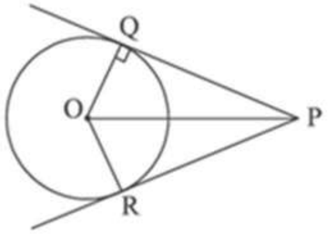
Set 430/4/2

SOLUTIONS

MATHEMATICS (Basic)

SECTION A

*This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each.* *20×1=20*

1. The 20th term of the A.P. : $10\sqrt{2}, 6\sqrt{2}, 2\sqrt{2}, \ldots$ is : (a) $-76+10\sqrt{2}$ (b) $-62\sqrt{2}$ (c) $-66\sqrt{2}$ (d) $86\sqrt{2}$	
Ans. (c) $-66\sqrt{2}$	1
2. If $\sec\theta - \tan\theta = 2$, then $\sec\theta + \tan\theta$ is equal to : (a) $\frac{1}{2}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2	
Ans (a) $\frac{1}{2}$	1
3. PQ and PR are tangents to the circle of radius 3 cm and centre O. If length of each tangent is 4 cm, then perimeter of Δ OQP is : (a) 5 cm (b) 12 cm (c) 9 cm (d) 8 cm  	
Ans. (b) 12 cm $\frac{3}{8}$	1
4. The value of m for which lines $14x + my = 20$ and $-3x + 2y = 16$ are parallel, is : (a) $-\frac{3}{14}$ (b) $-\frac{7}{3}$ (c) $-\frac{28}{3}$ (d) $-\frac{3}{28}$	
Ans. (c) $-\frac{28}{3}$	1
5. If n^{th} term of an A.P. is $5n-6$, then its common difference is : (a) -6 (b) $5n$ (c) 5 (d) 6	
Ans. (c) 5	1

6. The distance between the points (2, –7) and (–2, –1) is :

(a) 10 (b) $2\sqrt{13}$ (c) 8 (d) $4\sqrt{13}$

Ans. (b) $2\sqrt{13}$ 1

7. The length of arc subtending an angle of 210° at the centre of the circle, is $\frac{44}{3}$ cm. The radius of the circle is :

(a) $2\sqrt{2}$ cm (b) 4 cm (c) 8 cm (d) $\frac{1}{4}$ cm

Ans. (b) 4 cm 1

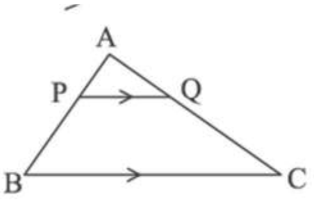
8. A card is drawn at random from a well shuffled deck of 52 playing cards. The probability that drawn card shows number '9' is :

(a) $\frac{1}{26}$ (b) $\frac{4}{13}$ (c) $\frac{1}{52}$ (d) $\frac{1}{13}$

Ans. (d) $\frac{1}{13}$ 1

9. In Δ ABC, PQ || BC. It is given that AP = 2.4 cm, PB = 3.6 cm and BC = 5.4 cm. PQ is equal to :

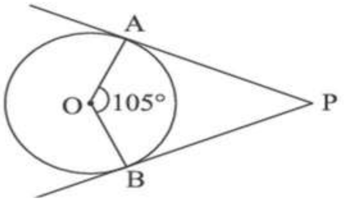
(a) 2.7 cm (b) 1.8 cm
(c) 3.6 cm (d) 2.16 cm



Ans. (d) 2·16 cm 1

10. PA and PB are tangents to a circle with centre O. If $\angle AOB = 105°$ then $\angle OAP + \angle APB$ is equal to :

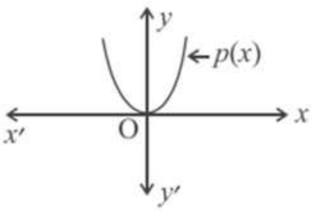
(a) 75° (b) 175°
(c) 180° (d) 165°



Ans. (d) 165^0 1

11. Two dice are rolled together. The probability that at least one of them shows a six, is :

(a) $\frac{12}{36}$ (b) $\frac{5}{36}$ (c) $\frac{11}{36}$ (d) $\frac{6}{36}$

Ans. (c) $\frac{11}{36}$	1
12. The curved surface area of a cone with base radius 7 cm, is 550 cm^2. The slant height of the cone is : (a) 25 cm (b) 14 cm (c) 20 cm (d) 24 cm	
Ans. (a) 25 cm	1
13. If tan A = $\frac{1}{2}$, then sin A is equal to : (a) $\frac{2}{\sqrt{5}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{5}}$ (d) 1	
Ans. (c) $\frac{1}{\sqrt{5}}$	1
14. If $\sqrt{2}\sin\theta = 1$, then $\cot\theta \times \operatorname{cosec}\theta$ is equal to : (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\frac{1}{2}$	
Ans. (c) $\sqrt{2}$	1
15. Three coins are tossed together. The probability that only one coin shows tail, is : (a) $\frac{1}{2}$ (b) $\frac{3}{8}$ (c) $\frac{7}{8}$ (d) 1	
Ans. (b) $\frac{3}{8}$	1
16. In the given figure, graph of $p(x)$ is shown. Number of distinct zeroes of $p(x)$ is : (a) 0 (b) 1 (c) 2 (d) many  	
Ans. (b) 1	1
17. Two right circular cylinders of equal volumes have their heights in the ratio 1:2. The ratio of their radii is : (a) $\sqrt{2}:1$ (b) $1:2$ (c) $1:4$ (d) $1:\sqrt{2}$	

Ans. (a) $\sqrt{2} : 1$ 1

18. α, β are zeroes of the polynomial $2x^2 + 5x + 1$. The value of $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is :

(a) $-\frac{5}{4}$ (b) 5 (c) $\frac{5}{4}$ (d) -5

Ans. (d) – 5 1

Directions :

Question numbers 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below :

(a) Both, Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both, Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** If E is an event such that $P(E) = \frac{1}{999}$, then $P(\overline{E}) = 0.001$.

Reason (R) : $P(E) + P(\overline{E}) = 1$

Ans. (d) Assertion (A) is false, but Reason (R) is true. 1

20. **Assertion (A) :** Median marks of students in a class test is 16. It means half of the class got marks less than 16.

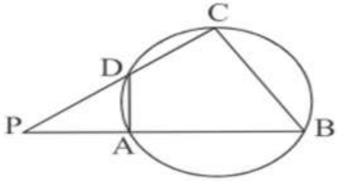
Reason (R) : Median divides the distribution in two equal parts.

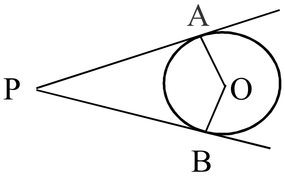
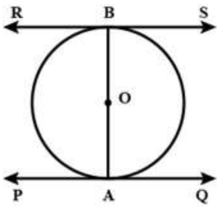
Ans. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). 1

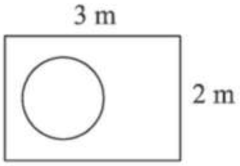
SECTION B

*This section has **5** Very Short Answer (VSA) type questions of **2** marks each.* *5×2=10*

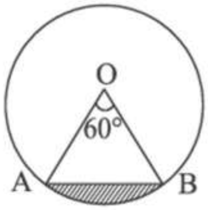
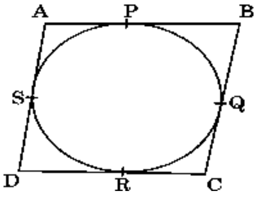
21. Two chords BA and CD intersect at point P outside the circle. Prove that ΔPDA ~ ΔPBC



Solution: $\angle PDA + \angle ADC = 180^\circ$ (linear pair) …….. (i)	$\frac{1}{2}$
Also $\angle ADC + \angle ABC = 180^\circ$ (opposite angles of cyclic quadrilateral) …. (ii)	
From (i) and (ii) $\angle PDA = \angle ABC$	1
Also $\angle P$ is common therefore $\Delta PDA \sim \Delta PBC$ (By AA similarity criterion)	$\frac{1}{2}$
22. **(A)** Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre. **OR** **(B)** Prove that the tangents drawn at the ends of a diameter of a circle are parallel.	
Solution: 	Correct figure $\frac{1}{2}$
(A) Since tangent ⊥ radius at point of contact	
$\therefore \angle OAP = \angle OBP = 90^\circ$	$\frac{1}{2}$
Since $\angle APB + \angle PBO + \angle BOA + \angle OAP = 360^\circ$	$\frac{1}{2}$
therefore $\angle APB + \angle BOA = 360^\circ - 90^\circ - 90^\circ = 180^0$	$\frac{1}{2}$
Hence $\angle AOB$ and $\angle APB$ are supplementary	
OR	
	Correct figure $\frac{1}{2}$
(B) Let AB be the diameter of the circle.	
Since tangent ⊥ radius at point of contact	
therefore $\angle OAP = \angle OBR = 90^\circ$	$\frac{1}{2}$
$\Rightarrow \angle OAP + \angle OBQ = 180^\circ$	$\frac{1}{2}$
$\Rightarrow$ Co-interior angles are supplementary	

$\Rightarrow$ PQ ∥ RS Hence tangents at the ends of a diameter of a circle are parallel.	$\frac{1}{2}$
23. Find the ratio in which point P (–1, *m*) divides the line segment joining the points A(2, 5) and B(–5, –2). Hence, find the value of *m*.	
Solution: Let the required ratio be k:1	
$-1 = \frac{-5k + 2}{k + 1}$	1
$\Rightarrow \quad k = \frac{3}{4}$	$\frac{1}{2}$
$\Rightarrow$ ratio is 3 : 4	
$\Rightarrow \quad m = 2$	$\frac{1}{2}$
24. Show that 45^n can not end with the digit 0, *n* being a natural number. Write the prime number '*a*' which on multiplying with 45^n makes the product end with the digit 0.	
Solution: $45^n = 3^{2n} \text{ x } 5^n$	1
To end with digit 0, 45^n should have prime factors 2 and 5 both. So it cannot end with digit 0.	$\frac{1}{2}$
45^n should be multiplied by 2 $\Rightarrow$ a = 2	$\frac{1}{2}$
25. **(A)** A coin is dropped at random on the rectangular region shown in the figure. What is the probability that it will land inside the circle with radius 0.7 m ?  **OR** **(B)** A die is thrown twice. What is the probability that (i) difference between two numbers obtained is 3 ? (ii) sum of the numbers obtained is 8 ?	
Solution: (A) Area of circle = $\frac{22}{7} \times \frac{7}{10} \times \frac{7}{10} = \frac{154}{100}$ sq m	$\frac{1}{2}$
Area of rectangle = 6 sq m	$\frac{1}{2}$
P(coin lands inside the circle) = $\frac{154}{600}$ or $\frac{77}{300}$	1

OR	
(B) (i) P(difference between two numbers obtained is 3) = $\frac{6}{36}$ or $\frac{1}{6}$	1
(ii) P (sum of numbers obtained is 8) = $\frac{5}{36}$	1
SECTION – C **Question Nos. 26 to 31 are short answer questions of 3 marks each.** **26.** **(A)** If points A(– 5, y), B(2, – 2), C(8, 4) and D(x, 5) taken in order, form a parallelogram ABCD, then find the values of x and y. Hence, find lengths of sides of the parallelogram. **OR** **(B)** A(6, – 3), B(0, 5) and C(– 2, 1) are vertices of ΔABC. Points P(3, 1) and Q(2, – 1) lie on sides AB and AC respectively. Check whether $\frac{AP}{PB} = \frac{AQ}{QC}$.	
Solution: (A) ABCD is a parallelogram ∴ Coordinates of mid pt. of BD = Coordinates of mid pt. of AC $\left(\frac{2+x}{2}, \frac{-2+5}{2}\right) = \left(\frac{8-5}{2}, \frac{4+y}{2}\right)$	1
Getting x = 1 and y = – 1	$\frac{1}{2}$ + $\frac{1}{2}$
AB = $\sqrt{7^2 + (-1)^2}$ = $\sqrt{50}$ or $5\sqrt{2}$	$\frac{1}{2}$
BC = $\sqrt{6^2 + 6^2}$ = $\sqrt{72}$ or $6\sqrt{2}$	$\frac{1}{2}$
OR	
(B) AP = $\sqrt{3^2 + (-4)^2}$ = 5	$\frac{1}{2}$
PB = $\sqrt{3^2 + (-4)^2}$ = 5	$\frac{1}{2}$
AQ = $\sqrt{4^2 + (-2)^2}$ = $2\sqrt{5}$	$\frac{1}{2}$
QC = $\sqrt{4^2 + (-2)^2}$ = $2\sqrt{5}$	$\frac{1}{2}$

So $\frac{AP}{PB} = 1$ and $\frac{AQ}{QC} = 1$ Therefore $\frac{AP}{PB} = \frac{AQ}{QC}$	*1*
27. A chord of a circle of radius 10 cm subtends an angle of 60° at the centre O. Find the area of the shaded region. (Use $\sqrt{3} = 1.73, \sqrt{2} = 1.41$ and $\pi = 3.14$) 	
Solution: Area of sector OAB = $\frac{60}{360} \times 3{\cdot}14 \times 10^2 = \frac{314}{6}$ sq. cm Area of ΔOAB = $\frac{\sqrt{3}}{4} \times 10^2 = \frac{173}{4}$ sq. cm Area of the shaded region = $\frac{314}{6} - \frac{173}{4} = \frac{109}{12}$ or 9·08 sq. cm	1 1 1
28. Prove that the parallelogram circumscribing a circle is a rhombus.	
Solution:  As lengths of tangents drawn from an external point to a circle are equal AP = AS ....(i) BP = BQ ....(ii) DR = DS ....(iii) CR = CQ ....(iv) Adding the above equations we get, AB + CD = AD + BC $\Rightarrow$ 2AB = 2 AD [$\because$ Opp. Sides of a Parallelogram are equal] $\Rightarrow$ AB = AD $\Rightarrow$ ABCD is a rhombus	Correct figure $\frac{1}{2}$ 1 1 $\frac{1}{2}$

29. **(A)** Find the sum of the A.P. $7, 10\frac{1}{2}, 14, \ldots\ldots 84.$

OR

(B) If the sum of first n terms of an A.P. is given by $S_n = \frac{n}{2}(2n+8)$. Then, find its first term and common difference. Hence, find its 15th term.

Solution: (A) $a = 7,\ d = \frac{21}{2} - 7 = \frac{7}{2}$	1
$84 = 7 + (n-1) \times \frac{7}{2} \Rightarrow n = 23$	1
$S_{23} = \frac{23}{2}(7 + 84) = \frac{2093}{2}$	1
OR	
(B) $S_1 = a = 5$	$\frac{1}{2}$
$S_2 = 12$	$\frac{1}{2}$
Therefore $d = 2$	1
Hence $a_{15} = 33$	1

30. Find the zeroes of the polynomial $p(x) = 4x^2 - 4x - 3$ and verify the relationship between zeroes and its coefficients.

Solution: $p(x) = 4x^2 - 4x - 3 = (2x+1)(2x-3)$	
$\therefore$ zeroes of p(x) are $\frac{-1}{2}$ and $\frac{3}{2}$	1
Sum of zeroes $= \frac{-1}{2} + \frac{3}{2} = 1 = \frac{-(-4)}{4} = \frac{coefficient\ of\ x}{coefficient\ of\ x}$	1
Product of zeroes $= \frac{-1}{2} \times \frac{3}{2} = \frac{-3}{4} = \frac{constant}{coefficient\ of\ x}$	1

31. Prove that $\sqrt{5}$ is an irrational number.

Solution: (a)Let $\sqrt{5}$ be a rational number such that	
$\sqrt{5} = \frac{p}{q}$ (p and q are co-prime numbers, $q \neq 0$)	$\frac{1}{2}$
$\sqrt{5}\,q = p \Rightarrow 5q^2 = p^2$	
5 divides p^2 $\Rightarrow$ 5 divides p as well	1
p = 5m (for some integer m)	
$5q^2 = 25m^2 \quad \Rightarrow q^2 = 5m^2$	
5 divides q^2 $\Rightarrow$ 5 divides q as well	

p and q have a common factor 5 which is a contradiction as p and q are co-prime. 1

∴ our assumption is wrong

Hence, $\sqrt{5}$ is an irrational number. $\frac{1}{2}$

SECTION – D

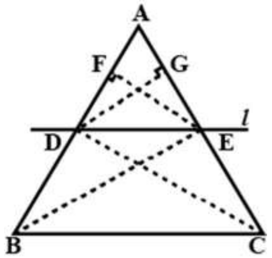
Question Nos. 32 to 35 are long answer questions of 5 marks each.

32. **(A)** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

OR

(B) In a Δ ABC, P and Q are points on AB and AC respectively such that PQ || BC. Prove that the median AD, drawn from A to BC, bisects PQ.

Solution: (A)



Correct figure $\frac{1}{2}$

Given: In Δ ABC, DE || BC

To Prove: $\frac{\textbf{AD}}{\textbf{DB}} = \frac{\textbf{AE}}{\textbf{EC}}$

Construction: Join BE, DC, Draw DM ⊥ AC and EN ⊥ AB

1

Proof : $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}\times AD\times EF}{\frac{1}{2}\times DB\times EF} \Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB}$ ……….(i) 1

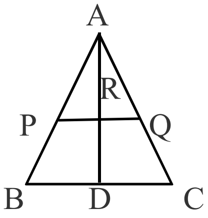
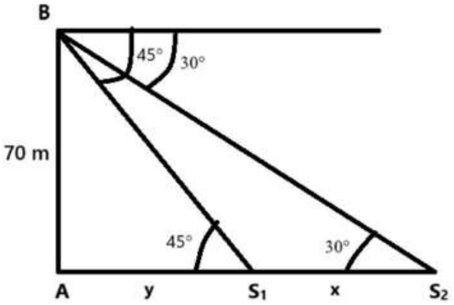
and $\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2}\times AE\times DG}{\frac{1}{2}\times EC\times DG} \Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{AE}{EC}$ ………………….. (ii) 1

As ΔBDE and ΔCDE are on the same base DE and between the same parallels DE and BC.

∴ ar(ΔBDE) = ar(ΔCDE) ..……………..(iii) 1

From (i), (ii) and (iii), we get $\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$ $\frac{1}{2}$

OR

(B) 	Correct figure $\frac{1}{2}$
Given PQ ∥ BC ⇒ PR ∥ BD Therefore in Δ APR and Δ ABD, ∠ PAR = ∠ BAD and ∠ APR = ∠ ABD ∴ Δ ARP ~ Δ ADB (By AA similarity criterion)	1
$\Rightarrow \frac{AR}{AD} = \frac{PR}{BD}$ ———— (i)	$\frac{1}{2}$
Similarly Δ ARQ ~ Δ ADC $\Rightarrow \frac{AR}{AD} = \frac{RQ}{DC}$ ———— (ii)	1
Using (i) and (ii) $\frac{PR}{BD} = \frac{RQ}{DC}$	$\frac{1}{2}$
AD is the median ∴ BD = DC ⇒ PR = RQ	1
i.e. AD bisects PQ	$\frac{1}{2}$
33. As observed from the top of a 70 m high lighthouse from the sea level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same sides of the lighthouse, find the distance between the two ships. (Use $\sqrt{3} = 1.73$)	
Solution: 	correct figure 1
Let AB be the light house & S_1 and S_2 be the position of the two ships.	
In ΔBAS_1, $\tan 45° = \frac{70}{y} = 1 \Rightarrow y = 70$ m	$1\frac{1}{2}$
In ΔBAS_2, $\tan 30° = \frac{1}{\sqrt{3}} = \frac{70}{y + x} \Rightarrow x + y = 70\sqrt{3}$	$1\frac{1}{2}$

$\Rightarrow x = 70\,(1{\cdot}73 - 1) = 51{\cdot}1$ m Hence the distance between the the two ships is 51·1 m.	*1*

34. **(A)** It is given that $p^2x^2 + (p^2 - q^2)x - q^2 = 0$; $(p \neq 0)$

(i) Show that the discriminant (D) of above equation is a perfect square.

(ii) Find the roots of the equation.

OR

(B) Three consecutive positive integers are such that the sum of the square of smallest and product of other two is 67. Find the numbers, using quadratic equation.

Solution: (A) (i) Discriminant $= (p^2 - q^2)^2 + 4p^2q^2$	1
$= (p^2 + q^2)^2$	1
(ii) $\therefore x = \dfrac{-(p^2 - q^2) \pm \sqrt{(p^2 + q^2)^2}}{2p^2}$	1
$= \dfrac{q^2}{p^2}, -1$	1+ 1
OR	
(B) Let the three consecutive positive integers be x, x + 1, x + 2	1
A.T.Q. $x^2 + (x + 1)(x + 2) = 67$	1
$\Rightarrow 2x^2 + 3x - 65 = 0$	
$\Rightarrow (2x + 13)(x - 5) = 0$	1
$\Rightarrow x = 5,$	1
$x = \dfrac{-13}{2}$ (rejected)	
So the three consecutive positive integers are 5, 6 and 7	1

35. Find 'median' and 'mode' of the following data :

Class	100-105	105-110	110-115	115-120	120-125	125-130
Frequency	6	8	10	4	9	3

Solution:

Class	f	cf
100 – 105	6	6
105 – 110	8	14
110 – 115	10	24
115 – 120	4	28
120 – 125	9	37
125 – 130	3	40

Correct table $1\frac{1}{2}$

Median class is 110 – 115

Median = $110 + \frac{5}{10}(20 - 14)$ $\quad 1$

= 113 $\quad \frac{1}{2}$

Modal class is 110 – 115

Mode = $110 + \frac{10 - 8}{20 - 8 - 4} \times 5$ $\quad 1\frac{1}{2}$

= 111·25 $\quad \frac{1}{2}$

SECTION – E

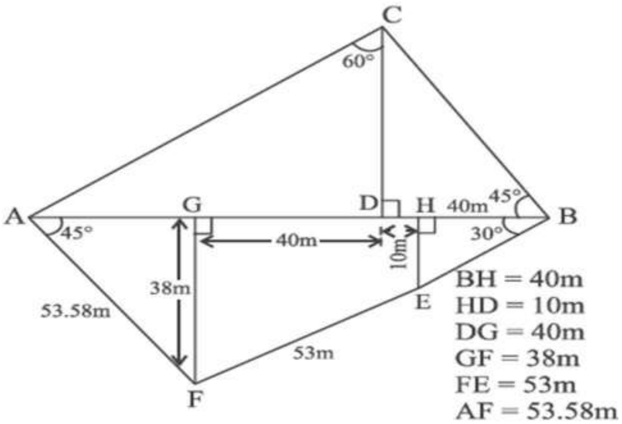
Question Nos. 36 to 38 are case-based questions of 4 marks each.

36. Rahim and Nadeem are two friends whose plots are adjacent to each other. Rahim's son made a drawing of the plots with necessary details.

It is decided that Rahim will fence the triangular plot ABC and Nadeem will fence along the sides AF, FE and BE.

Observe the diagram carefully and answer the following questions :

(Use $\sqrt{2} = 1.41$ and $\sqrt{3} = 1.73$)



(i) Find length BC.

(ii) Find length AG.

(iii) (a) Calculate perimeter of ΔABC.

OR

(iii) (b) Calculate length of (AF + FE + EB).

Solution: (i) $\cos 45^\circ = \frac{50}{BC} \Rightarrow BC = 50 \times 1{\cdot}41 = 70{\cdot}5$ m	1
(ii) $\tan 45^\circ = \frac{38}{AG} \Rightarrow AG = 38$ m	1
(iii) (a) $\sin 60^\circ = \frac{78}{AC} \Rightarrow AC = 89{\cdot}96$ m	1
Perimeter of $\Delta ABC = 70{\cdot}5 + 89{\cdot}96 + 38 + 50 + 40 = 288{\cdot}46$ m	1
OR	
(b) $\cos 30^\circ = \frac{40}{BE} \Rightarrow BE = 46{\cdot}13$ m	1
$AF + FE + EB = 53{\cdot}58 + 53 + 46{\cdot}13 = 152{\cdot}71$ m	1

37. Playing in a ball pool is good entertainment for kids. Suhana bought 600 new balls of diameter 7 cm to fill in the pool for her kids. The cuboidal box containing 600 balls has dimensions 42 cm × 91 cm × 50 cm ($l \times b \times h$).

Based on above information, answer the following questions :

(i) Find the volume of one ball.

(ii) 10 balls are painted with neon colours. Determine the area of painted surface.

(iii) (a) Find the volume of empty space in the box.

OR

(iii) (b) The lowermost layer of the balls covers the base of the box edge to edge when balls are placed evenly adjacent to each other. (A) How much area is covered by one ball? (B) How many balls are there in lowermost layer?

Solution: (i) Volume of one ball = $\frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \frac{539}{3}$ or 179.67 cu. cm	1
(ii) Area of painted surface = $10 \times 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 1540$ sq. cm	1
(iii) (a) Volume of box = 42 × 91 × 50 = 191100 cu. cm	$\frac{1}{2}$
Volume of 600 balls = $600 \times \frac{539}{3} = 107800$ cu. cm	$\frac{1}{2}$
Volume of empty space = 191100 – 107800 = 83300 cu. cm	1
OR	

(iii) (b) (A) Area covered by one ball = 7 × 7 = 49 sq. cm	1
(B) Number of balls in lowermost layer = 78	1

38. A telecommunication company came up with two plans– plan A and plan B for its customers.

The plans are represented by linear equations where 't' represents the time (in minutes) bought and 'C' represents the cost. The equations are :

Plan A : $3C = 20t$

Plan B : $3C = 10t + 300$

Based on above information, answer the following questions :

(i) If you purchase plan B, how much initial amount you have to pay ?

(ii) Charu purchased plan A. How many minutes she bought for ₹ 250 ?

(iii) (a) At how many minutes, do both the plans charge the same amount? What is that amount?

OR

(iii) (b) Which plan is better if you want to buy 60 minutes? Give reason for your answer.

Solution: (i) At $t = 0$, $3C = 300 \Rightarrow C = 100$	1
₹100 have to be paid initially.	
(ii) $3 \times 250 = 20t \Rightarrow t = 37{\cdot}5$ minutes	1
(iii) (a) $20t = 10t + 300 \Rightarrow t = 30$	1
Both plans charge same amount if a person buys 30 minutes.	
Amount = $\frac{20 \times 30}{3}$ = ₹ 200	1
OR	
(iii) (b) At $t = 60$, cost under plan A = ₹ 400	$\frac{1}{2}$
At $t = 60$, cost under plan B = ₹ 300	$\frac{1}{2}$
Plan B is better as ₹ 100 are saved.	1