Set 430/6/2

SOLUTIONS

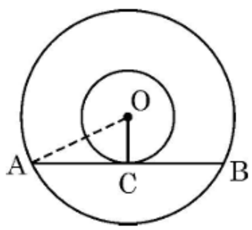
MATHEMATICS (BASIC)

Section – A $20 \times 1 = 20$

(Multiple Choice Questions)

Section-A consists of **20** Multiple Choice Questions of **1** mark each.

1. In two concentric circles centred at O, a chord AB of the larger circle touches the smaller circle at C. If OA = 3.5 cm, OC = 2.1 cm, then AB is equal to



(A) 5.6 cm (B) 2.8 cm
(C) 3.5 cm (D) 4.2 cm

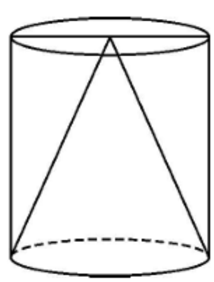
Ans: (A) 5.6 cm	1

2. Three coins are tossed together. The probability that exactly one coin shows head, is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$
(C) 1 (D) $\frac{3}{8}$

Ans: (D) $\frac{3}{8}$	1

3. The volume of air in a hollow cylinder is 450 cm^3. A cone of same height and radius as that of cylinder is kept inside it. The volume of empty space in the cylinder is



(A) 225 cm^3 (B) 150 cm^3
(C) 250 cm^3 (D) 300 cm^3

Ans: (D) 300 cm^3	

4. In ΔABC, $\angle B = 90°$. If $\frac{AB}{AC} = \frac{1}{2}$, then $\cos C$ is equal to

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Ans: (C) $\frac{\sqrt{3}}{2}$	1

5. 15th term of the A.P. $\frac{13}{3}, \frac{9}{3}, \frac{5}{3}, ...$ is

(A) 23 (B) $\frac{-53}{3}$

(C) -11 (D) $\frac{-43}{3}$

Ans: (D) $\frac{-43}{3}$	1

6. If probability of happening of an event is 57%, then probability of non-happening of the event is

(A) 0.43 (B) 0.57

(C) 53% (D) $\frac{1}{57}$

Ans: (A) 0.43	1

7. A quadratic polynomial having zeroes 0 and -2, is

(A) $x(x-2)$ (B) $4x(x+2)$

(C) $x^2 + 2$ (D) $2x^2 + 2x$

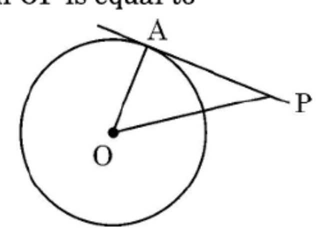
Ans: (B) 4x(x+2)	1

8. OAB is sector of a circle with centre O and radius 7 cm. If length of arc $\widehat{AB} = \frac{22}{3}$ cm, then $\angle AOB$ is equal to

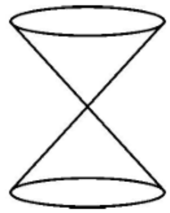
(A) $\left(\frac{120}{7}\right)^\circ$ (B) 45°

(C) 60° (D) 30°

Ans: (C) 60°	1

9. To calculate mean of a grouped data, Rahul used assumed mean method. He used $d = (x - A)$, where A is assumed mean. Then $\overline{x}$ is equal to (A) $A + \overline{d}$ (B) $A + h\overline{d}$ (C) $h(A + \overline{d})$ (D) $A - h\overline{d}$	
Ans: (A) $A + \overline{d}$	1
10. If the sum of first n terms of an A.P. is given by $S_n = \frac{n}{2}(3n + 1)$, then the first term of the A.P. is (A) 2 (B) $\frac{3}{2}$ (C) 4 (D) $\frac{5}{2}$	
Ans: (A) 2	1
11. ABCD is a rectangle with its vertices at (2, –2), (8, 4), (4, 8) and (– 2, 2) taken in order. Length of its diagonal is (A) $4\sqrt{2}$ (B) $6\sqrt{2}$ (C) $4\sqrt{26}$ (D) $2\sqrt{26}$	
Ans: (D) $2\sqrt{26}$	1
12. In the given figure, PA is tangent to a circle with centre O. If $\angle APO = 30°$ and OA = 2.5 cm, then OP is equal to  (A) 2.5 cm (B) 5 cm (C) $\frac{5}{\sqrt{3}}$ cm (D) 2 cm	
Ans: (B) 5 cm	1
13. Two dice are rolled together. The probability of getting an outcome (a, b) such that b = 2a, is (A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{1}{36}$ (D) $\frac{1}{9}$	
Ans: (B) $\frac{1}{12}$	1

14. Two identical cones are joined as shown in the figure. If radius of base is 4 cm and slant height of the cone is 6 cm, then height of the solid is



(A) 8 cm (B) $4\sqrt{5}$ cm

(C) $2\sqrt{5}$ cm (D) 12 cm

Ans: (B) $4\sqrt{5}$ cm — 1

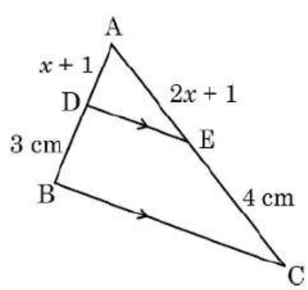
15. If $\sin\theta = \frac{1}{9}$, then $\tan\theta$ is equal to

(A) $\frac{1}{4\sqrt{5}}$ (B) $\frac{4\sqrt{5}}{9}$

(C) $\frac{1}{8}$ (D) $4\sqrt{5}$

Ans: (A) $\frac{1}{4\sqrt{5}}$ — 1

16. In ΔABC, $DE || BC$. If $AE = (2x + 1)$ cm, $EC = 4$ cm, $AD = (x + 1)$ cm and $DB = 3$ cm, then value of x is



(A) 1 (B) $\frac{1}{2}$

(C) -1 (D) $\frac{1}{3}$

Ans: (B) $\frac{1}{2}$ — 1

17. The value of k for which the system of equations $3x - 7y = 1$ and $kx + 14y = 6$ is inconsistent, is (A) -6 (B) $\frac{2}{3}$ (C) 6 (D) $\frac{-3}{2}$	
Ans: (A) -6	1
18. The line $2x - 3y = 6$ intersects x – axis at (A) $(0, -2)$ (B) $(0, 3)$ (C) $(-2, 0)$ (D) $(3, 0)$	
Ans: (D) (3,0)	1
(Assertion – Reason based questions) **Directions :** In question numbers **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option : (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not correct explanation for Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true. 19. **Assertion (A) :** $\Delta ABC \sim \Delta PQR$ such that $\angle A = 65°$, $\angle C = 60°$. Hence $\angle Q = 55°$. **Reason (R) :** Sum of all angles of a triangle is 180°.	
Ans: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not correct explanation of Assertion (A).	1
20. **Assertion (A) :** $(a + \sqrt{b}) \cdot (a - \sqrt{b})$ is a rational number, where a and b are positive integers. **Reason (R) :** Product of two irrationals is always rational.	
Ans: (C) Assertion (A) is true, but Reason (R) is false.	1

Section – B	
(Very Short Answer Type Questions) **5 × 2 = 10**	
Q. Nos. **21** to **25** are Very Short Answer type questions of **2** marks each.	
21. (a) Evaluate : $\frac{\cos 45^\circ}{\tan 30^\circ + \sin 60^\circ}$ **OR** (b) Verify that $\sin 2A = \frac{2\tan A}{1 + \tan^2 A}$, for A = 30°.	
Solution: (a) $\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{3}}+\frac{\sqrt{3}}{2}}$ $= \frac{2\sqrt{3}}{5\sqrt{2}}$ or $\frac{\sqrt{6}}{5}$	1½ ½
OR	
(b) LHS = $\sin 60^\circ = \frac{\sqrt{3}}{2}$ RHS = $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$ $= \frac{\sqrt{3}}{2}$ = LHS	½ 1 ½
22. A box contains 120 discs, which are numbered from 1 to 120. If one disc is drawn at random from the box, find the probability that (i) it bears a 2– digit number (ii) the number is a perfect square.	
Solution : (i) P(2 - digit number) = $\frac{90}{120}$ or $\frac{3}{4}$ (ii) P(the number is a perfect square) = $\frac{10}{120}$ or $\frac{1}{12}$	1 1
23. Using prime factorisation, find the HCF of 144, 180 and 192.	

Solution : $144 = 2^4 \times 3^2$, $180 = 2^2 \times 3^2 \times 5$, $192 = 2^6 \times 3$ HCF (144, 180, 192) = $2^2 \times 3 = 12$	1½ ½

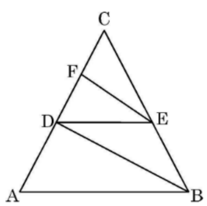
24. (a) Solve the equation $4x^2 - 9x + 3 = 0$, using quadratic formula.

OR

(b) Find the nature of roots of the equation $3x^2 - 4\sqrt{3}x + 4 = 0$.

Solution: (a) Discriminant = 33 $\Rightarrow x = \frac{9 \pm \sqrt{33}}{8}$ **OR** (b) Discriminant = $(-4\sqrt{3})^2 - 4 \times 4 \times 3 = 0$ $\Rightarrow$ The given equation has real and equal roots	1 1 1 1

25. In the given figure, AB || DE and BD || EF. Prove that $DC^2 = CF \times AC$.



Solution : EF || BD $\Rightarrow \frac{CF}{DC} = \frac{CE}{CB}$ (i) DE || AB $\Rightarrow \frac{DC}{AC} = \frac{CE}{CB}$ (ii) Using (i) & (ii), $\frac{CF}{DC} = \frac{DC}{AC}$ $\Rightarrow DC^2 = CF \times AC$	1 ½ ½

Section – C

(Short Answer Type Questions) **6 × 3 = 18**

Q. Nos. **26** to **31** are Short Answer type questions of **3** marks each.

26. Three friends plan to go for a morning walk. They step off together and their steps measures 48 cm, 52 cm and 56 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps ten times ?

Solution : $48 = 2^4 \times 3$, $52 = 2^2 \times 13$, $56 = 2^3 \times 7$ LCM = $2^4 \times 3 \times 13 \times 7 = 4368$ $\Rightarrow$ Minimum distance each walked in complete steps ten times = 43680 cm	1½ 1 ½

27. Prove that $\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\cot^2\theta}\right)=\frac{1}{\sin^2\theta-\sin^4\theta}$

Solution : LHS $=\frac{1+\tan^2\theta}{\tan^2\theta}+\frac{1+\cot^2\theta}{\cot^2\theta}$

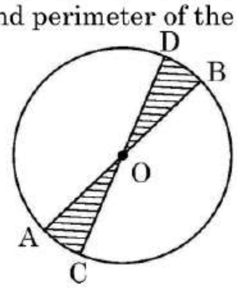
$=\frac{\sec^2\theta}{\tan^2\theta}+\frac{\text{cosec}^2\,\theta}{\cot^2\theta}$ 1

$=\frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta}$ 1

$=\frac{1}{\sin^2\theta(1-\sin^2\theta)}$ 1

$=\frac{1}{\sin^2\theta-\sin^4\theta}$ = RHS

28. AB and CD are diameters of a circle with centre O and radius 7 cm. If ∠BOD = 30°, then find the area and perimeter of the shaded region.



Solution:

Area of the shaded region = $2\left(\frac{30}{360}\times\frac{22}{7}\times 7\times 7\right)$ 1

$=\frac{77}{3}$ sq. cm or 25.67 sq. cm ½

Perimeter of the shaded region = $2\left(14+\frac{30}{360}\times 2\times\frac{22}{7}\times 7\right)$ 1

$=\frac{106}{3}$ cm or 35.33 cm ½

29. (a) Find the A.P. whose third term is 16 and seventh term exceeds the fifth term by 12. Also, find the sum of first 29 terms of the A.P.

OR

(b) Find the sum of first 20 terms of an A.P. whose n^{th} term is given by $a_n = 5 + 2n$. Can 52 be a term of this A.P. ?

Solution:

(a) $a + 2d = 16$ … (i) ½

$a + 6d = 12 + a + 4d$ …(ii) ½

Solving (i) and (ii) to get $d = 6$, $a = 4$ ½

$\therefore$ A.P. is 4, 10, 16, ... ½

$S_{29} = \frac{29}{2}[8 + 28 \times 6]$ ½

$= 2552$ ½

OR

(b) $a_n = 5 + 2n$

getting $a = 7$ and $d = 2$ 1

$S_{20} = \frac{20}{2}[14 + 19 \times 2]$

$= 520$ 1

$52 = 7 + (n - 1) \times 2$ ½

$\Rightarrow n = \frac{47}{2}$, which is not a natural number.

Therefore, 52 cannot be a term of this A.P. ½

30. (a) If α, β are zeroes of the polynomial $8x^2 - 5x - 1$, then form a quadratic polynomial in x whose zeroes are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$.

OR

(b) Find the zeroes of the polynomial $p(x) = 3x^2 + x - 10$ and verify the relationship between zeroes and its coefficients.

Solution:

(a) $p(x) = 8x^2 - 5x - 1$

$\alpha + \beta = \frac{5}{8}$, $\alpha\beta = \frac{-1}{8}$ ½ + ½

$\therefore$ sum of zeroes $= \frac{2}{\alpha} + \frac{2}{\beta} = -10$ ½

and product of zeroes $= \frac{2}{\alpha} \times \frac{2}{\beta} = -32$ ½

Required polynomial $= x^2 + 10x - 32$ 1

or $k(x^2 + 10x - 32)$ where k is any non-zero real number.

OR

(b) $p(x) = 3x^2 + x - 10 = (x + 2)(3x - 5)$	
Zeroes of p(x) are – 2 and $\frac{5}{3}$	1
Sum of zeroes = $-2 + \frac{5}{3} = \frac{-1}{3} = -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$	1
Product of zeroes = $-2 \times \frac{5}{3} = \frac{-10}{3} = \frac{\text{constant term}}{\text{coefficient of } x^2}$	1

31. The sum of a number and its reciprocal is $\frac{13}{6}$. Find the number.

Solution: Let the number be x	
A.T.Q. $x + \frac{1}{x} = \frac{13}{6}$	1
$\Rightarrow 6x^2 - 13x + 6 = 0$	1
$\Rightarrow (2x-3)(3x-2) = 0$	
$\Rightarrow x = \frac{3}{2}$ or $\frac{2}{3}$	1
∴ The required number is $\frac{3}{2}$ or $\frac{2}{3}$	

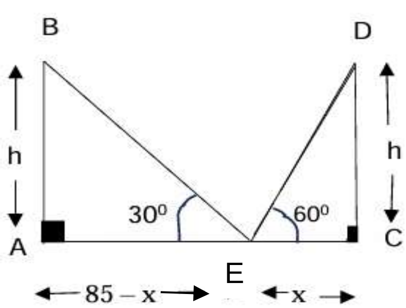
Section – D

(Long Answer Type Questions) **4 × 5 = 20**

Q. Nos. **32** to **35** are Long Answer type questions of **5** marks each.

32. Two poles of equal heights are standing opposite each other on either side of the road which is 85 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles. (Use $\sqrt{3} = 1.73$)

Solution:	Correct figure 1



Let AB and CD be the equal poles of height h metres

In Δ BAE, tan 30° = $\frac{h}{85-x}$ ⇒ 85–x = h$\sqrt{3}$ … (i) In Δ DCE, tan 60° = $\frac{h}{x}$ ⇒ h= x$\sqrt{3}$ … (ii) Using (i) & (ii) x = 21·25, 85 – x = 63.75 and h = 36·76 ∴ The height of the poles are 36.76 m and the distances of the points from the poles are 21.25 m and 63.75 m	1½ 1½ 1

33. (a) Solve the following pair of linear equations by graphical method :

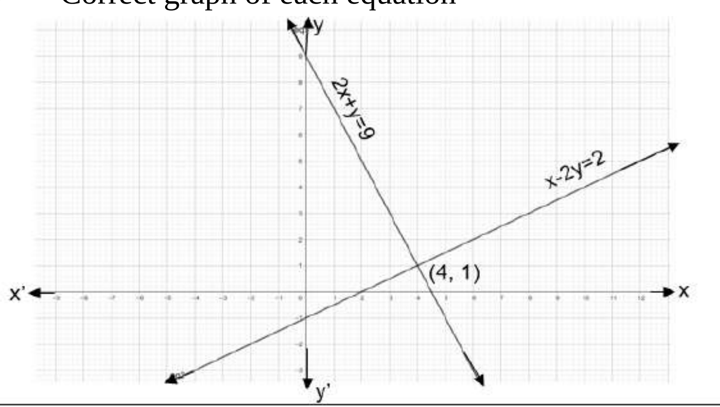
$2x + y = 9$ and $x - 2y = 2$

OR

(b) Nidhi received simple interest of ₹ 1,200 when invested ₹ x at 6% p.a. and ₹ y at 5% p.a. for 1 year. Had she invested ₹ x at 3% p.a. and ₹ y at 8% p.a. for that year, she would have received simple interest of ₹ 1,260. Find the values of x and y.

Solution:

(a) Correct graph of each equation — 2 + 2



Solution x = 4, y = 1 or (4, 1) — 1

OR

(b) A.T.Q.

$\frac{6}{100}x + \frac{5}{100}y = 1200 \Rightarrow 6x + 5y = 120000$ …(i) — 1½

$\frac{3}{100}x + \frac{8}{100}y = 1260 \Rightarrow 3x + 8y = 126000$ …(ii) — 1½

Solving (i) and (ii) we get, x = 10000 and y = 12000 — 1 + 1

34. Find 'mean' and 'mode' of the following data :

Class	0 – 15	15 – 30	30 – 45	45 – 60	60 – 75	75 – 90
Frequency	11	8	15	7	10	9

Solution:

Class	x_i	f_i	$u_i = \frac{x_i - 37.5}{15}$	$f_i u_i$
0 – 15	7.5	11	– 2	– 22
15 – 30	22.5	8	– 1	– 8
30 – 45	37.5	15	0	0
45 – 60	52.5	7	1	7
60 – 75	67.5	10	2	20
75 - 90	82.5	9	3	27
		$\sum f_i = 60$		$\sum f_i u_i = 24$

Correct table 1½

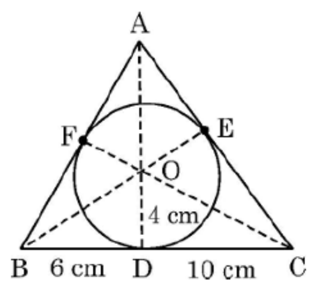
Mean = $37\cdot5 + 15 \times \frac{24}{60} = 43.5$ 1½

Modal class is 30 – 45

Mode = $30 + 15 \times \frac{15-8}{30-8-7}$ 1 ½

= 37 ½

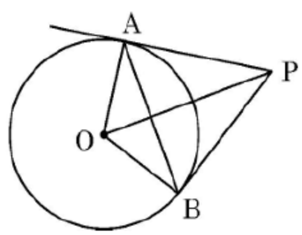
35. (a) The given figure shows a circle with centre O and radius 4 cm circumscribed by ΔABC. BC touches the circle at D such that BD = 6 cm, DC = 10 cm. Find the length of AE.



OR

(b) PA and PB are tangents drawn to a circle with centre O.

If $\angle AOB = 120°$ and OA = 10 cm, then



(i) Find $\angle OPA$. **1**

(ii) Find the perimeter of ΔOAP. **3**

(iii) Find the length of chord AB. **1**

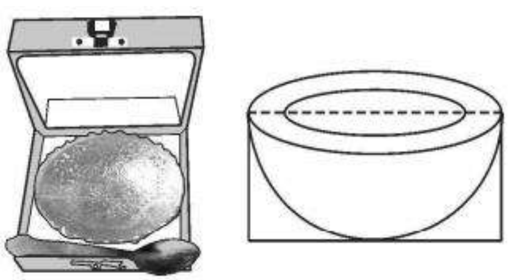
Solution:	Marks
(a) Let AE = x $\Rightarrow$ AF = x and CE = 10 cm, BF = 6 cm	1
(Lengths of tangents drawn from an external point to a circle are equal)	
$s = \frac{16 + 10 + x + 6 + x}{2} = 16 + x$	½
$\therefore$ Area of Δ ABC = $\sqrt{(16 + x)(x)(6)(10)}$ …(i)	1
Also, area of Δ ABC = $\frac{1}{2}[16 \times 4 + (10 + x)4 + (6 + x)4]$ …(ii)	1
Equating (i) & (ii), we get x = $\frac{64}{11}$ or 5·8	1½
x = – 16 (Rejected)	
Length of AE = $\frac{64}{11}$ cm or 5.8 cm	
OR	
(b) (i) $\angle$ OPA = 30°	1
(ii) In Δ OAP, sin 30° = $\frac{10}{OP} \Rightarrow$ OP = 20 cm	1
tan 30° = $\frac{10}{PA} \Rightarrow$ PA = $10\sqrt{3}$ cm	1
$\therefore$ Perimeter of Δ OPA = $(30 + 10\sqrt{3})$ cm	1
(iii) PA = PB and $\angle$ APB = 60°	½
Δ APB is an equilateral triangle	
$\therefore$ PA = AB = $10\sqrt{3}$ cm	½

Section – E

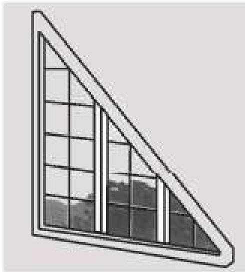
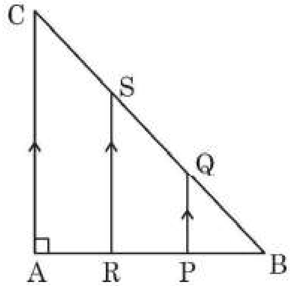
(Case-study based Questions) **3 × 4 = 12**

Q. Nos. **36** to **38** are Case-study based Questions of **4** marks each.

36.



A hemispherical bowl is packed in a cuboidal box. The bowl just fits in the box. Inner radius of the bowl is 10 cm. Outer radius of the bowl is 10.5 cm. Based on the above, answer the following questions : (i) Find the dimensions of the cuboidal box. **1** (ii) Find the total outer surface area of the box. **1** (iii) (a) Find the difference between the capacity of the bowl and the volume of the box. (use $\pi = 3.14$) **2** **OR** (iii) (b) The inner surface of the bowl and the thickness is to be painted. Find the area to be painted.	
Solution:	
(i) Diameter of bowl = 21 cm	
Dimensions of the box are 21 cm × 21 cm × 10·5 cm	1
(ii) Total surface area of the box = $2\left(441 + \frac{441}{2} + \frac{441}{2}\right)$ = 1764 sq. cm	½ + ½
(iii) (a) Capacity of bowl = $\frac{2}{3} \times 3{\cdot}14 \times 10^3$	½
$= \frac{6280}{3}$ cu. cm or 2093·33 cu. cm	½
Volume of box = $21 \times 21 \times \frac{21}{2} = \frac{9261}{2}$ cu. cm. or 4630·5 cu. cm	½
Required difference = $\frac{15223}{6}$ cu. cm or 2537·17 cu. cm	½
(NOTE: Here capacity is considered as volume to compute the difference.)	
OR	
(b) Required area = $2 \times \frac{22}{7} \times 10^2 + \frac{22}{7} \times (10{\cdot}5^2 - 10^2)$	1
$= \frac{4400}{7} + \frac{451}{14}$	
$= \frac{9251}{14}$ sq. cm or 660·79 sq. cm	1

37.  

A triangular window of a building is shown above. Its diagram represents a ΔABC with $\angle A = 90°$ and $AB = AC$. Points P and R trisect AB and $PQ \parallel RS \parallel AC$.

Based on the above, answer the following questions :

(i) Show that $\Delta BPQ \sim \Delta BAC$. **1**

(ii) Prove that $PQ = \frac{1}{3} AC$. **1**

(iii) (a) If AB = 3 m, find length BQ and BS. Verify that $BQ = \frac{1}{2} BS$. **2**

OR

(iii) (b) Prove that $BR^2 + RS^2 = \frac{4}{9} BC^2$.

Solution:

(i) In Δ BAC and Δ BPQ, $PQ \parallel AC$

$\therefore \angle BQP = \angle BCA$ and $\angle B$ is common

$\therefore \Delta BPQ \sim \Delta BAC$ (By AA similarity criterion) — 1

(ii) Since, $\Delta BPQ \sim \Delta BAC \Rightarrow \frac{PQ}{AC} = \frac{BP}{BA} = \frac{1}{3}$ — ½

$\Rightarrow \frac{PQ}{AC} = \frac{1}{3} \Rightarrow PQ = \frac{1}{3} AC$ — ½

(iii) (a) $\frac{BP}{BA} = \frac{PQ}{AC}$ (corresponding sides of similar triangles)

$\Rightarrow BP = PQ$ (as BA = AC)

$\therefore PQ = \frac{1}{3} \times 3 = 1$ m

Hence, $BQ = \sqrt{2}$ m — 1

getting $BS = 2\sqrt{2}$ m — ½

$\Rightarrow \frac{1}{2}BS = BQ$ (Hence verified) — ½

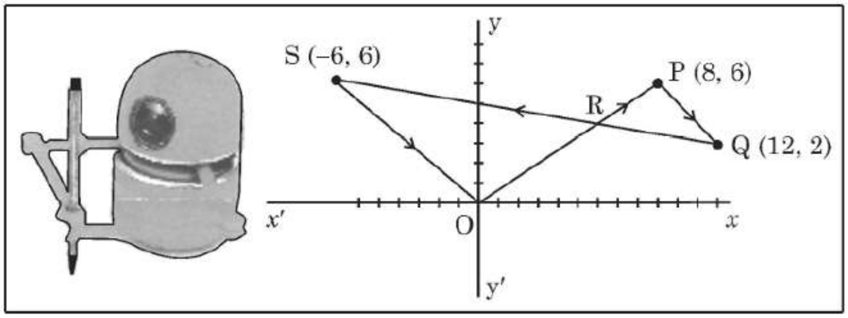
OR

(b) $BR^2 + RS^2 = \left(\frac{2}{3}AB\right)^2 + \left(\frac{2}{3}AC\right)^2$ — ½ + ½

$= \frac{4}{9}(AB^2 + AC^2)$ — ½

$= \frac{4}{9} BC^2$ — ½

38. Gurveer and Arushi built a robot that can paint a path as it moves on a graph paper. Some co-ordinate of points are marked on it. It starts from (0, 0), moves to the points listed in order (in straight lines) and ends at (0, 0).



Arushi entered the points P(8, 6), Q(12, 2) and S(– 6, 6) in order. The path drawn by robot is shown in the figure.

Based on the above, answer the following questions :

(i) Determine the distance OP. **1**

(ii) QS is represented by equation $2x + 9y = 42$. Find the co-ordinates of the point where it intersects y – axis. **1**

(iii) (a) Point R(4.8, y) divides the line segment OP in a certain ratio, find the ratio. Hence, find the value of y. **2**

OR

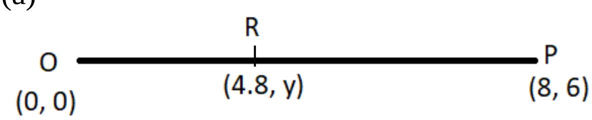
(iii) (b) Using distance formula, show that $\frac{PQ}{OS} = \frac{2}{3}$.

Solution:

(i) The distance OP = $\sqrt{64 + 36}$ = 10 — 1

(ii) $2x + 9y = 42$ intersects y-axis at $\left(0, \frac{14}{3}\right)$ — 1

(iii) (a)



Let OR : RP = k : 1, therefore $4 \cdot 8 = \frac{8k}{k+1}$ $\Rightarrow k = \frac{3}{2}$ — 1½

$\Rightarrow$ OR : RP = 3 : 2

$y = \frac{18}{5}$ — ½

OR

(b) PQ = $\sqrt{4^2 + (-4)^2} = \sqrt{32}$ or $4\sqrt{2}$ — ½

OS = $\sqrt{(-6)^2 + 6^2} = \sqrt{72}$ or $6\sqrt{2}$ — ½

$\therefore \frac{PQ}{OS} = \sqrt{\frac{32}{72}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$ — 1