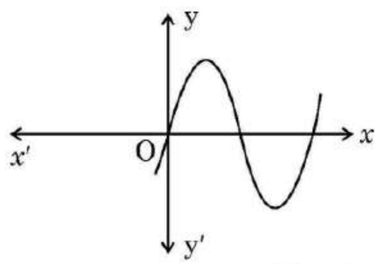
Set 430/6/1

SOLUTIONS
MATHEMATICS (BASIC)

SECTION A

*This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each.* $20\times1=20$

1. In the given figure, graph of polynomial $p(x)$ is shown. Number of zeroes of $p(x)$ is



(A) 3 (B) 2
(C) 1 (D) 4

Ans: (A) 3	1

2. 22^{nd} term of the A.P. : $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}$, ... is

(A) $\frac{45}{2}$ (B) -9
(C) $\frac{-39}{2}$ (D) -21

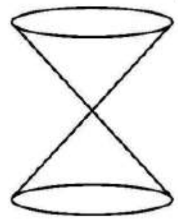
Ans: (C) $\frac{-39}{2}$	1

3. The line $2x - 3y = 6$ intersects $x-$axis at

(A) $(0, -2)$ (B) $(0, 3)$
(C) $(-2, 0)$ (D) $(3, 0)$

Ans: (D) (3,0)	1

4. Two identical cones are joined as shown in the figure. If radius of base is 4 cm and slant height of the cone is 6 cm, then height of the solid is



(A) 8 cm (B) $4\sqrt{5}$ cm

(C) $2\sqrt{5}$ cm (D) 12 cm

Ans: (B) $4\sqrt{5}$ cm	1

5. The value of k for which the system of equations $3x - 7y = 1$ and $kx + 14y = 6$ is inconsistent, is

(A) -6 (B) $\frac{2}{3}$

(C) 6 (D) $\frac{-3}{2}$

Ans: (A) -6	1

6. Two dice are rolled together. The probability of getting a sum more than 9 is

(A) $\frac{5}{6}$ (B) $\frac{5}{18}$

(C) $\frac{1}{6}$ (D) $\frac{1}{2}$

Ans: (C) $\frac{1}{6}$	1

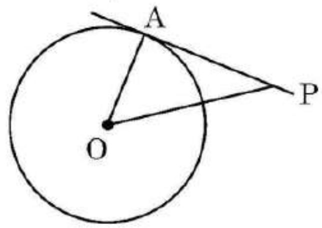
7. ABCD is a rectangle with its vertices at (2, –2), (8, 4), (4, 8) and (– 2, 2) taken in order. Length of its diagonal is

(A) $4\sqrt{2}$ (B) $6\sqrt{2}$

(C) $4\sqrt{26}$ (D) $2\sqrt{26}$

Ans: (D) $2\sqrt{26}$	1

8. In the given figure, PA is tangent to a circle with centre O. If $\angle APO = 30°$ and $OA = 2.5$ cm, then OP is equal to



(A) 2.5 cm (B) 5 cm

(C) $\frac{5}{\sqrt{3}}$ cm (D) 2 cm

Ans: (B) 5 cm	1

9. If probability of happening of an event is 57%, then probability of non-happening of the event is

(A) 0.43 (B) 0.57

(C) 53% (D) $\frac{1}{57}$

Ans: (A) 0.43	1

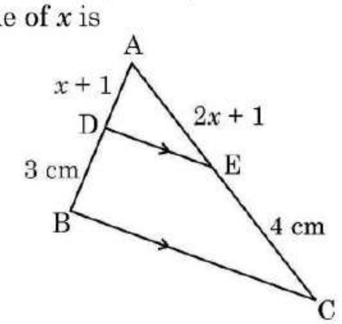
10. OAB is sector of a circle with centre O and radius 7 cm. If length of arc $\widehat{AB} = \frac{22}{3}$ cm, then $\angle AOB$ is equal to

(A) $\left(\frac{120}{7}\right)^{\circ}$ (B) 45°

(C) 60° (D) 30°

Ans: (C) 60°	1

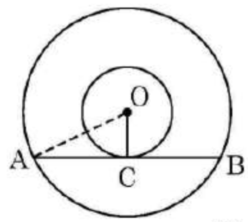
11. In ΔABC, $DE || BC$. If $AE = (2x + 1)$ cm, $EC = 4$ cm, $AD = (x + 1)$ cm and $DB = 3$ cm, then value of x is

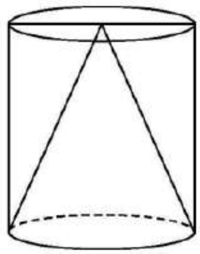


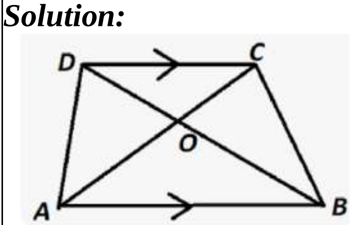
(A) 1 (B) $\frac{1}{2}$

(C) -1 (D) $\frac{1}{3}$

Ans: (B) $\frac{1}{2}$	1

12. Three coins are tossed together. The probability that exactly one coin shows head, is (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) 1 (D) $\frac{3}{8}$	
Ans: (D) $\frac{3}{8}$	1
13. In two concentric circles centred at O, a chord AB of the larger circle touches the smaller circle at C. If OA = 3.5 cm, OC = 2.1 cm, then AB is equal to  (A) 5.6 cm (B) 2.8 cm (C) 3.5 cm (D) 4.2 cm	
Ans: (A) 5.6 cm	1
14. If $\sqrt{3}\sin\theta = \cos\theta$, then value of θ is (A) $\sqrt{3}$ (B) 60° (C) $\frac{1}{\sqrt{3}}$ (D) 30°	
Ans: (D) 30°	1
15. To calculate mean of a grouped data, Rahul used assumed mean method. He used $d = (x - A)$, where A is assumed mean. Then $\overline{x}$ is equal to (A) $A + \overline{d}$ (B) $A + h\overline{d}$ (C) $h(A + \overline{d})$ (D) $A - h\overline{d}$	
Ans: (A) $A + \overline{d}$	1
16. If the sum of first n terms of an A.P. is given by $S_n = \frac{n}{2}(3n + 1)$, then the first term of the A.P. is (A) 2 (B) $\frac{3}{2}$ (C) 4 (D) $\frac{5}{2}$	
Ans: (A) 2	1

17. In ΔABC, $\angle B = 90°$. If $\frac{AB}{AC} = \frac{1}{2}$, then cos C is equal to (A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$	
Ans: (C) $\frac{\sqrt{3}}{2}$	1
18. The volume of air in a hollow cylinder is 450 cm^3. A cone of same height and radius as that of cylinder is kept inside it. The volume of empty space in the cylinder is  (A) 225 cm^3 (B) 150 cm^3 (C) 250 cm^3 (D) 300 cm^3	
Ans: (D) 300 cm^3	1
(Assertion – Reason based questions) **Directions :** In question numbers **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option : (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not correct explanation for Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true. 19. **Assertion (A) :** $(a + \sqrt{b}) \cdot (a - \sqrt{b})$ is a rational number, where a and b are positive integers. **Reason (R) :** Product of two irrationals is always rational.	
Ans: (C) Assertion (A) is true, but Reason (R) is false.	1

20. **Assertion (A) :** $\Delta ABC \sim \Delta PQR$ such that $\angle A = 65°$, $\angle C = 60°$. Hence $\angle Q = 55°$. **Reason (R) :** Sum of all angles of a triangle is 180°.	
Ans: (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).	1
SECTION B *This section has **5** Very Short Answer (VSA) type questions of **2** marks each. 5×2=10* 21. (a) Solve the equation $4x^2 - 9x + 3 = 0$, using quadratic formula. **OR** (b) Find the nature of roots of the equation $3x^2 - 4\sqrt{3}x + 4 = 0$.	
Solution: (a) Discriminant = 33 $\Rightarrow x = \frac{9 \pm \sqrt{33}}{8}$ **OR** (b) Discriminant = $(-4\sqrt{3})^2 - 4 \times 4 \times 3 = 0$ $\Rightarrow$ The given equation has real and equal roots	1 1 1 1
22. In a trapezium ABCD, AB||DC and its diagonals intersect at O. Prove that $\frac{OA}{OC} = \frac{OB}{OD}$.	
Solution:  Proving Δ OAB ~ Δ OCD (By AA similarity criterion) $\therefore \frac{OA}{OC} = \frac{OB}{OD}$ (corresponding sides of similar triangles are proportional)	1½ ½
23. A box contains 120 discs, which are numbered from 1 to 120. If one disc is drawn at random from the box, find the probability that (i) it bears a 2– digit number (ii) the number is a perfect square.	

Solution: (i) P(2 - digit number) $=\frac{90}{120}$ or $\frac{3}{4}$ (ii) P(the number is a perfect square) = $\frac{10}{120}$ or $\frac{1}{12}$	1 1

24. (a) Evaluate : $\dfrac{\cos 45°}{\tan 30° + \sin 60°}$

OR

(b) Verify that $\sin 2A = \dfrac{2 \tan A}{1 + \tan^2 A}$, for A = 30°.

Solution:	
(a) $\dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{3}}+\frac{\sqrt{3}}{2}}$	1½
$= \dfrac{2\sqrt{3}}{5\sqrt{2}}$ or $\dfrac{\sqrt{6}}{5}$	½
OR	
(b) LHS = $\sin 60° = \dfrac{\sqrt{3}}{2}$	½
RHS = $\dfrac{2 \tan 30°}{1 + \tan^2 30°} = \dfrac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$	1
$= \dfrac{\sqrt{3}}{2}$ = LHS	½

25. Using prime factorisation, find the HCF of 180, 140 and 210.

Solution:	
$180 = 2^2 \times 3^2 \times 5$, $140 = 2^2 \times 5 \times 7$, $210 = 2 \times 3 \times 5 \times 7$	1½
HCF (180, 140, 210) = 2 × 5 = 10	½

Section – C

(Short Answer Type Questions) **6 × 3 = 18**

Q. Nos. **26** to **31** are Short Answer type questions of **3** marks each.

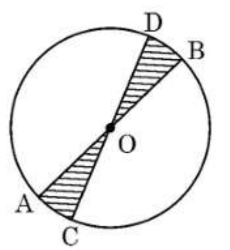
26. (a) If α, β are zeroes of the polynomial $8x^2 - 5x - 1$, then form a quadratic polynomial in x whose zeroes are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$. **OR** (b) Find the zeroes of the polynomial $p(x) = 3x^2 + x - 10$ and verify the relationship between zeroes and its coefficients.	
Solution: (a) $p(x) = 8x^2 - 5x - 1$ $\alpha + \beta = \frac{5}{8}$, $\alpha\beta = \frac{-1}{8}$ $\therefore$ sum of zeroes = $\frac{2}{\alpha} + \frac{2}{\beta} = -10$ and product of zeroes = $\frac{2}{\alpha} \times \frac{2}{\beta} = -32$ Required polynomial = $x^2 + 10x - 32$ or $k(x^2 + 10x - 32)$ where k is any non-zero real number. **OR** (b) $p(x) = 3x^2 + x - 10$ $= (x + 2)(3x - 5)$ Zeroes of p(x) are -2 and $\frac{5}{3}$ Sum of zeroes = $-2 + \frac{5}{3} = \frac{-1}{3} = -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$ Product of zeroes = $-2 \times \frac{5}{3} = \frac{-10}{3} = \frac{\text{constant term}}{\text{coefficient of } x^2}$	½ + ½ ½ ½ 1 1 1 1

27. Find length and breadth of a rectangular park whose perimeter is 100 m and area is 600 m^2.	
Solution: Let length and breadth of the park be a metres and b metres respectively. A.T.Q. $2(a + b) = 100$ …(i) and $ab = 600$ …(ii) using (i) & (ii) we get $a^2 - 50a + 600 = 0$ $\Rightarrow a = 30$ or 20 and $b = 20$ or 30 $\therefore$ length = 30 m, breadth = 20 m or length = 20 m, breadth = 30 m	½ ½ 1 ½ ½

28. Three measuring rods are of lengths 120 cm, 100 cm and 150 cm. Find the least length of a fence that can be measured an exact number of times, using any of the rods. How many times each rod will be used to measure the length of the fence ?

Solution:	
$120 = 2^3 \times 3 \times 5$, $100 = 2^2 \times 5^2$, $150 = 2 \times 3 \times 5^2$	1½
LCM (120, 100, 150) = $2^3 \times 3 \times 5^2 = 600$	1
∴ Least length of the fence is 600 cm.	
Each rod is used 5, 6 and 4 times respectively	½

29. AB and CD are diameters of a circle with centre O and radius 7 cm. If ∠BOD = 30°, then find the area and perimeter of the shaded region.



Solution:	
Area of the shaded region = $2\left(\frac{30}{360} \times \frac{22}{7} \times 7 \times 7\right)$	1
$= \frac{77}{3}$ sq. cm or 25.67 sq. cm	½
Perimeter of the shaded region = $2\left(14 + \frac{30}{360} \times 2 \times \frac{22}{7} \times 7\right)$	1
$= \frac{106}{3}$ cm or 35.33 cm	½

30. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \sec\theta\,\mathrm{cosec}\,\theta + 1$.

Solution:	
LHS = $\dfrac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \dfrac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$	½
$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \dfrac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$	½

$= \dfrac{\sin^3\theta - \cos^3\theta}{(\sin\theta - \cos\theta)\sin\theta\cos\theta}$	1
$= \dfrac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta}$	½
$= \sec\theta\ \text{cosec}\,\theta + 1 = \text{RHS}$	½

31. (a) Find the A.P. whose third term is 16 and seventh term exceeds the fifth term by 12. Also, find the sum of first 29 terms of the A.P.

OR

(b) Find the sum of first 20 terms of an A.P. whose n^{th} term is given by $a_n = 5 + 2n$. Can 52 be a term of this A.P. ?

Solution:	
(a) $a + 2d = 16$ … (i)	½
$a + 6d = 12 + a + 4d$ …(ii)	½
Solving (i) and (ii) to get $d = 6$, $a = 4$	½
∴ A.P. is 4, 10, 16, ...	½
$S_{29} = \dfrac{29}{2}[8 + 28 \times 6]$	½
$= 2552$	½
OR	
(b) $a_n = 5 + 2n$	
getting $a = 7$ and $d = 2$	1
$S_{20} = \dfrac{20}{2}[14 + 19 \times 2]$	
$= 520$	1
$52 = 7 + (n - 1) \times 2$	½
$\Rightarrow n = \dfrac{47}{2}$, which is not a natural number.	
Therefore, 52 cannot be a term of this A.P.	½

Section – D

(Long Answer Type Questions) **4 × 5 = 20**

Q. Nos. **32** to **35** are Long Answer type questions of **5** marks each.

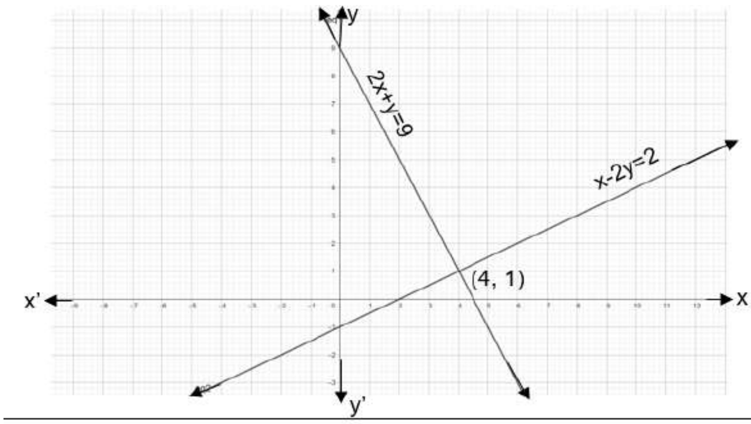
32. (a) Solve the following pair of linear equations by graphical method :

$2x + y = 9$ and $x - 2y = 2$

OR
(b) Nidhi received simple interest of ₹ 1,200 when invested ₹ x at 6% p.a. and ₹ y at 5% p.a. for 1 year. Had she invested ₹ x at 3% p.a. and ₹ y at 8% p.a. for that year, she would have received simple interest of ₹ 1,260. Find the values of x and y.

Solution:

(a) Correct graph of each equation — 2 + 2



Solution x = 4, y = 1 or (4, 1) — 1

OR

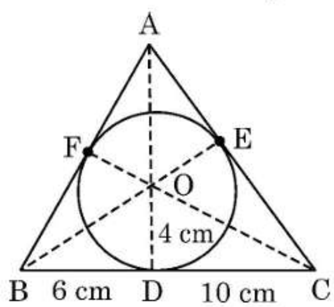
(b) A.T.Q.

$\frac{6}{100}x + \frac{5}{100}y = 1200 \Rightarrow 6x + 5y = 120000$ …(i) — 1½

$\frac{3}{100}x + \frac{8}{100}y = 1260 \Rightarrow 3x + 8y = 126000$ …(ii) — 1½

Solving (i) and (ii) we get, x = 10000 and y = 12000 — 1 + 1

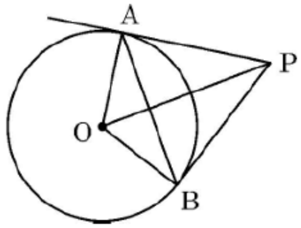
33. (a) The given figure shows a circle with centre O and radius 4 cm circumscribed by ΔABC. BC touches the circle at D such that BD = 6 cm, DC = 10 cm. Find the length of AE.



OR

(b) PA and PB are tangents drawn to a circle with centre O.

If $\angle AOB = 120°$ and OA = 10 cm, then



(i) Find $\angle OPA$. **1**

(ii) Find the perimeter of ΔOAP. **3**

(iii) Find the length of chord AB. **1**

Solution:

(a) Let AE = x $\Rightarrow$ AF = x and CE = 10 cm, BF = 6 cm — 1

(Lengths of tangents drawn from an external point to a circle are equal)

$s = \dfrac{16 + 10 + x + 6 + x}{2} = 16 + x$ — ½

$\therefore$ Area of Δ ABC = $\sqrt{(16 + x)\,(x)\,(6)\,(10)}$ …(i) — 1

Also, area of Δ ABC = $\dfrac{1}{2}[16 \times 4 + (10 + x)4 + (6 + x)4]$ …(ii) — 1

Equating (i) & (ii), we get $x = \dfrac{64}{11}$ or 5·8 — 1½

x = – 16 (Rejected)

Length of AE = $\dfrac{64}{11}$ cm or 5.8 cm

OR

(b) (i) $\angle$ OPA = 30° — 1

(ii) In Δ OAP, $\sin 30° = \dfrac{10}{OP} \Rightarrow OP = 20$ cm — 1

$\tan 30° = \dfrac{10}{PA} \Rightarrow PA = 10\sqrt{3}$ cm — 1

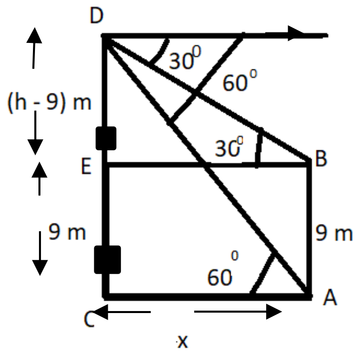
$\therefore$ Perimeter of Δ OPA = $(30 + 10\sqrt{3}\,)$ cm — 1

(iii) PA = PB and $\angle$ APB = 60° — ½

Δ APB is an equilateral triangle

$\therefore$ PA = AB = $10\sqrt{3}$ cm — ½

34. The angles of depression of the top and the foot of a 9 m tall building from the top of a multi-storeyed building are 30° and 60° respectively. Find the height of the multi-storeyed building and the distance between the two buildings. (Use $\sqrt{3} = 1.73$)

Solution: Let CD be the multi storeyed building of height h metres and AB be the 9 m tall building.  In Δ DEB, $\tan 30° = \frac{h-9}{x} \Rightarrow x = (h-9)\sqrt{3}$ (i) In Δ DCA, $\tan 60° = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ (ii) Solving (i) & (ii) we get, x = 7·79, h = 13·5 ∴ The height of the multi-storeyed building is 13.5 m and the distance between the two buildings is 7.79 m	Correct figure 1 1 + ½ 1 + ½ ½ + ½

35. Find 'mean' and 'mode' of the following data :

Class	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45
Frequency	12	10	15	11	7	5

Solution:

Class	x_i	f_i	$u_i = \frac{x_i - 27.5}{5}$	$f_i u_i$
15 – 20	17·5	12	– 2	– 24
20 – 25	22·5	10	– 1	– 10
25 – 30	27·5	15	0	0
30 – 35	32·5	11	1	11
35 – 40	37·5	7	2	14
40 – 45	42·5	5	3	15
		$\sum f_i = 60$		$\sum f_i u_i = 6$

Correct table 1½

Mean $= 27{\cdot}5 + 5 \times \dfrac{6}{60} = 28$ — 1½

Modal class is 25 – 30

Mode $= 25 + 5 \times \dfrac{15 - 10}{30 - 10 - 11}$ — 1½

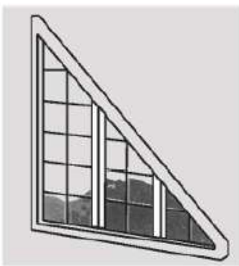
$= 27{\cdot}78$ — ½

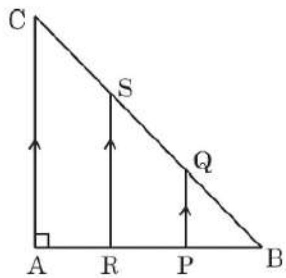
Section – E

(Case-study based Questions) **3 × 4 = 12**

Q. Nos. **36** to **38** are Case-study based Questions of 4 marks each.

36.





A triangular window of a building is shown above. Its diagram represents a ΔABC with ∠A = 90° and AB = AC. Points P and R trisect AB and PQ ∥ RS ∥ AC.

Based on the above, answer the following questions :

(i) Show that ΔBPQ ~ ΔBAC. **1**

(ii) Prove that $PQ = \frac{1}{3} AC$. **1**

(iii) (a) If AB = 3 m, find length BQ and BS. Verify that $BQ = \frac{1}{2} BS$. **2**

OR

(iii) (b) Prove that $BR^2 + RS^2 = \frac{4}{9} BC^2$.

Solution:

(i) In Δ BAC and Δ BPQ, PQ ∥ AC

∴ ∠ BQP = ∠ BCA and ∠ B is common

∴ Δ BPQ ~ Δ BAC (By AA similarity criterion) — 1

(ii) Since, Δ BPQ ~ Δ BAC $\Rightarrow \frac{PQ}{AC} = \frac{BP}{BA} = \frac{1}{3}$ — ½

$\Rightarrow \frac{PQ}{AC} = \frac{1}{3} \Rightarrow PQ = \frac{1}{3} AC$ — ½

(iii) (a) $\frac{BP}{BA} = \frac{PQ}{AC}$ (corresponding sides of similar triangles)

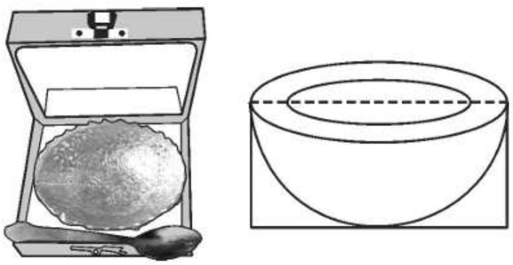
$\Rightarrow$ BP = PQ (as BA = AC)

∴ $PQ = \frac{1}{3} \times 3 = 1$ m

Hence, $BQ = \sqrt{2}$ m — 1

getting BS = $2\sqrt{2}$ m	½
$\Rightarrow \frac{1}{2}$BS = BQ (Hence verified)	½
OR	
(b) $BR^2 + RS^2 = \left(\frac{2}{3}AB\right)^2 + \left(\frac{2}{3}AC\right)^2$	½ + ½
$= \frac{4}{9}(AB^2 + AC^2)$	½
$= \frac{4}{9}BC^2$	½

37.



A hemispherical bowl is packed in a cuboidal box. The bowl just fits in the box. Inner radius of the bowl is 10 cm. Outer radius of the bowl is 10.5 cm.

Based on the above, answer the following questions :

(i) Find the dimensions of the cuboidal box. **1**

(ii) Find the total outer surface area of the box. **1**

(iii) (a) Find the difference between the capacity of the bowl and the volume of the box. (use $\pi = 3.14$) **2**

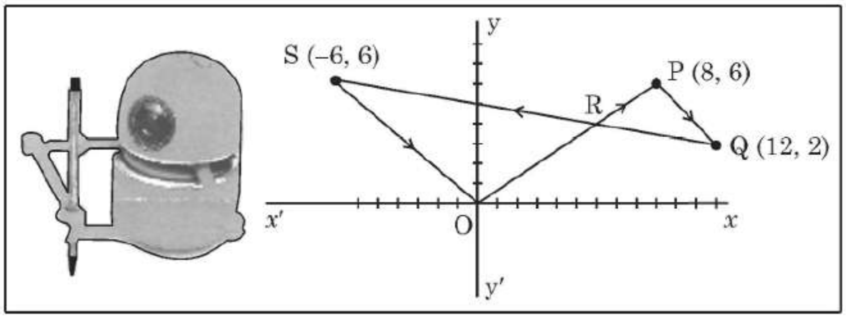
OR

(iii) (b) The inner surface of the bowl and the thickness is to be painted. Find the area to be painted.

Solution: (i) Diameter of bowl = 21 cm	
Dimensions of the box are 21 cm × 21 cm × 10·5 cm	1
(ii) Total surface area of the box = $2\left(441 + \frac{441}{2} + \frac{441}{2}\right)$ = 1764 sq. cm	½ + ½
(iii) (a) Capacity of bowl = $\frac{2}{3} \times 3{\cdot}14 \times 10^3$	½
$= \frac{6280}{3}$ cu. cm or 2093·33 cu. cm	½
Volume of box = $21 \times 21 \times \frac{21}{2} = \frac{9261}{2}$ cu. cm. or 4630·5 cu. cm	½
Required difference = $\frac{15223}{6}$ cu. cm or 2537·17 cu. cm	½
(NOTE: Here capacity is considered as volume to compute the difference.)	
OR	
(b) Required area = $2 \times \frac{22}{7} \times 10^2 + \frac{22}{7} \times (10{\cdot}5^2 - 10^2)$	1

$= \frac{4400}{7} + \frac{451}{14}$ $= \frac{9251}{14}$ sq. cm or 660·79 sq. cm	1

38. Gurveer and Arushi built a robot that can paint a path as it moves on a graph paper. Some co-ordinate of points are marked on it. It starts from (0, 0), moves to the points listed in order (in straight lines) and ends at (0, 0).



Arushi entered the points P(8, 6), Q(12, 2) and S(– 6, 6) in order. The path drawn by robot is shown in the figure.

Based on the above, answer the following questions :

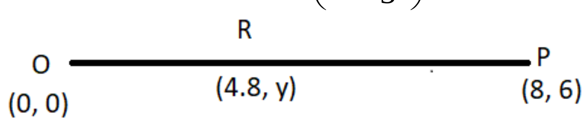
(i) Determine the distance OP. **1**

(ii) QS is represented by equation $2x + 9y = 42$. Find the co-ordinates of the point where it intersects y – axis. **1**

(iii) (a) Point R(4.8, y) divides the line segment OP in a certain ratio, find the ratio. Hence, find the value of y. **2**

OR

(iii) (b) Using distance formula, show that $\frac{PQ}{OS} = \frac{2}{3}$.

Solution: (i) The distance OP = $\sqrt{64 + 36} = 10$	1
(ii) $2x + 9y = 42$ intersects y-axis at $\left(0, \frac{14}{3}\right)$	1
(iii) (a) 	
Let OR : RP = k : 1, therefore $4{\cdot}8 = \frac{8k}{k+1}$ $\Rightarrow k = \frac{3}{2}$	1½
$\Rightarrow$ OR : RP = 3 : 2	
$y = \frac{18}{5}$	½
OR	
(b) PQ = $\sqrt{4^2 + (-4)^2} = \sqrt{32}$ or $4\sqrt{2}$	½
OS = $\sqrt{(-6)^2 + 6^2} = \sqrt{72}$ or $6\sqrt{2}$	½
$\therefore \frac{PQ}{OS} = \sqrt{\frac{32}{72}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$	1