Set 430/5/1

SOLUTIONS
MATHEMATICS (Basic)

Section – A $20 \times 1 = 20$

(Multiple Choice Questions)

Section-A consists of **20** Multiple Choice Questions of **1** mark each.

1. $(2-5\sqrt{3})^2$ is

(A) a negative integer (B) an irrational number

(C) a rational number (D) a positive integer

Ans: (B) an irrational number	1

2. The value of k for which the roots of the quadratic equation $6x^2 + 4kx + k = 0$ are real and equal, is

(A) 0 (B) $\frac{3}{4}$

(C) $\frac{-3}{2}$ (D) $\frac{2}{3}$

Ans: (A) 0	1

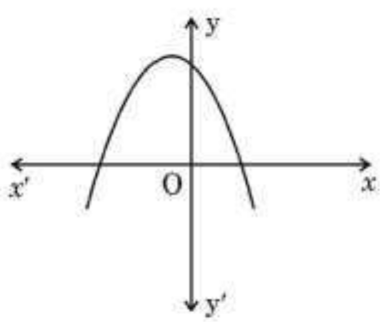
3. The distance between the points (2, 3) and (–2, –3) is

(A) $4\sqrt{13}$ (B) $\sqrt{40}$

(C) $2\sqrt{13}$ (D) 5

Ans: (C) $2\sqrt{13}$	1

4. Observe the given graph of polynomial p(x). The number of zeroes of p(x) is

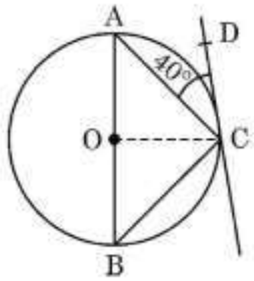


(A) 0 (B) 1

(C) 3 (D) 2

Ans: (D) 2	1

5. In the given figure, AB is diameter of the circle with centre O. CD is tangent to the circle so that $\angle ACD = 40^\circ$. The value of $\angle CBA$ is



(A) 50° (B) 40°

(C) 80° (D) 45°

Ans: (B) 40°	1

6. 10^{th} term of the A.P. : –12, –19, –26, ... is

(A) –75 (B) –65

(C) 51 (D) –82

Ans: (A) – 75	1

7. The roots of the equation $x^2 - 8 = 0$ are

(A) rational and distinct (B) irrational and distinct

(C) real and equal (D) not real

Ans: (B) irrational and distinct	1

8. The point $(x, 0)$ divides the line segment joining the points (–4, 5) and (0, –10) in the ratio

(A) 1 : 3 (B) 2 : 1

(C) 1 : 1 (D) 1 : 2

Ans: (D) 1 : 2	1

9. A black card is lost from a deck of 52 playing cards. Rest of the cards are shuffled and one card is drawn at random from the available cards. The probability that drawn card is 'king of hearts', is

(A) $\frac{1}{52}$ (B) $\frac{1}{4}$

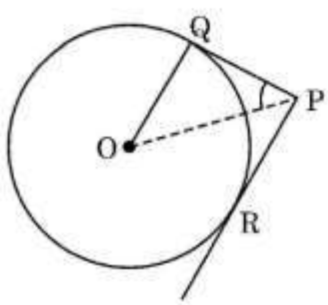
(C) $\frac{1}{51}$ (D) $\frac{1}{26}$

Ans: (C) $\frac{1}{51}$	1

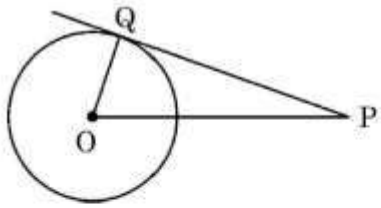
10. The largest possible cone is just fitted inside a hollow cube of edge 25 cm. The radius of the base of the cone is

(A) 5 cm (B) 12.5 cm

(C) 25 cm (D) 10 cm

Ans: (B) 12·5 cm	1
11. If E is an event such that P(E) = 0.1, then $P(\overline{E})$ is equal to (A) 0.9 (B) $\frac{1}{2}$ (C) 0.99 (D) –1	
Ans: (A) 0·9	1
12. If tan A = 1, then 3 sin A + cos A is equal to (A) $4\sqrt{2}$ (B) 4 (C) $2\sqrt{2}$ (D) $4 \times 45°$	
Ans: (C) $2\sqrt{2}$	1
13. A quadratic polynomial having only zero (–2) is (A) $(x-2)^2$ (B) x^2-2 (C) x^2+2x (D) $(x+2)^2$	
Ans: (D) $(x + 2)^2$	1
14. PQ and PR are tangents to a circle with centre O such that OQ = QP. The value of ∠OPQ is equal to  (A) 45° (B) 30° (C) 60° (D) 90°	
Ans: (A) 45°	1
15. Which of the following depends on all observations of a given data ? (A) Median (B) Mean (C) Range (D) Mode	
Ans: (B) Mean	1

16. PQ is tangent to the circle centred at O. If OQ = 3 cm, PQ = 5 cm, then OP is equal to



(A) 5 cm (B) 4 cm

(C) $\sqrt{15}$ cm (D) $\sqrt{34}$ cm

Ans: (D) $\sqrt{34}$ cm	1

17. Two dice are rolled together. The probability that only one die shows number 4, is

(A) $\frac{11}{36}$ (B) $\frac{1}{3}$

(C) $\frac{5}{18}$ (D) $\frac{1}{4}$

Ans: (C) $\frac{5}{18}$	1

18. An arc of length 22 cm subtends an angle of x° at the centre of the circle. If radius of circle is 36 cm, the value of x is

(A) 35 (B) 40

(C) 60 (D) 30

Ans: (A) 35	1

(Assertion – Reason based questions)

Directions : Question numbers **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

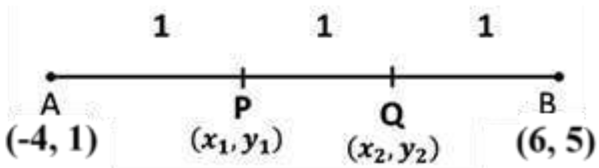
(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** When a hemisphere of same radius (r) is carved out from one side of a solid wooden cylinder, the total surface area of remaining solid is increased by $2\pi r^2$. **Reason (R) :** Curved surface area of hemisphere is $2\pi r^2$.	
Ans: (D) Assertion (A) is false, but Reason (R) is true.	1
20. **Assertion (A) :** In a right angle triangle ABC, $\angle B = 90^\circ$. Therefore the value of cos (A + C) is equal to 0. **Reason (R) :** $A + B + C = 180^\circ$ and $\cos 90^\circ = 0$.	
Ans: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	1
Section – B **(Very Short Answer Type Questions)** **5 × 2 = 10** Q. Nos. **21 to 25** are Very Short Answer type questions of **2** marks each. 21. Check whether $15^n \times 2^n$, n being a natural number, ends with the digit zero.	
Solution: $15^n \times 2^n = 5^n \times 3^n \times 2^n$ $\Rightarrow$ 2 and 5 both are the factors of the given number $\therefore$ the given number ends with the digit zero.	1 1
22. (a) Evaluate : $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 90^\circ$. **OR** (b) Verify that $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$ for $A = 30^\circ$.	
Solution: (a) $2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2$ $= \frac{7}{4}$ **OR** (b) $\text{LHS} = \cos 60^\circ = \frac{1}{2}$ $\text{RHS} = \frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}}$ $= \frac{1}{2} = \text{LHS}$	1½ ½ ½ 1 ½

23. Find the coordinates of the points of trisection of line segment joining the points (–4, 1) and (6, 5).

Solution:



AP : PB = 1 : 2

$\therefore x_1 = \frac{-8+6}{3} = \frac{-2}{3},\ y_1 = \frac{5+2}{3} = \frac{7}{3}$

Coordinates of point P are $\left(\frac{-2}{3}, \frac{7}{3}\right)$ — ½ + ½

AQ : QB = 2 : 1

$\therefore x_2 = \frac{12-4}{3} = \frac{8}{3},\ y_2 = \frac{10+1}{3} = \frac{11}{3}$

Coordinates of point Q are $\left(\frac{8}{3}, \frac{11}{3}\right)$ — ½ + ½

24. (a) A bag contains 40 marbles out of which some are white and others are black. If the probability of drawing a black marble is $\frac{3}{5}$, then find the number of white marbles.

OR

(b) In a pre-primary class, a teacher put cards numbered 20 to 59 in a bowl. A student picked up a card at random and read the number. Find the probability that the number read was (i) a prime number (ii) a perfect square.

Solution: (a) Let the number of black marbles be n.

P (drawing a black marble) $= \frac{n}{40}$ — ½

$\therefore \frac{3}{5} = \frac{n}{40} \Rightarrow n = 24$ — 1

Hence, number of white marbles = 16 — ½

OR

(b) Total number of cards = 40

(i) P (a prime number) $= \frac{9}{40}$ — 1

(ii) P (no. is perfect square) $= \frac{3}{40}$ — 1

25. Using distance formula, prove that the points (1, 5), (2, 3) and (3, 1) are collinear.

Solution: Let A(1, 5), B(2, 3) and C(3, 1) be the points

$AB = \sqrt{1^2 + (-2)^2} = \sqrt{5}$ — ½

$BC = \sqrt{1^2 + (-2)^2} = \sqrt{5}$	½
$AC = \sqrt{2^2 + (-4)^2} = \sqrt{20}$ or $2\sqrt{5}$	½
$\therefore$ AB + BC = AC, therefore points A, B and C are collinear.	½

Section – C

(Short Answer Type Questions) **$6 \times 3 = 18$**

Q. Nos. **26** to **31** are Short Answer type questions of **3** marks each.

26. (a) If α, β are zeroes of the polynomial $3x^2 - 8x + 4$, then form a quadratic polynomial in x whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

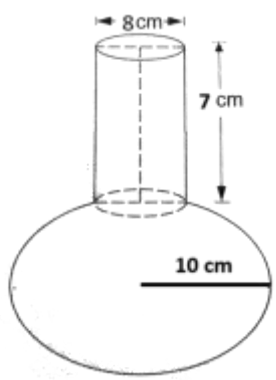
OR

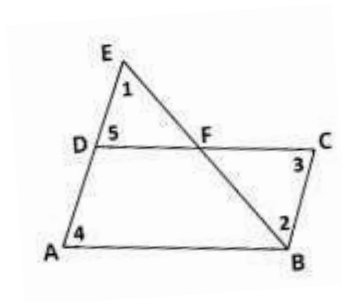
(b) Find zeroes of the polynomial $6x^2 - 7x - 3$ and verify the relationship between zeroes and its coefficients.

Solution:	
(a) $p(x) = 3x^2 - 8x + 4$	
$\alpha + \beta = \frac{8}{3}, \alpha\beta = \frac{4}{3}$	½ + ½
$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = 2$	½
and $\frac{1}{\alpha\beta} = \frac{3}{4}$	½
$\therefore$ required polynomial is $x^2 - 2x + \frac{3}{4}$	1
or $k(4x^2 - 8x + 3)$, where k is a non-zero real number.	
OR	
(b) $p(x) = 6x^2 - 7x - 3 = (2x - 3)(3x + 1)$	
Zeroes of p(x) are $\frac{3}{2}$ and $-\frac{1}{3}$	1
Sum of zeroes = $\frac{3}{2} - \frac{1}{3} = \frac{7}{6} = -\frac{\text{coefficient of } x}{\text{coeffiecient of } x^2}$	1
Product of zeroes = $\frac{3}{2} \times \frac{-1}{3} = \frac{-3}{6} = \frac{\text{constant term}}{\text{coeffiecient of } x^2}$	1

27. The sum of the squares of two consecutive even numbers is 452. Find the numbers.

Solution: Let the two consecutive even numbers be x and x + 2.	½
A.T.Q.	
$x^2 + (x + 2)^2 = 452$	1
$\Rightarrow x^2 + 2x - 224 = 0$	1
$\Rightarrow (x + 16)(x - 14) = 0$	
$\Rightarrow x = 14$	

Required numbers are 14 and 16.	½
28. Prove that $(\text{cosec } A + \sin A)^2 + (\sec A + \cos A)^2 = 7 + \tan^2 A + \cot^2 A$.	
Solution: $\text{LHS} = \text{cosec}^2 A + \sin^2 A + 2 \text{ cosec } A \sin A + \sec^2 A + \cos^2 A + 2 \cos A \sec A$ $= (\sin^2 A + \cos^2 A) + (1 + \tan^2 A) + (1 + \cot^2 A) + 4$ $= 7 + \tan^2 A + \cot^2 A = \text{RHS}$	1 1 1
29. The traffic lights at three different road crossings change after every 45 seconds, 75 seconds and 60 seconds respectively. If they change together at 5.00 a.m., then at what time they will change together next ?	
Solution: $45 = 3^2 \times 5, 75 = 3 \times 5^2, 60 = 2^2 \times 3 \times 5$ $\text{LCM}(45, 75, 60) = 2^2 \times 3^2 \times 5^2 = 900$ 900 seconds = 15 minutes Lights will change together at 5:15 a.m. again	1½ ½ ½ ½
30. (a) A spherical glass vessel has a cylindrical neck 7 cm long and 8 cm in diameter. The radius of spherical part is 10 cm. Find the volume of the vessel. **OR** (b) From each end of a solid cylinder of height 20 cm and base radius 7 cm, a cone of base radius 2.1 cm and height 5 cm is scooped out. Find the volume of the remaining solid.	
Solution:  (a) Volume of the vessel $= \frac{4}{3} \times \frac{22}{7} \times 10 \times 10 \times 10 + \frac{22}{7} \times 4 \times 4 \times 7$ $= 4190{\cdot}4 + 352 = 4542{\cdot}48$ cu. cm **OR** (b) Volume of cylinder $= \frac{22}{7} \times 7 \times 7 \times 20 = 3080$ cu. cm Volume of cones $= 2 \times \frac{1}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times 5 = 46{\cdot}2$ cu. cm	1 + 1 1 1 1

Volume of remaining solid = 3080 – 46·2 = 3033·8 cu. cm	1
31. Point E lies on the extended side AD of parallelogram ABCD. BE intersects CD at F. Show that (i) ΔDFE ~ ΔCFB (ii) ΔAEB ~ ΔCBF.	
Solution:  (i) In △DFE and △CFB $\angle 5 = \angle 3$ (Alternate Interior Angle) $\angle 1 = \angle 2$ (Alternate Interior Angle) ∴ By AA similarity criterion, △DFE ~ △CFB (ii) In △AEB and △CBF $\angle 1 = \angle 2$ (Alternate Interior Angle) $\angle 4 = \angle 3$ (Opposite angles of a parallelogram) ∴ By AA similarity criterion, △AEB ~ △CBF	Correct Figure ½ 1 ½ ½ ½
Section – D **(Long Answer Type Questions)** **4 × 5 = 20** Q. Nos. **32** to **35** are Long Answer type questions of **5** marks each. 32. Determine graphically whether the following pair of linear equations $2x + 3y = 12$ and $x - y = 1$ has unique solution or infinitely many solutions.	
Solution: Correct graph of each equation	2 + 2

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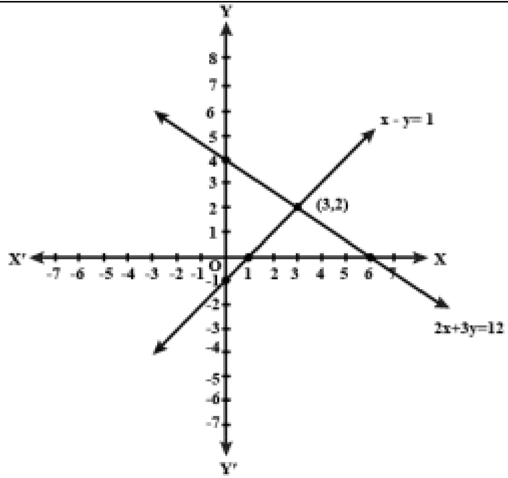

Since lines are intersecting at a point so, equations have unique solution.
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33. (a) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

OR

(b) It is given that sides AB and AC and median AD of ΔABC are respectively proportional to sides PQ and PR and median PM of another ΔPQR. Show that ΔABC ~ ΔPQR.
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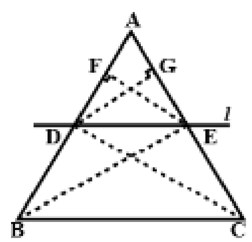
Solution:


Given: In Δ ABC, DE || BC
To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$
Construction: Join BE, DC, Draw DG ⊥ AC and EF ⊥ AB

Proof: $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF} = \frac{AD}{DB}$ ……….(i)
and $\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} = \frac{AE}{EC}$ ………. (ii)
As ΔBDE and ΔCDE are on the same base DE and between the same parallels DE and BC.
$\therefore$ ar(ΔBDE) = ar(ΔCDE) …………..(iii)
From (i), (ii) and (iii), we get $\frac{AD}{DB} = \frac{AE}{EC}$

OR
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Correct
Figure
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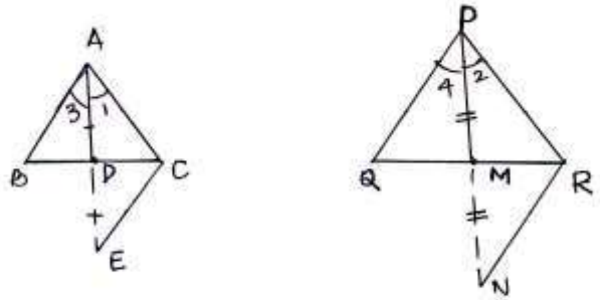
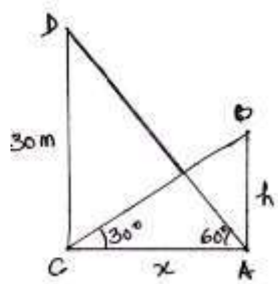


	Correct Figure 1
Extend AD to E and PM to N such that AD = DE and PM = MN.	
Proving $\Delta DAB \cong \Delta DEC$ (By SAS congruency criterion)	1
Similarly, $\Delta MPQ \cong \Delta MNR$	
$\therefore AB = CE$ and $PQ = NR$ (by cpct)	½
Given $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$	
$\Rightarrow \frac{CE}{NR} = \frac{AE/2}{PN/2} = \frac{AC}{PR}$	
$\Rightarrow \frac{CE}{NR} = \frac{AE}{PN} = \frac{AC}{PR}$	1
Hence $\Delta CAE \sim \Delta RPN$ (By SSS similarity criterion)	½
$\Rightarrow \angle 1 = \angle 2$, similarly $\angle 3 = \angle 4$	½
Adding, we get $\angle 1 + \angle 3 = \angle 2 + \angle 4$	
or $\angle BAC = \angle QPR$	½
Hence, $\Delta ABC \sim \Delta PQR$ (By SAS similarity criterion)	

34. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 30 m high, find the height of the building and distance between the building and the tower. (Use $\sqrt{3} = 1.73$)

Solution:



	Correct Figure 1
Let AB be the building and CD be the tower.	
In $\triangle ACD$, $\tan 60° = \frac{30}{x} \Rightarrow x = 10\sqrt{3}$ ....(i)	1 + ½
In $\triangle CAB$, $\tan 30° = \frac{h}{x} \Rightarrow x = h\sqrt{3}$ .... (ii)	1 + ½
Using (i) and (ii) $h = 10$, $x = 10 \times 1{\cdot}73 = 17{\cdot}3$	½ + ½
$\therefore$ Height of the building = 10 m and distance between the building and the tower = 17·3 m	

35. (a) Find 'mean' and 'mode' of the following data :

Class	10-25	25-40	40-55	55-70	70-85	85-100
Number of Students	12	10	15	13	8	12

OR

(b) The following table shows the ages of patients admitted in a hospital during a year :

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of Patients	7	10	21	22	15	5

Find 'mode' and 'median' of the above data.

Solution: (a)

CI	x_i	f_i	$u_i = \frac{x_i - 47\cdot5}{15}$	f_iu_i
10 – 25	17·5	12	– 2	– 24
25 – 40	32·5	10	– 1	– 10
40 – 55	47·5	15	0	0
55 – 70	62·5	13	1	13
70 – 85	77·5	8	2	16
85 – 100	92·5	12	3	36
		70		31

Correct Table 1½

$$\text{Mean} = 47\cdot5 + 15 \times \frac{31}{70} = 54\cdot14$$ 1½

Modal class is 40 – 55

$$\text{Mode} = 40 + 15 \times \frac{15 - 10}{30 - 10 - 13}$$ 1½

$$= 50\cdot71$$ ½

OR

(b)

C I	5-15	15-25	25-35	35-45	45-55	55-65
f	7	10	21	22	15	5
cf	7	17	38	60	75	N = 80

Correct Table 1

Median class is 35 – 45

$$\text{Median} = 35 + \frac{10}{22} \times (40 - 38)$$ 1½

$$= 35\cdot91$$ ½

Modal class is 35 – 45

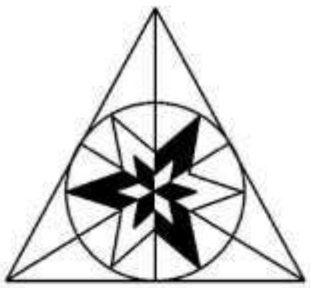
$\text{Mode} = 35 + \dfrac{22-21}{44-21-15} \times 10$	1½
$= 36{\cdot}25$	½

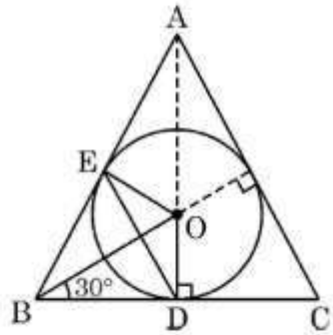
Section – E

(Case-study based Questions) **3 × 4 = 12**

Q. Nos. **36** to **38** are Case-study based Questions of **4** marks each.

36.





In a Fine Arts class, students were asked to design triangular tiles in geometric pattern.

Neelima made a circular design inside an equilateral triangle ABC. The radius of the circle is 4 cm. Observe the diagram and answer the following questions :

(i) Determine the length OB.

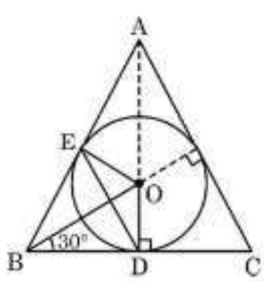
(ii) Is DE || CA ? Give reason for your answer.

(iii) (a) Write all angles of quadrilateral OEBD and show that it is a cyclic quadrilateral.

OR

(iii) (b) Find the perimeter of ΔABC. (Use $\sqrt{3} = 1.73$)

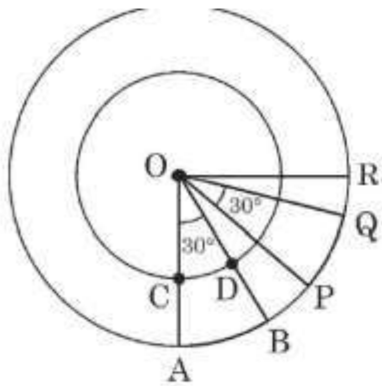
Solution:



(i) In $\triangle ODB$, $\sin 30° = \dfrac{4}{OB} \Rightarrow OB = 8$ cm	1
(ii) Yes, DE || CA	½
Δ ABC is an equilateral triangle and AD$\perp$ BC	
$\Rightarrow$ D is the mid point of BC	½
Similarly, E is the mid point of AB, so DE || CA	
(iii) (a) $\angle EBD = 60° \Rightarrow \angle EOD = 120°$	½
$\angle OEB = \angle ODB = 90°$	½
(radius is perpendicular to the tangent through the point of contact)	

	$\angle OEB + \angle ODB = 90° + 90° = 180°$	½
	∴ quad. OEBD is a cyclic quad.	½
	OR	
(iii) (b)	In $\triangle OBD$, $\cos 30° = \frac{BD}{8} \Rightarrow BD = 6{\cdot}92$ cm	1
	$BC = 2\ BD = 13.84$ cm	
	∴ Perimeter of Δ ABC = 41·52 cm	1

37.



A farmer has put up a decorative windmill in his farm in which there are eight blades of equal width and equally placed in a circular arrangement. A circular wire goes through them.

The diagram shows two blades OAB and OPQ in a quarter circle with centre O. $\angle AOB = \angle POQ = 30°$, OA = 28 cm, OC = 21 cm.

O is the centre of both the circles.

(i) Determine the measure of $\angle BOP$.

(ii) Find length of arc CD.

(iii) (a) Find the area of region CABD.

OR

(iii) (b) Find perimeter of region CABD.

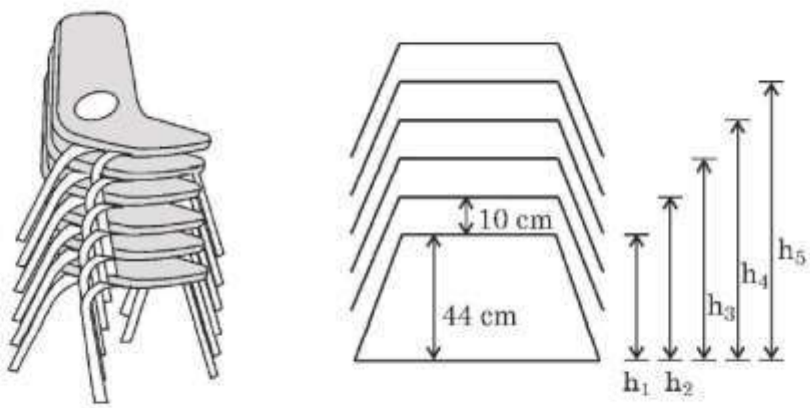
Solution: (i)	$\angle AOC = 90°$ and blades are equally placed	
	$\therefore \angle BOP = \frac{1}{2}(90° - 60°) = 15°$	1
(ii)	Length of arc CD $= \frac{30}{360} \times 2 \times \frac{22}{7} \times 21 = 11$ cm	1
(iii) (a)	Area CABD $= \frac{30}{360} \times \frac{22}{7} \times (28^2 - 21^2)$	1
	$= 89.8$ sq. cm	1

OR

(iii) (b) Length of arc AB = $\frac{30}{360} \times 2 \times \frac{22}{7} \times 28 = \frac{44}{3} = 14{\cdot}67$ cm — 1

Perimeter of CABD = $14{\cdot}67 + 11 + 2 \times (28 - 21) = 39{\cdot}67$ cm — 1

38. A tent house owner provides furniture on rent. He stacks chairs in his shop to save space.



In the diagram, the height of seat of chair from ground is represented by $h_1, h_2, h_3, ...$. The height of first seat is 44 cm from ground level and gap between every two seats is 10 cm.

(i) Write the values of h_1, h_2, h_3, h_4 and h_5 in this order only.

(ii) Show that the above values form an A.P. Write its first term and common difference.

(iii) (a) If chairs can be stacked up to the maximum height of 160 cm, then find the maximum number of chairs in a stack.

OR

(iii) (b) Is it possible to stack 15 chairs if maximum height of the stack can not be more than 180 cm ? Justify your answer.

Solution: (i) $h_1 = 44$, $h_2 = 54$, $h_3 = 64$, $h_4 = 74$, $h_5 = 84$	1
(ii) Since gap between heights of seats of every two adjacent chairs is same	½
∴ $h_1, h_2, h_3, ...$ form an A.P.	
Here, a = 44 and d = 10	½

(iii) (a) $160 = 44 + (n - 1) \times 10$ $\Rightarrow n = 12 \cdot 6$ $\therefore$ maximum 12 chairs can be stacked up. **OR** (iii) (b) $h_{15} = 44 + 14 \times 10$ $= 184$ cm 184 cm $>$ 180 cm $\therefore$ 15 chairs cannot be stacked up	1 ½ ½ 1 ½ ½