Set 430/5/3

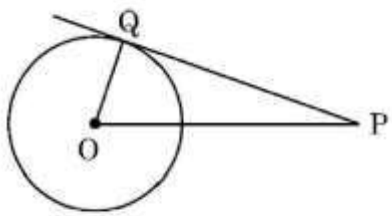
SOLUTIONS
MATHEMATICS (Basic)

Section – A **20 × 1 = 20**

(Multiple Choice Questions)

Section-A consists of **20** Multiple Choice Questions of **1** mark each.

1. PQ is tangent to the circle centred at O. If OQ = 3 cm, PQ = 5 cm, then OP is equal to



(A) 5 cm (B) 4 cm

(C) $\sqrt{15}$ cm (D) $\sqrt{34}$ cm

Ans: (D) $\sqrt{34}$ cm	1

2. Which of the following depends on all observations of a given data ?

(A) Median (B) Mean

(C) Range (D) Mode

Ans: (B) Mean	1

3. A quadratic polynomial having only zero (–2) is

(A) $(x-2)^2$ (B) x^2-2

(C) x^2+2x (D) $(x+2)^2$

Ans: (D) $(x + 2)^2$	1

4. If sin A = cos A, then $\frac{1-\tan A}{1+\tan A}$ is equal to

(A) 1 (B) –1

(C) 0 (D) not defined

Ans: (C) 0	1

5. A black card is lost from a deck of 52 playing cards. Rest of the cards are shuffled and one card is drawn at random from the available cards. The probability that drawn card is 'king of hearts', is

(A) $\frac{1}{52}$ (B) $\frac{1}{4}$

(C) $\frac{1}{51}$ (D) $\frac{1}{26}$

Ans: (C) $\frac{1}{51}$	1

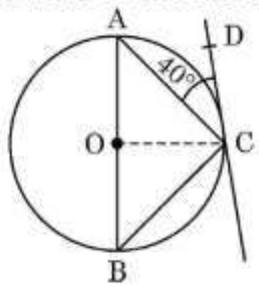
6. The point $(x, 0)$ divides the line segment joining the points $(-4, 5)$ and $(0, -10)$ in the ratio

(A) 1 : 3 (B) 2 : 1

(C) 1 : 1 (D) 1 : 2

Ans: (D) 1 : 2	1

7. In the given figure, AB is diameter of the circle with centre O. CD is tangent to the circle so that $\angle ACD = 40^\circ$. The value of $\angle CBA$ is

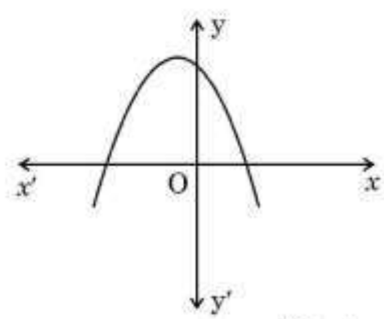


(A) 50° (B) 40°

(C) 80° (D) 45°

Ans: (B) 40°	1

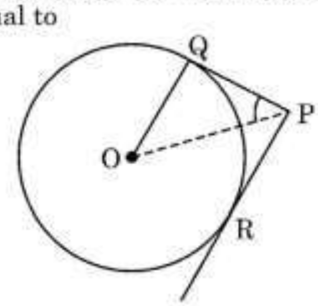
8. Observe the given graph of polynomial $p(x)$. The number of zeroes of $p(x)$ is



(A) 0 (B) 1

(C) 3 (D) 2

Ans: (D) 2	1

9. Two dice are rolled together. The probability that only one die shows number 4, is (A) $\frac{11}{36}$ (B) $\frac{1}{3}$ (C) $\frac{5}{18}$ (D) $\frac{1}{4}$	
Ans: (C) $\frac{5}{18}$	1
10. If the distance between the points (3, 0) and (2, y) is $\sqrt{5}$, then the value(s) of y is : (A) 2, –2 (B) 2, 0 (C) 2, 1 (D) –2, 0	
Ans: (A) 2, −2	1
11. PQ and PR are tangents to a circle with centre O such that OQ = QP. The value of ∠OPQ is equal to  (A) 45° (B) 30° (C) 60° (D) 90°	
Ans: (A) 45°	1
12. The roots of the equation $x^2 - 8 = 0$ are (A) rational and distinct (B) irrational and distinct (C) real and equal (D) not real	
Ans: (B) irrational and distinct	1
13. 10^{th} term of the A.P. : –12, –19, –26, ... is (A) –75 (B) –65 (C) 51 (D) –82	
Ans: (A) – 75	1
14. If E is an event such that P(E) = 0.1, then P($\overline{E}$) is equal to (A) 0.9 (B) $\frac{1}{2}$ (C) 0.99 (D) –1	

Ans: (A) 0·9	1
15. The largest possible cone is just fitted inside a hollow cube of edge 25 cm. The radius of the base of the cone is (A) 5 cm (B) 12.5 cm (C) 25 cm (D) 10 cm	
Ans: (B) 12·5 cm	1
16. An arc of length 'l' subtends an angle of 15° at the centre of a circle of radius 8.4 cm. The value of l is (A) 22 cm (B) 2.2 cm (C) 9.24 cm (D) 4.2 cm	
Ans: (B) 2.2 cm	1
17. The value of k for which the roots of the quadratic equation $6x^2 + 4kx + k = 0$ are real and equal, is (A) 0 (B) $\frac{3}{4}$ (C) $\frac{-3}{2}$ (D) $\frac{2}{3}$	
Ans: (A) 0	1
18. $\sqrt{2}(\sqrt{2} - 1)$ is (A) an integer (B) a rational number (C) an irrational number (D) equal to 1	
Ans: (C) an irrational number	1

(Assertion – Reason based questions)

Directions : Question numbers **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** In a right angle triangle ABC, ∠B = 90°. Therefore the value of cos (A + C) is equal to 0. **Reason (R) :** A + B + C = 180° and cos 90° = 0.		
Ans: (A)	Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	1
20. **Assertion (A) :** When a hemisphere of same radius (r) is carved out from one side of a solid wooden cylinder, the total surface area of remaining solid is increased by $2\pi r^2$. **Reason (R) :** Curved surface area of hemisphere is $2\pi r^2$.		
Ans: (D)	Assertion (A) is false, but Reason (R) is true.	1

Section – B

(Very Short Answer Type Questions) **5 × 2 = 10**

Q. Nos. **21 to 25** are Very Short Answer type questions of **2** marks each.

21. (a) A bag contains 40 marbles out of which some are white and others are black. If the probability of drawing a black marble is $\frac{3}{5}$, then find the number of white marbles.

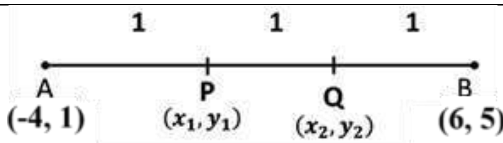
OR

(b) In a pre-primary class, a teacher put cards numbered 20 to 59 in a bowl. A student picked up a card at random and read the number. Find the probability that the number read was (i) a prime number (ii) a perfect square.

Solution: (a) Let the number of black marbles be n.	
P (drawing a black marble) = $\frac{n}{40}$	½
$\therefore \frac{3}{5} = \frac{n}{40} \Rightarrow n = 24$	1
Hence, the number of white marbles = 16	½
OR	
(b) Total number of cards = 40	
(i) P (a prime number) = $\frac{9}{40}$	1
(ii) P (no. is perfect square) = $\frac{3}{40}$	1

22. Find the coordinates of the points of trisection of line segment joining the points (–4, 1) and (6, 5).

Solution:	



AP : PB = 1 : 2 $\therefore x_1 = \frac{-8+6}{3} = \frac{-2}{3}$, $y_1 = \frac{5+2}{3} = \frac{7}{3}$ Coordinates of point P are $\left(\frac{-2}{3}, \frac{7}{3}\right)$ AQ : QB = 2 : 1 $\therefore x_2 = \frac{12-4}{3} = \frac{8}{3}$, $y_2 = \frac{10+1}{3} = \frac{11}{3}$ Coordinates of point Q are $\left(\frac{8}{3}, \frac{11}{3}\right)$	½ + ½ ½ + ½
23. Using distance formula, show that the points (–1, 3), (6, 2) and (3, –1) are vertices of a right-angled triangle.	
Solution: Let A(– 1, 3), B(6, 2) and C(3, – 1) are given points AB = $\sqrt{(6+1)^2 + (2-3)^2} = \sqrt{50}$ BC = $\sqrt{(3-6)^2 + (-1-2)^2} = \sqrt{18}$ CA = $\sqrt{(-1-3)^2 + (3+1)^2} = \sqrt{32}$ Since $AB^2 = BC^2 + CA^2$ $\Rightarrow \Delta$ ABC is right angled at C	 ½ ½ ½ ½
24. Check whether $15^n \times 2^n$, n being a natural number, ends with the digit zero.	
Solution: $15^n \times 2^n = 5^n \times 3^n \times 2^n$ $\Rightarrow$ 2 and 5 both are the factors of the given number $\therefore$ The given number ends with the digit zero	1 1
25. (a) If sec A = $\frac{25}{7}$, then find the value of cosec A and tan A. **OR** (b) Verify that sin (A + B) = sin A cos B + cos A sin B for A = 60° and B = 30°.	
Solution: (a) cosec A = $\frac{25}{24}$ and tan A = $\frac{24}{7}$ **OR** (b) LHS = sin (60° + 30°) = sin 90° = 1 RHS = sin 60° cos 30° + cos 60° sin 30°	1 1 ½ 1

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$ $= 1 =$ LHS	½

Section – C

(Short Answer Type Questions) **6 × 3 = 18**

Q. Nos. **26** to **31** are Short Answer type questions of **3** marks each.

26. Prove that : $(\sin\theta + \sec\theta)^2 + (\cos\theta + \text{cosec}\,\theta)^2 = (1 + \sec\theta\,\text{cosec}\,\theta)^2$.

Solution:	
LHS $= \sin^2\theta + \sec^2\theta + 2\sin\theta\sec\theta + \cos^2\theta + \text{cosec}^2\theta + 2\cos\theta\,\text{cosec}\,\theta$	1
$= 1 + \left(\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}\right) + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$	½
$= 1 + \frac{1}{\sin^2\theta\cos^2\theta} + \frac{2}{\sin\theta\cos\theta}$	½
$= 1 + \sec^2\theta\,\text{cosec}^2\theta + 2\sec\theta\,\text{cosec}\,\theta$	½
$= (1 + \sec\theta\,\text{cosec}\,\theta)^2 =$ RHS	½

27. The traffic lights at three different road crossings change after every 45 seconds, 75 seconds and 60 seconds respectively. If they change together at 5.00 a.m., then at what time they will change together next ?

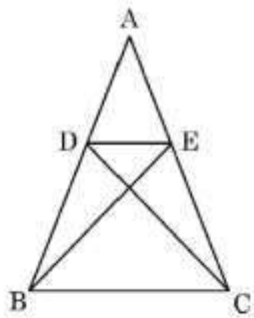
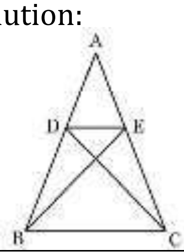
Solution: $45 = 3^2 \times 5, 75 = 3 \times 5^2, 60 = 2^2 \times 3 \times 5$	1½
LCM (45, 75, 60) $= 2^2 \times 3^2 \times 5^2 = 900$	½
900 seconds = 15 minutes	½
Lights will change together at 5:15 a.m. again	½

28. (a) If α, β are zeroes of the polynomial $3x^2 - 8x + 4$, then form a quadratic polynomial in x whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

OR

(b) Find zeroes of the polynomial $6x^2 - 7x - 3$ and verify the relationship between zeroes and its coefficients.

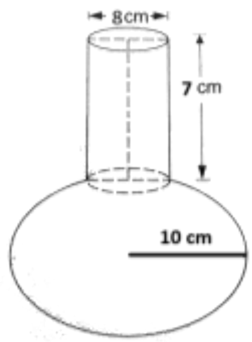
Solution:	
(a) $p(x) = 3x^2 - 8x + 4$	
$\alpha + \beta = \frac{8}{3}, \alpha\beta = \frac{4}{3}$	½ + ½
$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = 2$	½
and $\frac{1}{\alpha\beta} = \frac{3}{4}$	½
$\therefore$ required polynomial is $x^2 - 2x + \frac{3}{4}$	1

or $k(4x^2 - 8x + 3)$, where k is a non-zero real number **OR** (b) $p(x) = 6x^2 - 7x - 3 = (2x - 3)(3x + 1)$ Zeroes of p(x) are $\frac{3}{2}$ and $-\frac{1}{3}$ Sum of zeroes $= \frac{3}{2} - \frac{1}{3} = \frac{7}{6} = -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$ Product of zeroes $= \frac{3}{2} \times \frac{-1}{3} = \frac{-3}{6} = \frac{\text{constant term}}{\text{coefficient of } x^2}$	 1 1 1
29. It is given that $\Delta ACD \cong \Delta ABE$. Prove that $\Delta ADE \sim \Delta ABC$. 	
Solution:  Given, $\Delta ACD \cong \Delta ABE$, $\therefore$ by CPCT $AC = AB$ and $AD = AE$ $\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$ and $\angle A$ is common Hence, by SAS similarity criterion $\Delta ADE \sim \Delta ABC$	1 1 1
30. The sum of the squares of two consecutive odd numbers is 514. Find the numbers.	
Solution: Let two consecutive odd numbers be x and x + 2 A.T.Q. $x^2 + (x + 2)^2 = 514$ $\Rightarrow 2x^2 + 4x - 510 = 0$ or $x^2 + 2x - 255 = 0$ $\Rightarrow (x + 17)(x - 15) = 0$ $\Rightarrow x = 15$ Required numbers are 15 and 17	½ 1 1 ½
31. (a) A spherical glass vessel has a cylindrical neck 7 cm long and 8 cm in diameter. The radius of spherical part is 10 cm. Find the volume of the vessel.	

OR

(b) From each end of a solid cylinder of height 20 cm and base radius 7 cm, a cone of base radius 2.1 cm and height 5 cm is scooped out. Find the volume of the remaining solid.

Solution:



(a) Volume of the vessel $= \frac{4}{3} \times \frac{22}{7} \times 10 \times 10 \times 10 + \frac{22}{7} \times 4 \times 4 \times 7$	1 + 1
$= 4190{\cdot}4 + 352 = 4542{\cdot}48$ cu. cm	1
OR	
(b) Volume of cylinder $= \frac{22}{7} \times 7 \times 7 \times 20 = 3080$ cu. cm	1
Volume of cones $= 2 \times \frac{1}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times 5 = 46{\cdot}2$ cu. cm	1
Volume of remaining solid $= 3080 - 46{\cdot}2$ $= 3033{\cdot}8$ cu. cm	1

Section – D

(Long Answer Type Questions) **4 × 5 = 20**

Q. Nos. **32** to **35** are Long Answer type questions of **5** marks each.

32. (a) Find 'mean' and 'mode' of the following data :

Class	10-25	25-40	40-55	55-70	70-85	85-100
Number of Students	12	10	15	13	8	12

OR

(b) The following table shows the ages of patients admitted in a hospital during a year :

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of Patients	7	10	21	22	15	5

Find 'mode' and 'median' of the above data.

Solution: (a)

CI	x_i	f_i	$u_i = \frac{x_i - 47\cdot5}{15}$	f_iu_i
10 – 25	17·5	12	– 2	– 24
25 – 40	32·5	10	– 1	– 10
40 – 55	47·5	15	0	0
55 – 70	62·5	13	1	13
70 – 85	77·5	8	2	16
85 – 100	92·5	12	3	36
		70		31

Correct Table 1½

$\text{Mean} = 47\cdot5 + 15 \times \frac{31}{70} = 54\cdot14$ 1½

Modal class is 40 – 55

$\text{Mode} = 40 + 15 \times \frac{15 - 10}{30 - 10 - 13}$ 1½

$= 50\cdot71$ ½

OR

(b)

C I	5-15	15-25	25-35	35-45	45-55	55-65
f	7	10	21	22	15	5
cf	7	17	38	60	75	N = 80

Correct table 1

Median class is 35 – 45

$\text{Median} = 35 + \frac{10}{22} \times (40 - 38)$ 1½

$= 35\cdot91$ ½

Modal class is 35 – 45

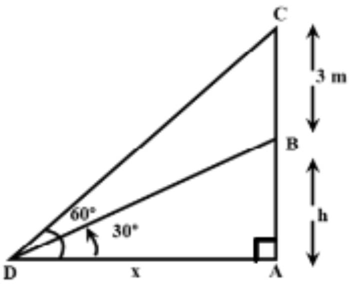
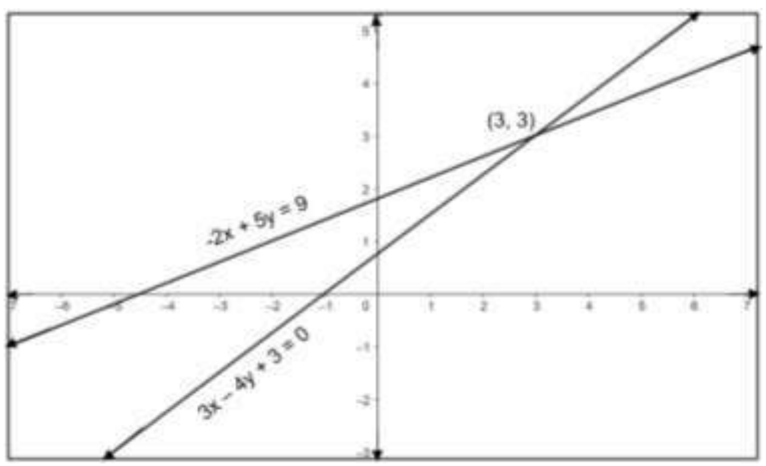
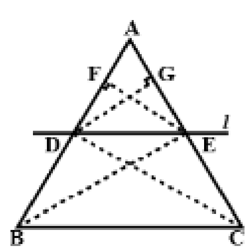
$\text{Mode} = 35 + \frac{22 - 21}{44 - 21 - 15} \times 10$ 1½

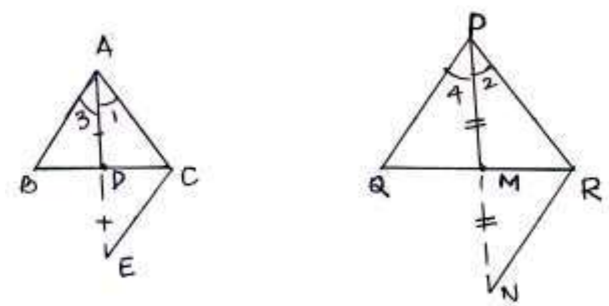
$= 36\cdot25$ ½

33. A statue 3 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 30°. Find the height of the pedestal and its distance from the point of observation on ground. (Use $\sqrt{3}$ = 1.73)

Solution: Let height of the pedestal be AB = h m and the distance between pedestal and the point of observation on the ground be AD = x m and

Correct Figure 1

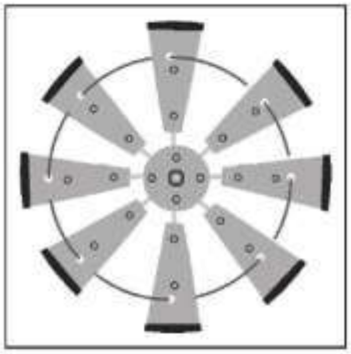
 the height of the statue be BC. In ΔDAB, $\tan 30° = \frac{h}{x} \Rightarrow x = h\sqrt{3}$ In ΔDAC, $\tan 60° = \frac{h+3}{x} \Rightarrow 2h = 3$ Solving equations to get $h = 1{\cdot}5$ and $x = 2{\cdot}6$ The height of the pedestal = 1.5 m and distance from the point of observation = 2.6 m	1½ 1½ 1
34. Solve the following pair of equations using graphical method : $3x - 4y + 3 = 0$ and $-2x + 5y = 9$	
Solution: Correct graph of each equation  Getting solution x = 3 , y = 3 or (3, 3)	2+2 1
35. (a) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. **OR** (b) It is given that sides AB and AC and median AD of ΔABC are respectively proportional to sides PQ and PR and median PM of another ΔPQR. Show that ΔABC ~ ΔPQR.	
Solution:  	Correct Figure ½

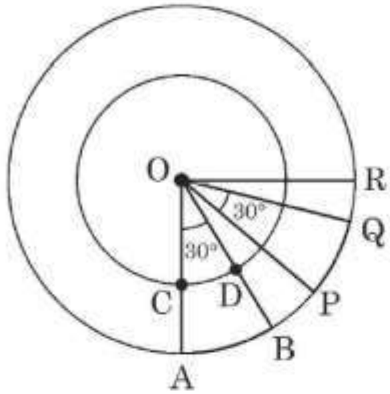
Given: In Δ ABC, DE || BC To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$ Construction: Join BE, DC, Draw DG $\perp$ AC and EF $\perp$ AB	1
Proof: $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF} = \frac{AD}{DB}$ ……….(i)	1
and $\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} = \frac{AE}{EC}$ ……….. (ii)	1
As ΔBDE and ΔCDE are on the same base DE and between the same parallels DE and BC. $\therefore$ ar(ΔBDE) = ar(ΔCDE) ……….(iii)	1
From (i), (ii) and (iii), we get $\frac{AD}{DB} = \frac{AE}{EC}$	½
OR	
	Correct Figure 1
Extend AD to E and PM to N such that AD = DE and PM = MN. Proving Δ DAB $\cong$ Δ DEC (By SAS congruency criterion)	1
Similarly, Δ MPQ $\cong$ Δ MNR $\therefore$ AB = CE and PQ = NR (by cpct)	½
Given $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ $\Rightarrow \frac{CE}{NR} = \frac{AE/2}{PN/2} = \frac{AC}{PR}$ $\Rightarrow \frac{CE}{NR} = \frac{AE}{PN} = \frac{AC}{PR}$	1
Hence Δ CAE ~ Δ RPN (By SSS similarity criterion) $\Rightarrow \angle 1 = \angle 2$, similarly $\angle 3 = \angle 4$	½
Adding, we get $\angle 1 + \angle 3 = \angle 2 + \angle 4$ or $\angle BAC = \angle QPR$	½
Hence, Δ ABC ~ Δ PQR (By SAS similarity criterion)	½

Section – E

(Case-study based Questions) **3 × 4 = 12**

Q. Nos. **36** to **38** are Case-study based Questions of **4** marks each.

36. 



A farmer has put up a decorative windmill in his farm in which there are eight blades of equal width and equally placed in a circular arrangement. A circular wire goes through them.

The diagram shows two blades OAB and OPQ in a quarter circle with centre O. ∠AOB = ∠POQ = 30°, OA = 28 cm, OC = 21 cm.

O is the centre of both the circles.

(i) Determine the measure of ∠BOP.

(ii) Find length of arc CD.

(iii) (a) Find the area of region CABD.

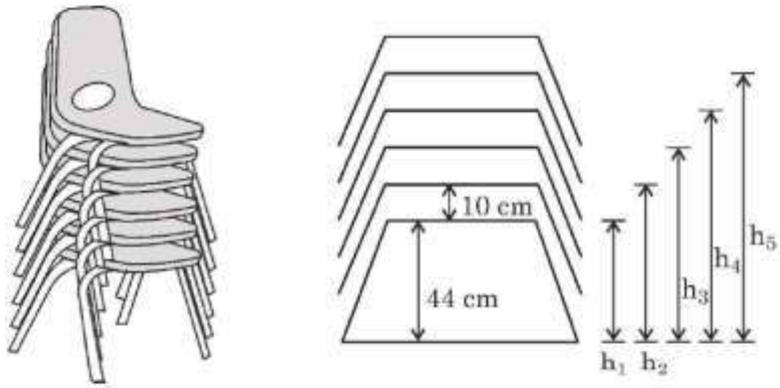
OR

(iii) (b) Find perimeter of region CABD.

Solution: (i) ∠AOC = 90° and blades are equally placed	
$\therefore \angle BOP = \frac{1}{2}(90° - 60°) = 15°$	1
(ii) Length of arc CD $= \frac{30}{360} \times 2 \times \frac{22}{7} \times 21 = 11$ cm	1
(iii) (a) Area CABD $= \frac{30}{360} \times \frac{22}{7} \times (28^2 - 21^2)$	1
$= 89.8$ sq. cm	1
OR	
(iii) (b) Length of arc AB $= \frac{30}{360} \times 2 \times \frac{22}{7} \times 28 = \frac{44}{3} = 14{\cdot}67$ cm	1

Perimeter of CABD = 14·67 + 11 + 2 × (28 – 21) = 39·67 cm	1

37. A tent house owner provides furniture on rent. He stacks chairs in his shop to save space.



In the diagram, the height of seat of chair from ground is represented by h_1, h_2, h_3, ... The height of first seat is 44 cm from ground level and gap between every two seats is 10 cm.

(i) Write the values of h_1, h_2, h_3, h_4 and h_5 in this order only.

(ii) Show that the above values form an A.P. Write its first term and common difference.

(iii) (a) If chairs can be stacked up to the maximum height of 160 cm, then find the maximum number of chairs in a stack.

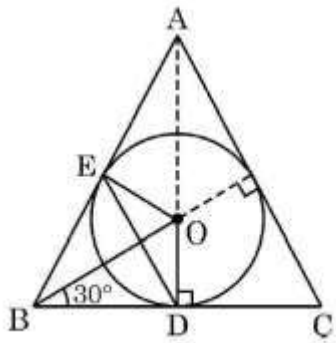
OR

(iii) (b) Is it possible to stack 15 chairs if maximum height of the stack can not be more than 180 cm ? Justify your answer.

Solution:	
(i) $h_1 = 44, h_2 = 54, h_3 = 64, h_4 = 74, h_5 = 84$	1
(ii) Since gap between heights of seats of every two adjacent chairs is same therefore h_1, h_2, h_3, ... form an A.P.	½
Here, a = 44, d = 10	½
(iii) (a) 160 = 44 + (n – 1) × 10	1
⇒ n = 12·6	½
Therefore maximum 12 chairs can be stacked up.	½
OR	

(iii) (b) $h_{15} = 44 + 14 \times 10$ $= 184$ cm $184 \text{ cm} > 180 \text{ cm}$ ∴ 15 chairs cannot be stacked up	1 ½ ½

38.



In a Fine Arts class, students were asked to design triangular tiles in geometric pattern.

Neelima made a circular design inside an equilateral triangle ABC. The radius of the circle is 4 cm. Observe the diagram and answer the following questions :

Neelima made a circular design inside an equilateral triangle ABC. The radius of the circle is 4 cm. Observe the diagram and answer the following questions :

(i) Determine the length OB.

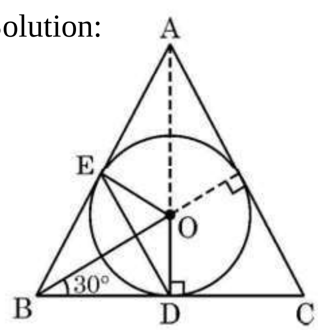
(ii) Is DE || CA ? Give reason for your answer.

(iii) (a) Write all angles of quadrilateral OEBD and show that it is a cyclic quadrilateral.

OR

(iii) (b) Find the perimeter of ΔABC. (Use $\sqrt{3} = 1.73$)

Solution:



(i) In $\triangle ODB$, $\sin 30° = \frac{4}{OB} \Rightarrow OB = 8$ cm	1
(ii) Yes, DE || CA Δ ABC is an equilateral triangle and $AD \perp BC$ ⇒ D is the mid point of BC Similarly, E is the mid point of AB, so DE || CA	 ½ ½
(iii) (a) $\angle EBD = 60° \Rightarrow \angle EOD = 120°$ $\angle OEB = \angle ODB = 90°$ (radius is perpendicular to the tangent through the point of contact)	½ ½

$\angle$ OEB + $\angle$ ODB = 90° + 90° = 180° $\therefore$ quad. OEBD is a cyclic quad.	½ ½
OR	
(iii) (b) In $\triangle$OBD, $\cos 30° = \frac{BD}{8} \Rightarrow BD = 6{\cdot}92$ cm	1
BC = 2 BD = 13.84 cm	
$\therefore$ Perimeter of Δ ABC = 41·52 cm	1