \frac{4}{9}(AB^2 + AC^2)$	½
$= \frac{4}{9} BC^2$	½

37. Gurveer and Arushi built a robot that can paint a path as it moves on a graph paper. Some co-ordinate of points are marked on it. It starts from (0, 0), moves to the points listed in order (in straight lines) and ends at (0, 0).



Arushi entered the points P(8, 6), Q(12, 2) and S(– 6, 6) in order. The path drawn by robot is shown in the figure.

Based on the above, answer the following questions :

(i) Determine the distance OP. **1**

(ii) QS is represented by equation $2x + 9y = 42$. Find the co-ordinates of the point where it intersects y – axis. **1**

(iii) (a) Point R(4.8, y) divides the line segment OP in a certain ratio, find the ratio. Hence, find the value of y. **2**

OR

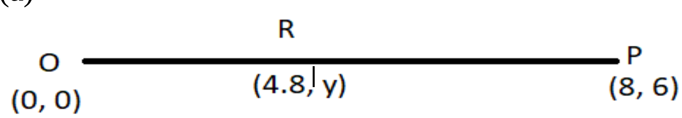
(iii) (b) Using distance formula, show that $\frac{PQ}{OS} = \frac{2}{3}$.

Solution:

(i) The distance OP = $\sqrt{64 + 36} = 10$ — 1

(ii) $2x + 9y = 42$ intersects y-axis at $\left(0, \frac{14}{3}\right)$ — 1

(iii) (a)



Let OR : RP = k : 1, therefore $4{\cdot}8 = \frac{8k}{k+1} \Rightarrow k = \frac{3}{2}$ — 1½

$\Rightarrow$ OR : RP = 3 : 2

$y = \frac{18}{5}$ — ½

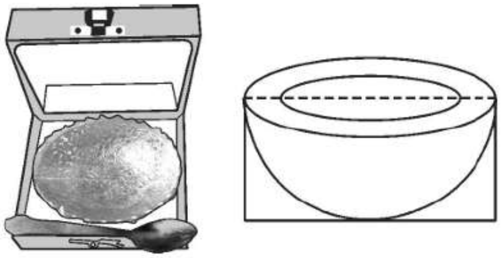
OR

(b) PQ = $\sqrt{4^2 + (-4)^2} = \sqrt{32}$ or $4\sqrt{2}$ — ½

OS = $\sqrt{(-6)^2 + 6^2} = \sqrt{72}$ or $6\sqrt{2}$ — ½

$\therefore \frac{PQ}{OS} = \sqrt{\frac{32}{72}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$	1

38.



A hemispherical bowl is packed in a cuboidal box. The bowl just fits in the box. Inner radius of the bowl is 10 cm. Outer radius of the bowl is 10.5 cm.

Based on the above, answer the following questions :

(i) Find the dimensions of the cuboidal box. **1**

(ii) Find the total outer surface area of the box. **1**

(iii) (a) Find the difference between the capacity of the bowl and the volume of the box. (use $\pi = 3.14$) **2**

OR

(iii) (b) The inner surface of the bowl and the thickness is to be painted. Find the area to be painted.

Solution:

(i) Diameter of bowl = 21 cm	
Dimensions of the box are 21 cm × 21 cm × 10·5 cm	1
(ii) Total surface area of the box = $2\left(441 + \frac{441}{2} + \frac{441}{2}\right)$ = 1764 sq. cm	½ + ½
(iii) (a) Capacity of bowl = $\frac{2}{3} \times 3{\cdot}14 \times 10^3$	½
= $\frac{6280}{3}$ cu. cm or 2093·33 cu. cm	½
Volume of box = $21 \times 21 \times \frac{21}{2} = \frac{9261}{2}$ cu. cm. or 4630·5 cu. cm	½
Required difference = $\frac{15223}{6}$ cu. cm or 2537·17 cu. cm	½
(NOTE: Here capacity is considered as volume to compute the difference.)	
OR	
(b) Required area = $2 \times \frac{22}{7} \times 10^2 + \frac{22}{7} \times (10{\cdot}5^2 - 10^2)$	1
$= \frac{4400}{7} + \frac{451}{14}$	
$= \frac{9251}{14}$ sq. cm or 660·79 sq. cm	1