Set 430/5/2

SOLUTIONS

MATHEMATICS (Basic)

Section – A $\quad 20 \times 1 = 20$

(Multiple Choice Questions)

Section-A consists of **20** Multiple Choice Questions of **1** mark each.

1. The point $(x, 0)$ divides the line segment joining the points $(-4, 5)$ and $(0, -10)$ in the ratio

(A) 1 : 3 (B) 2 : 1

(C) 1 : 1 (D) 1 : 2

Ans: (D) 1 : 2	1

2. A black card is lost from a deck of 52 playing cards. Rest of the cards are shuffled and one card is drawn at random from the available cards. The probability that drawn card is 'king of hearts', is

(A) $\frac{1}{52}$ (B) $\frac{1}{4}$

(C) $\frac{1}{51}$ (D) $\frac{1}{26}$

Ans: (C) $\frac{1}{51}$	1

3. $(2 - 5\sqrt{3})^2$ is

(A) a negative integer (B) an irrational number

(C) a rational number (D) a positive integer

Ans: (B) an irrational number	1

4. The 16th term of the A.P. : $5\sqrt{3}, 2\sqrt{3}, -\sqrt{3}, \ldots$ is

(A) $-25\sqrt{3}$ (B) $-40\sqrt{3}$

(C) $50\sqrt{3}$ (D) $-45 + 5\sqrt{3}$

Ans: (B) $-40\sqrt{3}$	1

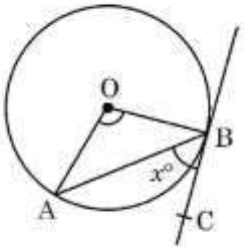
5. The roots of the equation $x^2 - 8 = 0$ are

(A) rational and distinct (B) irrational and distinct

(C) real and equal (D) not real

Ans: (B) irrational and distinct	1

6. Tangent BC makes an angle of x° with the chord AB of circle centered at O. If $\angle AOB = 100^\circ$, then value of x is



(A) 40

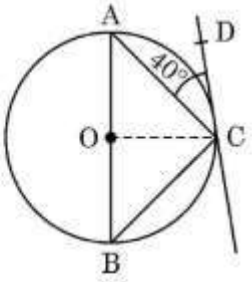
(B) 80

(C) 90

(D) 50

Ans: (D) 50	1

7. In the given figure, AB is diameter of the circle with centre O. CD is tangent to the circle so that $\angle ACD = 40^\circ$. The value of $\angle CBA$ is



(A) 50°

(B) 40°

(C) 80°

(D) 45°

Ans: (B) 40°	1

8. A quadratic polynomial having only zero (–2) is

(A) $(x-2)^2$

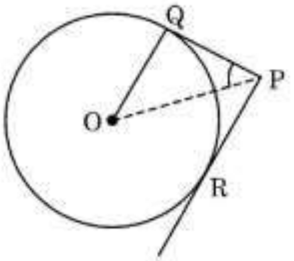
(B) $x^2 - 2$

(C) $x^2 + 2x$

(D) $(x+2)^2$

Ans: (D) $(x + 2)^2$	1

9. PQ and PR are tangents to a circle with centre O such that OQ = QP. The value of ∠OPQ is equal to



(A) 45°
(B) 30°
(C) 60°
(D) 90°

Ans: (A) 45°	1

10. If tan A = 1, then 3 sin A + cos A is equal to

(A) $4\sqrt{2}$
(B) 4
(C) $2\sqrt{2}$
(D) 4 × 45°

Ans: (C) $2\sqrt{2}$	1

11. Which of the following depends on all observations of a given data ?

(A) Median
(B) Mean
(C) Range
(D) Mode

Ans: (B) Mean	1

12. The value of k for which roots of quadratic equation $kx(x-2)+6=0$ are real and equal, is

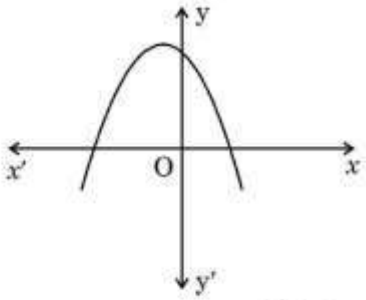
(A) 0 only
(B) 0, 6
(C) 6 only
(D) –6 only

Ans: (C) 6 only	1

13. An arc of length 22 cm subtends an angle of x° at the centre of the circle. If radius of circle is 36 cm, the value of x is

(A) 35
(B) 40
(C) 60
(D) 30

Ans: (A) 35	1

14. Two dice are rolled together. The probability that only one die shows number 4, is (A) $\frac{11}{36}$ (B) $\frac{1}{3}$ (C) $\frac{5}{18}$ (D) $\frac{1}{4}$	
Ans: (C) $\frac{5}{18}$	1
15. The distance between the points (2, 3) and (–2, –3) is (A) $4\sqrt{13}$ (B) $\sqrt{40}$ (C) $2\sqrt{13}$ (D) 5	
Ans: (C) $2\sqrt{13}$	1
16. Observe the given graph of polynomial p(x). The number of zeroes of p(x) is  (A) 0 (B) 1 (C) 3 (D) 2	
Ans: (D) 2	1
17. If E is an event such that P(E) = 1%, then P($\overline{E}$) is equal to (A) 0.09 (B) 0.99 (C) $\frac{1}{99}$ (D) 0.90	
Ans: (B) 0.99	1
18. The largest possible cone is just fitted inside a hollow cube of edge 25 cm. The radius of the base of the cone is (A) 5 cm (B) 12.5 cm (C) 25 cm (D) 10 cm	
Ans: (B) 12·5 cm	1

(Assertion – Reason based questions)

Directions : Question numbers **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** In a right angle triangle ABC, $\angle B = 90^\circ$. Therefore the value of $\cos(A + C)$ is equal to 0.

Reason (R) : $A + B + C = 180^\circ$ and $\cos 90^\circ = 0$.

Ans: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	1

20. **Assertion (A) :** When a hemisphere of same radius (r) is carved out from one side of a solid wooden cylinder, the total surface area of remaining solid is increased by $2\pi r^2$.

Reason (R) : Curved surface area of hemisphere is $2\pi r^2$.

Ans: (D) Assertion (A) is false, but Reason (R) is true.	1

Section – B

(Very Short Answer Type Questions) 5 × 2 = 10

Q. Nos. **21** to **25** are Very Short Answer type questions of **2** marks each.

21. Establish a relation between x and y such that point (x, y) is equidistant from points $(-2, 5)$ and $(3, 9)$.

Solution: $(x + 2)^2 + (y - 5)^2 = (x - 3)^2 + (y - 9)^2$ 1

$\Rightarrow x^2 + y^2 + 4x - 10y + 4 + 25 = x^2 + y^2 - 6x - 18y + 9 + 81$

$\Rightarrow 10x + 8y = 61$ 1

22. Using distance formula, prove that the points (1, 5), (2, 3) and (3, 1) are collinear.

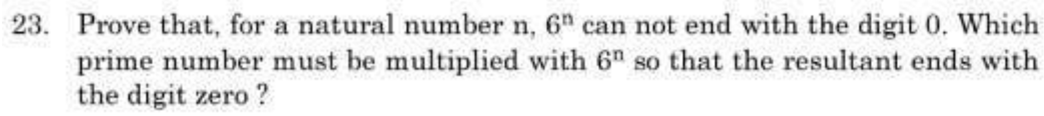
Solution: Let A(1, 5), B(2, 3) and C(3, 1) be the points

$AB = \sqrt{1^2 + (-2)^2} = \sqrt{5}$ ½

$BC = \sqrt{1^2 + (-2)^2} = \sqrt{5}$ ½

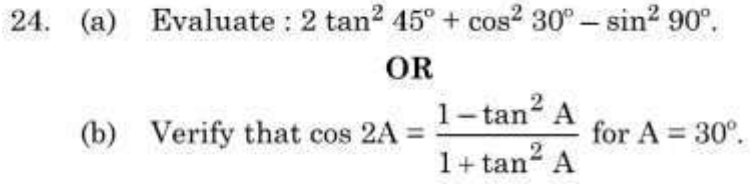
$AC = \sqrt{2^2 + (-4)^2} = \sqrt{20}$ or $2\sqrt{5}$ ½

∴ AB + BC = AC, therefore points A, B and C are collinear. ½



23. Prove that, for a natural number n, 6^n can not end with the digit 0. Which prime number must be multiplied with 6^n so that the resultant ends with the digit zero ?

Solution: $6^n = 2^n \times 3^n$	1
To end with the digit 0, 6^n should have 2 and 5 both as prime factors.	
$\therefore 6^n$ cannot end with the digit 0.	½
To end with digit 0, 6^n should be multiplied by the prime number 5.	½

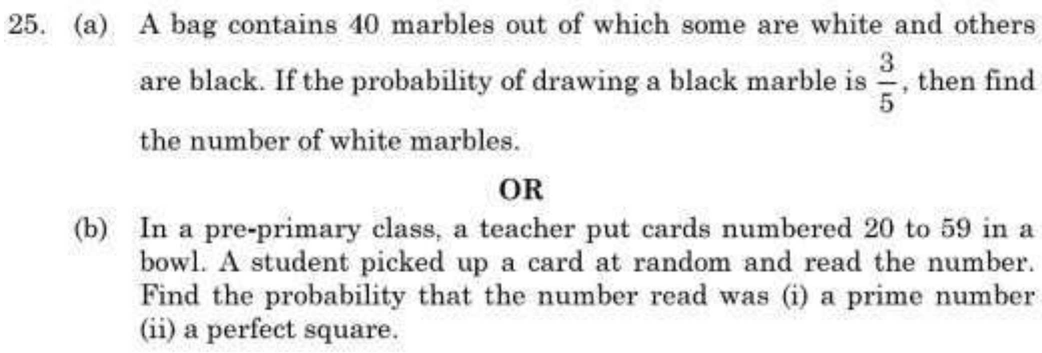


24. (a) Evaluate : $2\tan^2 45° + \cos^2 30° - \sin^2 90°$.

OR

(b) Verify that $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$ for A = 30°.

Solution: (a) $2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2$	1½
$= \frac{7}{4}$	½
OR	
(b) LHS = $\cos 60° = \frac{1}{2}$	½
RHS = $\frac{1-tan^2 30°}{1+tan^2 30°} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}}$	1
$= \frac{1}{2}$ = LHS	½



25. (a) A bag contains 40 marbles out of which some are white and others are black. If the probability of drawing a black marble is $\frac{3}{5}$, then find the number of white marbles.

OR

(b) In a pre-primary class, a teacher put cards numbered 20 to 59 in a bowl. A student picked up a card at random and read the number. Find the probability that the number read was (i) a prime number (ii) a perfect square.

Solution: (a) Let the number of black marbles be n	
P(drawing a black marble) $= \frac{n}{40}$	½
$\therefore \frac{3}{5} = \frac{n}{40} \Rightarrow n = 24$	1

Hence, number of white marbles = 16	½
OR	
(b) Total number of cards = 40	
(i) P (a prime number) = $\frac{9}{40}$	1
(ii) P (no. is perfect square) = $\frac{3}{40}$	1

Section – C

(Short Answer Type Questions) **6 × 3 = 18**

Q. Nos. **26** to **31** are Short Answer type questions of **3** marks each.

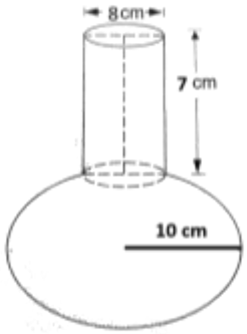
26. Find the smallest number which when increased by 20, is exactly divisible by 72, 90 and 150.

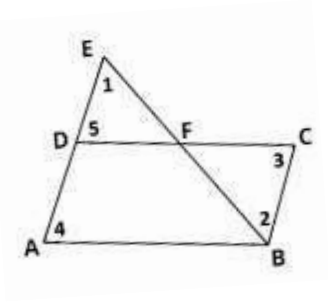
Solution: $72 = 2^3 \times 3^2$, $90 = 3^2 \times 2 \times 5$, $150 = 5^2 \times 2 \times 3$	1½
LCM (72, 90, 150) = $2^3 \times 3^2 \times 5^2 = 1800$	1
Required smallest number is 1800 – 20 = 1780	½

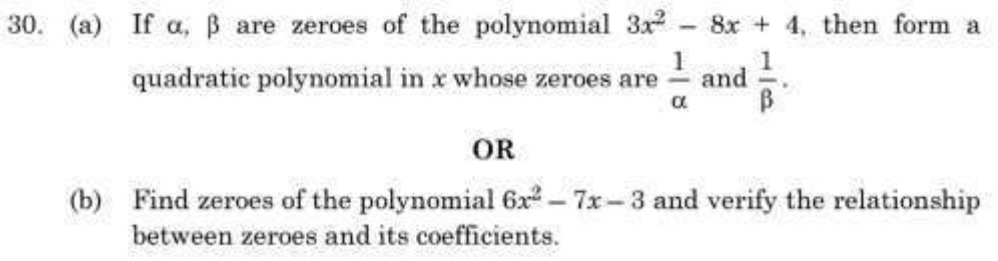
27. (a) A spherical glass vessel has a cylindrical neck 7 cm long and 8 cm in diameter. The radius of spherical part is 10 cm. Find the volume of the vessel.

OR

(b) From each end of a solid cylinder of height 20 cm and base radius 7 cm, a cone of base radius 2.1 cm and height 5 cm is scooped out. Find the volume of the remaining solid.

Solution:	
	
(a) Volume of the vessel = $\frac{4}{3} \times \frac{22}{7} \times 10 \times 10 \times 10 + \frac{22}{7} \times 4 \times 4 \times 7$	1 + 1
= 4190·4 + 352 = 4542·48 cu. cm	1
OR	
(b) Volume of cylinder = $\frac{22}{7} \times 7 \times 7 \times 20 = 3080$ cu. cm	1

Volume of cones = $2 \times \frac{1}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times 5$ = 46·2 cu. cm	1
Volume of remaining solid = 3080 – 46·2 = 3033·8 cu. cm	1
28. Point E lies on the extended side AD of parallelogram ABCD. BE intersects CD at F. Show that (i) ΔDFE ~ ΔCFB (ii) ΔAEB ~ ΔCBF.	
Solution: 	Correct Figure ½
(i) In △DFE and △CFB	
$\angle 5 = \angle 3$ (Alternate Interior Angle)	1
$\angle 1 = \angle 2$ (Alternate Interior Angle)	
∴ By AA similarity criterion, △DFE ~ △CFB	½
(ii) In △AEB and △CBF	
$\angle 1 = \angle 2$ (Alternate Interior Angle)	
$\angle 4 = \angle 3$ (Opposite angles of a parallelogram)	½
∴ By AA similarity criterion, △AEB ~ △CBF	½
29. Prove that : $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \cos\theta + \sin\theta.$	
Solution: LHS $= \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$	½
$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$	1
$= \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\cos\theta-\sin\theta}$	1
$= \cos\theta + \sin\theta$ = RHS	½

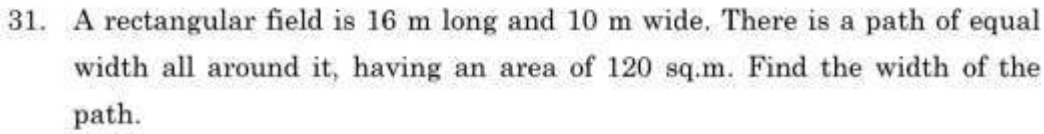


30. (a) If α, β are zeroes of the polynomial $3x^2 - 8x + 4$, then form a quadratic polynomial in x whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

OR

(b) Find zeroes of the polynomial $6x^2 - 7x - 3$ and verify the relationship between zeroes and its coefficients.

Solution: (a) $p(x) = 3x^2 - 8x + 4$	
$\alpha + \beta = \frac{8}{3}, \alpha\beta = \frac{4}{3}$	½ + ½
$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = 2$	½
and $\frac{1}{\alpha\beta} = \frac{3}{4}$	½
∴ required polynomial is $x^2 - 2x + \frac{3}{4}$	1
or $k(4x^2 - 8x + 3)$, where k is a non-zero real number.	
OR	
(b) $p(x) = 6x^2 - 7x - 3 = (2x - 3)(3x + 1)$	
Zeroes of p(x) are $\frac{3}{2}$ and $-\frac{1}{3}$	1
Sum of zeroes = $\frac{3}{2} - \frac{1}{3} = \frac{7}{6} = -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$	1
Product of zeroes = $\frac{3}{2} \times \frac{-1}{3} = \frac{-3}{6} = \frac{\text{constant term}}{\text{coefficient of } x^2}$	1



31. A rectangular field is 16 m long and 10 m wide. There is a path of equal width all around it, having an area of 120 sq.m. Find the width of the path.

Solution: Let the width of the path be x m.	
A. T. Q. $(16 + 2x)(10 + 2x) - 16 \times 10 = 120$	1
$\Rightarrow 4x^2 + 52x - 120 = 0$ or $x^2 + 13x - 30 = 0$	1
$\Rightarrow (x - 2)(x + 15) = 0$	
$\Rightarrow x = 2$	1
(Rejecting x = - 15)	
∴ Width of the path is 2 m.	

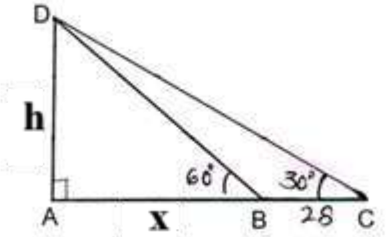
Section – D

(Long Answer Type Questions) **4 × 5 = 20**

Q. Nos. **32** to **35** are Long Answer type questions of **5** marks each.

32. From a point on the ground, the angle of elevation of the top of a tree observed by a person is 60°. When moved back by 28 m, in the same line, the angle of elevation from another point on ground becomes 30°. Find the height of the tree and its distance from the initial point. (Use $\sqrt{3}$ = 1.73)

Solution:



Correct Figure 1

Let the height AD of the tree be h m and its distance from the initial point B be x m

In ΔCAD, $\tan 30° = \dfrac{h}{x + 28} \Rightarrow x + 28 = h\sqrt{3}$ ………. (i) 1 + ½

In ΔBAD, $\tan 60° = \dfrac{h}{x} \quad \Rightarrow h = x\sqrt{3}$ ……….. (ii) 1 + ½

Solving to get, x = 14, h = 14 × 1·73 = 24·22 ½ + ½

Height of the tree = 24.22 m and distance from the initial point = 14 m.

33. (a) Find 'mean' and 'mode' of the following data :

Class	10-25	25-40	40-55	55-70	70-85	85-100
Number of Students	12	10	15	13	8	12

OR

(b) The following table shows the ages of patients admitted in a hospital during a year :

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of Patients	7	10	21	22	15	5

Find 'mode' and 'median' of the above data.

Solution: (a)

CI	x_i	f_i	$u_i = \frac{x_i - 47\cdot5}{15}$	f_iu_i
10 – 25	17·5	12	– 2	– 24
25 – 40	32·5	10	– 1	– 10
40 – 55	47·5	15	0	0
55 – 70	62·5	13	1	13

Correct Table 1½

70 – 85	77·5	8	2	16
85 – 100	92·5	12	3	36
		70		31

1½

$$\text{Mean} = 47{\cdot}5 + 15 \times \frac{31}{70} = 54{\cdot}14$$

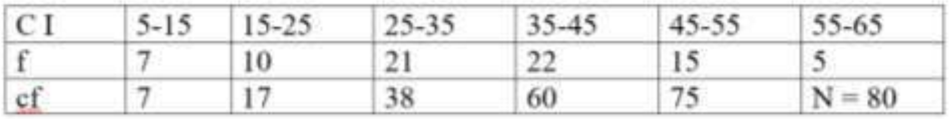
Modal class is 40 – 55

$$\text{Mode} = 40 + 15 \times \frac{15 - 10}{30 - 10 - 13}$$ 1½

$= 50{\cdot}71$ ½

OR

(b)



C I	5-15	15-25	25-35	35-45	45-55	55-65
f	7	10	21	22	15	5
cf	7	17	38	60	75	N = 80

Correct Table 1

Median class is 35 – 45

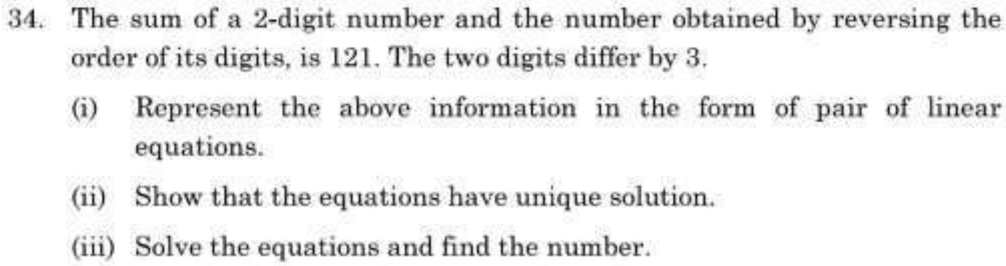
$$\text{Median} = 35 + \frac{10}{22} \times (40 - 38)$$ 1½

$= 35{\cdot}91$ ½

Modal class is 35 – 45

$$\text{Mode} = 35 + \frac{22 - 21}{44 - 21 - 15} \times 10$$ 1½

$= 36{\cdot}25$ ½



34. The sum of a 2-digit number and the number obtained by reversing the order of its digits, is 121. The two digits differ by 3.
    (i) Represent the above information in the form of pair of linear equations.
    (ii) Show that the equations have unique solution.
    (iii) Solve the equations and find the number.

Solution: Let the unit digit be y and tens digit be x (x > y) ½

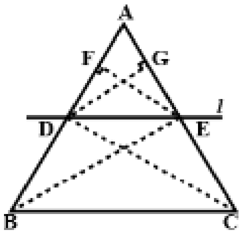
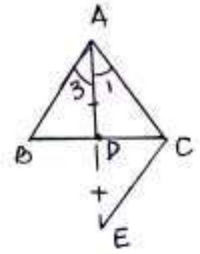
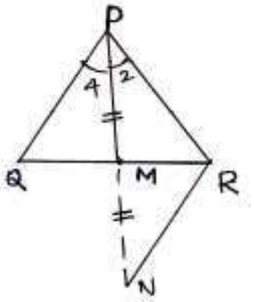
The two-digit number will be 10x + y ½

A.T.Q. (10x + y) + (10y + x) = 121

(i) $\Rightarrow x + y = 11$ (1)

and $x - y = 3$ (2) 1

(ii) $\frac{1}{1} \neq \frac{1}{-1}$ therefore equations have unique solution 1

(iii) Solving equations (1) and (2), we get x = 7, y = 4 ∴ Number is 74 47 may be considered as the correct answer if y > x.	1 1
35. (a) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. **OR** (b) It is given that sides AB and AC and median AD of ΔABC are respectively proportional to sides PQ and PR and median PM of another ΔPQR. Show that ΔABC ~ ΔPQR.	
Solution:  Given: In Δ ABC, DE || BC To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$ Construction: Join BE, DC, Draw DG ⊥ AC and EF ⊥ AB Proof: $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}\times AD\times EF}{\frac{1}{2}\times DB\times EF} = \frac{AD}{DB}$ ……….(i) and $\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2}\times AE\times DG}{\frac{1}{2}\times EC\times DG} = \frac{AE}{EC}$ ……… (ii) As ΔBDE and ΔCDE are on the same base DE and between the same parallels DE and BC. ∴ ar(ΔBDE) = ar(ΔCDE) ..……………..(iii) From (i), (ii) and (iii), we get $\frac{AD}{DB} = \frac{AE}{EC}$ **OR**   Extend AD to E and PM to N such that AD = DE and PM = MN. Proving Δ DAB ≅ Δ DEC (By SAS congruency criterion)	Correct Figure ½ 1 1 1 1 ½ Correct Figure 1 1

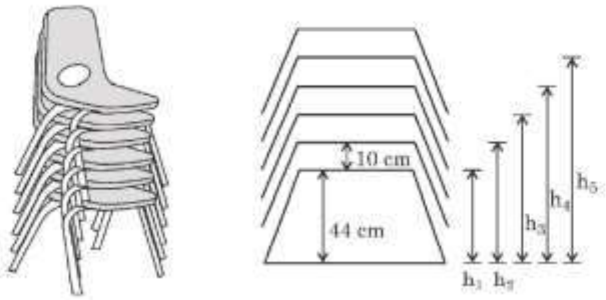
Similarly, $\Delta MPQ \cong \Delta MNR$ $\therefore AB = EC$ and $PQ = NR$ (by cpct)	½
Given $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ $\Rightarrow \frac{CE}{NR} = \frac{AE/2}{PN/2} = \frac{AC}{PR}$ $\Rightarrow \frac{CE}{NR} = \frac{AE}{PN} = \frac{AC}{PR}$	1
Hence $\Delta CAE \sim \Delta RPN$ (By SSS similarity criterion)	½
$\Rightarrow \angle 1 = \angle 2$, similarly $\angle 3 = \angle 4$ Adding, we get $\angle 1 + \angle 3 = \angle 2 + \angle 4$ or $\angle BAC = \angle QPR$	½
Hence, $\Delta ABC \sim \Delta PQR$ (By SAS similarity criterion)	½

Section – E

(Case-study based Questions) **$3 \times 4 = 12$**

Q. Nos. **36** to **38** are Case-study based Questions of **4** marks each.

36. A tent house owner provides furniture on rent. He stacks chairs in his shop to save space.



In the diagram, the height of seat of chair from ground is represented by $h_1, h_2, h_3, \ldots$. The height of first seat is 44 cm from ground level and gap between every two seats is 10 cm.

(i) Write the values of h_1, h_2, h_3, h_4 and h_5 in this order only.

(ii) Show that the above values form an A.P. Write its first term and common difference.

(iii) (a) If chairs can be stacked up to the maximum height of 160 cm, then find the maximum number of chairs in a stack.

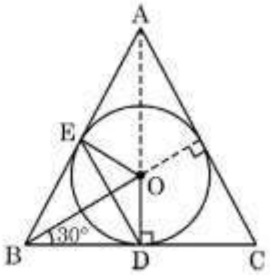
OR

(iii) (b) Is it possible to stack 15 chairs if maximum height of the stack can not be more than 180 cm ? Justify your answer.

Solution: (i) $h_1 = 44, h_2 = 54, h_3 = 64, h_4 = 74, h_5 = 84$	1
(ii) Since gap between heights of seats of every two adjacent chairs is same $\therefore h_1, h_2, h_3, \ldots$ form an A.P.	½

Here, a = 44 and d = 10	½
(iii) (a) $160 = 44 + (n - 1) \times 10$	1
$\Rightarrow n = 12 \cdot 6$	½
∴ maximum 12 chairs can be stacked up.	½
OR	
(iii) (b) $h_{15} = 44 + 14 \times 10$	1
$= 184$ cm	
184 cm > 180 cm	½
∴ 15 chairs cannot be stacked up	½

37.



In a Fine Arts class, students were asked to design triangular tiles in geometric pattern.

Neelima made a circular design inside an equilateral triangle ABC. The radius of the circle is 4 cm. Observe the diagram and answer the following questions :

(i) Determine the length OB.

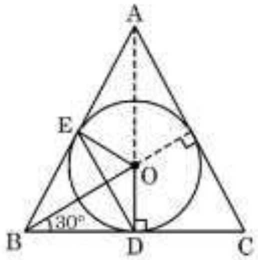
(ii) Is DE || CA ? Give reason for your answer.

(iii) (a) Write all angles of quadrilateral OEBD and show that it is a cyclic quadrilateral.

OR

(iii) (b) Find the perimeter of ΔABC. (Use $\sqrt{3} = 1.73$)

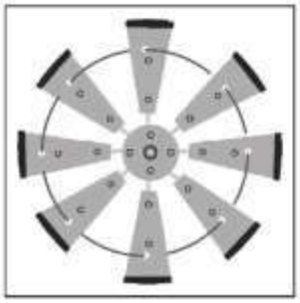
Solution:

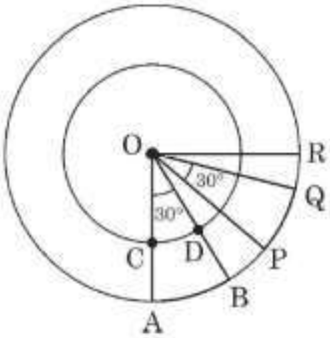


(i)	In △ODB, $\sin 30° = \frac{4}{OB} \Rightarrow OB = 8$ cm	1
(ii)	Yes, DE || CA	½

Δ ABC is an equilateral triangle and AD ⊥ BC	
⇒ D is the mid point of BC	½
Similarly, E is the mid point of AB, so DE ∥ CA	
(iii) (a) ∠ EBD = 60° ⇒ ∠ EOD = 120°	½
∠ OEB = ∠ ODB = 90°	½
(radius is perpendicular to the tangent through the point of contact)	
∠ OEB + ∠ ODB = 90° + 90° = 180°	½
∴ quad. OEBD is a cyclic quad.	½
OR	
(iii) (b) In △OBD, cos 30° = $\frac{\mathrm{BD}}{8}$ ⇒ BD = 6·92 cm	1
BC = 2 BD = 13.84 cm	
∴ Perimeter of Δ ABC = 41·52 cm	1

38.





A farmer has put up a decorative windmill in his farm in which there are eight blades of equal width and equally placed in a circular arrangement. A circular wire goes through them.

The diagram shows two blades OAB and OPQ in a quarter circle with centre O. ∠AOB = ∠POQ = 30°, OA = 28 cm, OC = 21 cm.

O is the centre of both the circles.

(i) Determine the measure of ∠BOP.

(ii) Find length of arc CD.

(iii) (a) Find the area of region CABD.

OR

(iii) (b) Find perimeter of region CABD.

Solution: (i) ∠AOC = 90° and blades are equally placed ∴ ∠BOP = $\frac{1}{2}$(90° – 60°) = 15°	1
(ii) Length of arc CD = $\frac{30}{360} \times 2 \times \frac{22}{7} \times 21$ = 11 cm	1

(iii) (a) Area of region CABD = $\frac{30}{360} \times \frac{22}{7} \times (28^2 - 21^2)$	1
$= 89.8$ sq. cm	1
OR	
(iii) (b) Length of arc AB = $\frac{30}{360} \times 2 \times \frac{22}{7} \times 28 = \frac{44}{3} = 14{\cdot}67$ cm	1
Perimeter of region CABD = $14{\cdot}67 + 11 + 2 \times (28 - 21) = 39{\cdot}67$ cm	1