Set 430/3/2

SOLUTIONS
MATHEMATICS (BASIC)

SECTION A

*This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each.* *$20\times1=20$*

1. The distance of point (– 3, 4) from y-axis is :

(A) – 3 (B) 3

(C) 4 (D) 5

Ans: (B) 3 1

2. The value of $\dfrac{\cot^2 A - \operatorname{cosec}^2 A}{\sin 30^\circ + \cos 60^\circ}$ is :

(A) 1 (B) – 1

(C) $\dfrac{2}{1+\sqrt{3}}$ (D) $\dfrac{-2}{1+\sqrt{3}}$

Ans: (B) -1 1

3. Which types of triangles are always similar ?

(A) Right-angled triangles

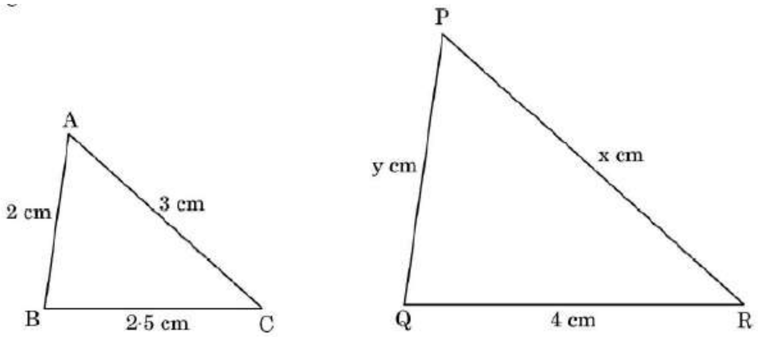
(B) Acute-angled triangles

(C) Isosceles triangles

(D) Equilateral triangles

Ans: (D) Equilateral triangles 1

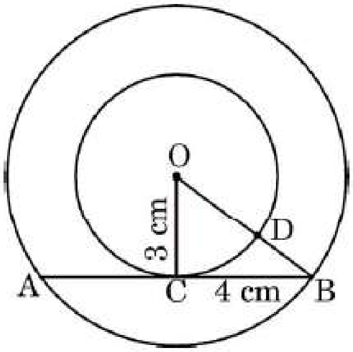
4. What values of x and y will make Δ ABC similar to Δ QRP in the figures given below ?



(A) $x = 6, \quad y = 5$
(B) $x = 5, y = 6$
(C) $x = 6, \quad y = 6$
(D) $x = 12, \quad y = 3{\cdot}2$

Ans: (B) $x = 5, y = 6$ 1

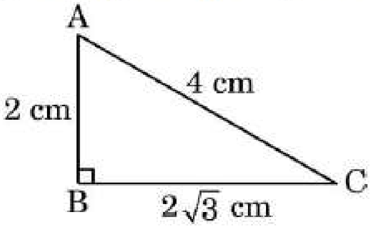
5. In the given figure, chord AB of the larger circle touches the smaller circle at C. If both the circles have the same centre O, then the length of BD is :



(A) 1 cm
(B) 2 cm
(C) 3 cm
(D) 4 cm

Ans: (B) 2 cm 1

6. In the given figure, the angle of elevation of point A from point C is :



(A) 30°
(B) 45°
(C) 60°
(D) Cannot be determined

Ans: (A) 30° 1

7. The angle of the sector of a circle whose area is one-eighth of the area of the circle is :

(A) $22\frac{1}{2}^{\circ}$
(B) 45°
(C) 60°
(D) 90°

Ans: (B) 45° 1

8. The perimeter of a quadrant of a circle of circumference 22 cm is :

(A) 29 cm

(B) 22 cm

(C) 12·5 cm

(D) 5·5 cm

Ans: (C) 12·5 cm 1

9. A cone and cylinder have same height and same radius. The volume of the cone and the volume of the cylinder are in the ratio :

(A) 1 : 1 (B) 1 : 3

(C) 3 : 1 (D) 1 : 2

Ans: (B) 1 : 3 1

10. The following table shows the marks scored by 23 students of a class.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of Students	5	3	4	8	3

The lower limit of the modal class is :

(A) 10 (B) 20

(C) 30 (D) 40

Ans: (C) 30 1

11. For a distribution, if mean = 15 and mode = 12, then its median is :

(A) 12

(B) 13

(C) 14

(D) 15

Ans: (C) 14 1

12. A pair of dice is thrown simultaneously. Let E denote the event that "The sum of numbers obtained on both dice is at least 9." The number of outcomes in favour of event E is :

(A) 4

(B) 6

(C) 10

(D) 26

Ans: (C) 10 1

13. If $p = 2^3 \times 3^2 \times 5$ and $q = 2^2 \times 3^3$, then the LCM of p and q is :

(A) $2^3 \times 3^3$

(B) $2^2 \times 3^2$

(C) $2^2 \times 3^2 \times 5$

(D) $2^3 \times 3^3 \times 5$

Ans: (D) $2^3 \times 3^3 \times 5$ 1

14. 3^n, where n is a natural number, cannot end with the digit :

(A) 3 (B) 5

(C) 7 (D) 9

Ans: (B) 5 1

15. A prime number has :

(A) exactly two prime factors

(B) exactly one prime factor

(C) at least one prime factor

(D) at least two prime factors

Ans: (B) exactly one prime factor 1

16. For what value(s) of k, is the system of equations $kx + 2y = 3$ and $2x + y = 5$ inconsistent ?

(A) k = Any real number (B) $k \neq 2$

(C) $k \neq 4$ (D) $k = 4$

Ans: (D) $k = 4$ 1

17. If $(\sqrt{x} + 1)^2 = x^2 + 2\sqrt{x}$ is expressed as a quadratic equation in the form of $ax^2 + bx + c = 0$, then the value of $a - b + c$ is :

(A) -1 (B) 0

(C) 1 (D) 2

Ans: (C) 1 1

18. If point (1, 2) divides the line segment joining the points (3, 5) and (2p, q) in the ratio 1 : 1, then (p, q) is equal to :

(A) $\left(-\frac{1}{2}, -1\right)$ (B) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

(C) $(-1, -1)$ (D) $\left(-1, -\frac{1}{2}\right)$

Ans: (A) $\left(-\frac{1}{2},-1\right)$	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* Every quadratic equation has two real roots.

Reason (R) : A quadratic polynomial can have at most two zeroes.

Ans: (D) Assertion (A) is false, but Reason (R) is True.	1

20. *Assertion (A) :* For an acute angle θ, $\cot \theta = 1 \Rightarrow \operatorname{cosec} \theta = 2$.

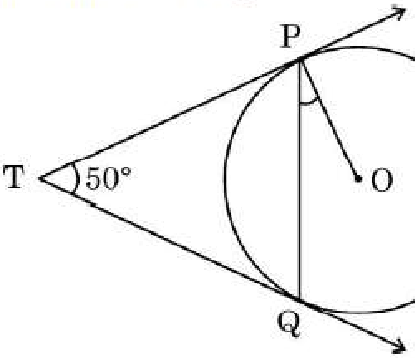
Reason (R) : $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$.

Ans: (D) Assertion (A) is false, but Reason (R) is True.	1

SECTION B

*This section has **5** Very Short Answer (VSA) type questions carrying **2** marks each.* *5×2=10*

21. In the given figure, TP and TQ are two tangents. If $\angle PTQ = 50°$, then find the measure of $\angle OPQ$.



Solution: TP = TQ (Tangents drawn from an exterior point to a circle are equal) Since angles opposite to equal sides of a triangle are equal	
$\therefore \angle TPQ = \angle TQP$	½
In Δ TPQ, $50° + 2\angle TPQ = 180°$	
$\Rightarrow \angle TPQ = 65°$	1
$\angle OPQ = 90° - 65° = 25°$	½

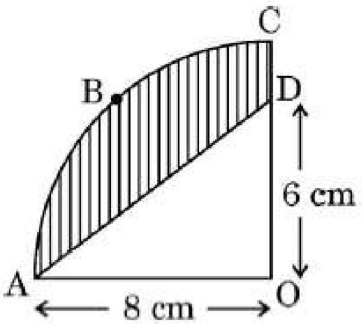
22. (a) If $\sin 3A = 1$, then find the value of $\cos 2A - \tan^2 45°$.

OR

(b) If $(\sec A + \tan A)(1 - \sin A) = k \cos A$, then find the value of k.

Solution: (a) $3A = 90° \Rightarrow A = 30°$	½
$\cos 2A - \tan^2 45° = \cos 60° - \tan^2 45°$ $= \frac{1}{2} - 1 = \frac{-1}{2}$	1½
OR	
(b) $\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A) = k \cos A$	½
$1 - \sin^2 A = k \cos^2 A$	½
$\cos^2 A = k \cos^2 A$	½
$k = 1$	½

23. In the given figure, OABC is a quadrant of a circle with centre O and radius 8 cm. If OD = 6 cm, then find then perimeter of the shaded region.



Solution:

Perimeter of the shaded region = length of Arc ABC + AD + CD

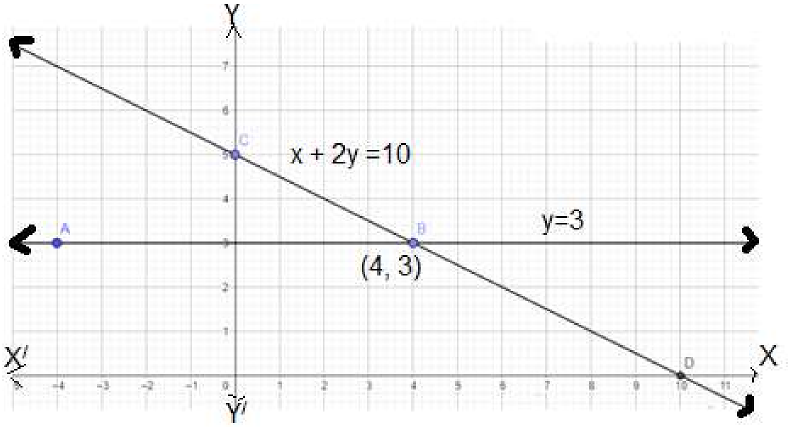
$= \frac{90}{360} \times 2 \times \frac{22}{7} \times 8 + \sqrt{6^2 + 8^2} + (8 - 6)$ 1½

$= \frac{172}{7}$ cm or 24·57 cm ½

24. Solve the following system of equations graphically :

x + 2y = 10 and y = 3

Solution: Correct graph of equations 1½

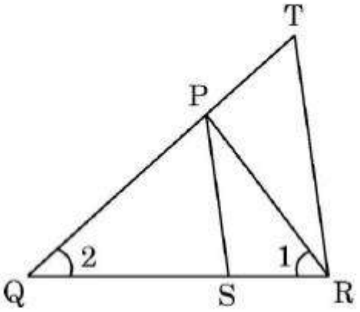


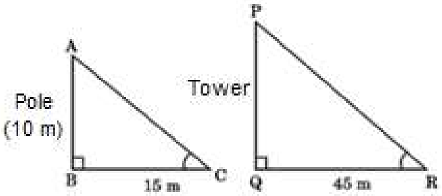
Solution: x = 4, y = 3 or (4, 3) ½

25. (a) A vertical pole of height 10 m casts a shadow of 15 m on the ground and at the same time, a tower casts a shadow of 45 m on the ground. Find the height of the tower.

OR

(b) In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Prove that $\Delta PQS \sim \Delta TQR$.



Solution: (a)	
	
$\because$ Δ ABC ~ Δ PQR	½
$\therefore \frac{10}{PQ} = \frac{15}{45} \Rightarrow$ PQ = 30	1½
Height of the tower = 30 m	
OR	
(b) In ΔPQR $\angle 1 = \angle 2 \Rightarrow$ PQ = PR	½
$\therefore$ In ΔPQS and ΔTQR	
$\frac{QR}{QS} = \frac{QT}{PR} \Rightarrow \frac{QR}{QS} = \frac{QT}{QP}$	½
also $\angle$Q = $\angle$Q (Common)	½
$\therefore$ Δ PQS ~ Δ TQR (by SAS similarity criterion)	½

SECTION C

*This section has **6** Short Answer (SA) type questions carrying **3** marks each.* *6×3=18*

26. (a) A fraction becomes $\frac{1}{3}$, when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$, when 8 is added to its denominator. Find the fraction.

OR

(b) Find the value of k for which the following pair of linear equations will have infinitely many solutions : $kx + 3y - (k-3) = 0$ and $12x + ky - k = 0$ Hence, find any two solutions of the given pair of equations.	
Solution: (a) Let the fraction be $\frac{x}{y}$	½
$\frac{x-1}{y} = \frac{1}{3} \quad \Rightarrow 3x - y = 3$..............(i)	½
$\frac{x}{y+8} = \frac{1}{4} \quad \Rightarrow 4x - y = 8$..............(ii)	½
On solving the equations (i) and (ii), we get $x = 5, y = 12$	1
Required fraction is $\frac{5}{12}$	½
OR	
(b) For infinitely many solutions: $\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$	1
$k^2 = 36$ and $k^2 - 3k = 3k$ $(k = \pm 6)$ and $(k = 6, 0)$ $\therefore\ k = 6$	1
For k =6, equations are $6x + 3y = 3$ and $12x + 6y = 6$ any two correct solutions	½+½
27. Prove the following trigonometric identity : $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$	
Solution: LHS = $\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$	½
$= 1 + \cos A$	1
$= \frac{(1+\cos A)(1-\cos A)}{(1-\cos A)}$	½
$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A}$ = RHS	1

28. A box contains 6 blue, 4 white and 8 red marbles. A marble is drawn at random from this box. Find the probability that the marble so drawn is :

(i) white

(ii) white or red

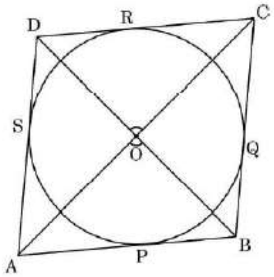
(iii) not red

Solution: (i) P (white marble) = $\frac{4}{18}$ or $\frac{2}{9}$ 1

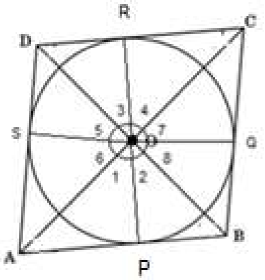
(ii) P (white or red marble) = $\frac{12}{18}$ or $\frac{2}{3}$ 1

(iii) P (not a red marble) = $\frac{10}{18}$ or $\frac{5}{9}$ 1

29. In the given figure, a circle is inscribed in a quadrilateral ABCD which touches the sides AB, BC, CD and DA at P, Q, R and S respectively. Prove that $\angle$ AOB + $\angle$ COD = 180°.



Solution:



Proving ΔOAP ≅ ΔOAS (by any congruency criterion) 1

$\Rightarrow \angle 1 = \angle 6$ (cpct) 1

Similarly $\angle 3 = \angle 5$, $\angle 4 = \angle 7$ and $\angle 2 = \angle 8$

Also $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ ½

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$\angle AOB + \angle COD = 180°$ ½

30. (a) Prove that $\sqrt{2}$ is an irrational number.

OR

(b) Find which among the following numbers a, b and c is/are composite numbers.

$$a = 7 \times 11 \times 13 + 13$$

$$b = 6 \times 5 \times 4 + 4$$

$$c = 7 \times 13 + 6$$

Solution: (a)Let $\sqrt{2}$ be a rational number such that $\sqrt{2} = \frac{p}{q}$ ½

(p and q are co-prime numbers, $q \neq 0$)

$\sqrt{2}\,q = p \Rightarrow 2q^2 = p^2$

2 divides p^2 $\Rightarrow$ 2 divides p as well 1

p = 2m (for some integer m)

$2q^2 = 4m^2 \quad \Rightarrow q^2 = 2m^2$

2 divides q^2 $\Rightarrow$ 2 divides q as well

p and q have a common factor 2 which is a contradiction as p and q are co-prime. 1

$\therefore$ our assumption is wrong

Hence, $\sqrt{2}$ is an irrational number ½

OR

(b) a and b are **only** composite numbers. 3

31. Find the zeroes of the polynomial $4x^2 + 4x + 1$ and verify the relationship between the zeroes and the coefficients of the given polynomial.

Solution: $4x^2 + 4x + 1$

$(2x + 1)\,(2x + 1)$

Zeroes are $-\frac{1}{2}$ and $-\frac{1}{2}$ 1

Sum of zeroes $= \frac{-1}{2} + \frac{-1}{2} = -1 = \frac{-4}{4} = \frac{-\mathit{Coefficient\ of}\ x}{\mathit{Coefficient\ of}\ x^2}$ 1

Product of zeroes $= \frac{-1}{2} \times \frac{-1}{2} = \frac{1}{4} = \frac{\mathit{constant\ term}}{\mathit{Coefficient\ of}\ x^2}$ 1

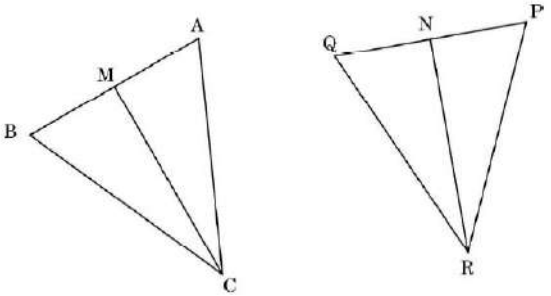
SECTION D

*This section has **4** Long Answer (LA) type questions carrying **5** marks each.* *4×5=20*

32. (a) State and Prove “Basic Proportionality Theorem”.

OR

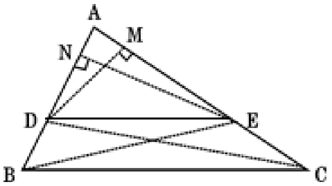
(b) In the given figure, CM and RN are respectively, the medians of Δ ABC and Δ PQR. If Δ ABC ~ Δ PQR, prove that :

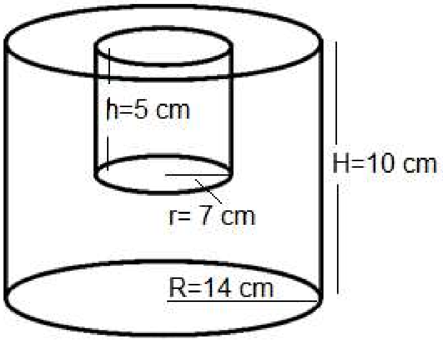


(i) Δ AMC ~ Δ PNR

(ii) ∠ BCM = ∠ QRN

(iii) Δ BMC ~ Δ QNR

Solution: (a) Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.	Correct Statement: 1 mark
Given: In Δ ABC, DE ∥ BC To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$  Construction: Draw DM ⊥ AC, EN ⊥ AB, join BE and CD	Given + To prove + Construction + Figure: 1 mark
Proof : $\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB}$ ……………………..(i)	1
$\frac{ar(\Delta ADE)}{ar(\Delta ECD)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$ ………..……….(ii)	1
as ΔDBE and ΔDCE lie on the same base DE and between same parallels BC and DE ∴ ar(ΔDBE) = ar(ΔECD) or $\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{ar(\Delta ADE)}{ar(\Delta ECD)}$ ………..(iii)	½
From (i), (ii) and (iii), we get $\frac{AD}{DB} = \frac{AE}{EC}$	½

OR	
(b) (i) $\Delta ABC \sim \Delta PQR \Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$	½
$\Rightarrow \frac{AC}{PR} = \frac{\frac{1}{2}AB}{\frac{1}{2}PQ} \Rightarrow \frac{AC}{PR} = \frac{AM}{PN}$	½
Also $\angle A = \angle P$	½
$\therefore \Delta AMC \sim \Delta PNR$ (by SAS similarity criterion)	½
(ii) $\Delta AMC \sim \Delta PNR$ (from part (i))	
$\therefore \angle ACM = \angle PRN$	½
Also $\angle ACB = \angle PRQ$ (as $\Delta ABC \sim \Delta PQR$)	
$\therefore \angle ACB - \angle ACM = \angle PRQ - \angle PRN$	½
$\Rightarrow \angle BCM = \angle QRN$	
(iii) $\Delta ABC \sim \Delta PQR \Rightarrow \frac{AB}{PQ} = \frac{BC}{QR}$	½
$\Rightarrow \frac{BC}{QR} = \frac{\frac{1}{2}AB}{\frac{1}{2}PQ} \Rightarrow \frac{BC}{QR} = \frac{BM}{QN}$	½
Also $\angle B = \angle Q$	½
$\therefore \Delta BMC \sim \Delta QNR$ (by SAS similarity criterion)	½
33. From a solid wooden cylinder of height 10 cm and radius 14 cm, a cylinder of radius 7 cm and height 5 cm is scooped out to form a cavity inside the solid cylinder. Find the total surface area of the remaining solid.	
Solution: 	

Total surface area of the remaining solid

= CSA of outer cylinder + CSA of inner cylinder + area of bases + Area of ring

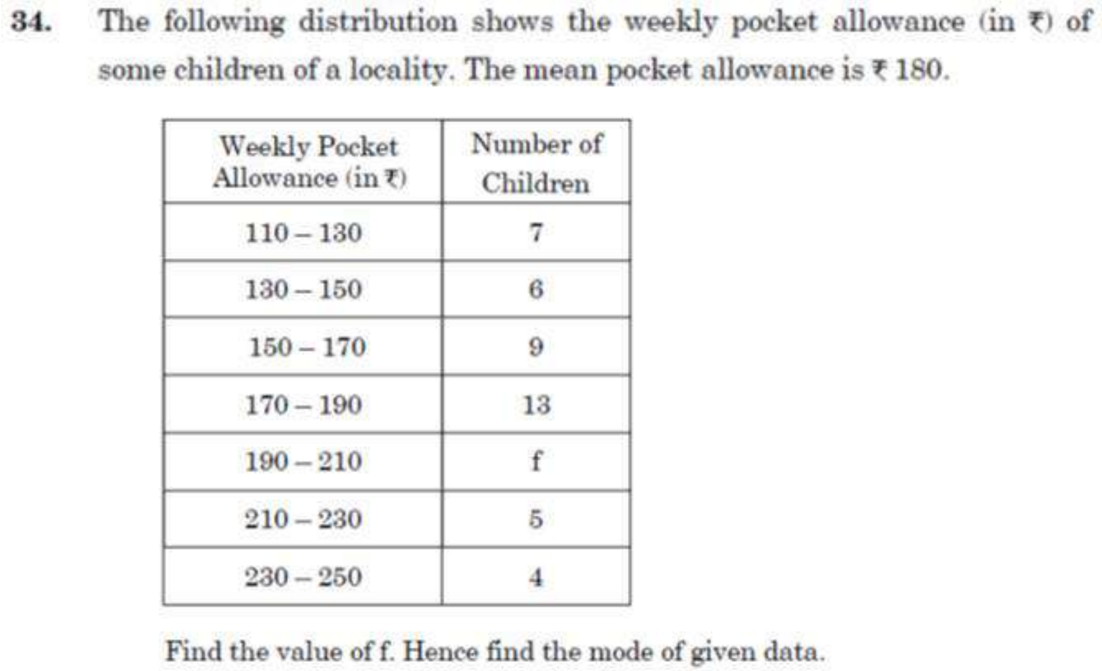
$$= 2\pi RH + 2\pi rh + (\pi R^2 + \pi r^2) + (\pi R^2 - \pi r^2)$$

$$= 2\pi RH + 2\pi rh + 2\pi R^2$$

$$= 2\times\frac{22}{7}\times14\times10 + 2\times\frac{22}{7}\times7\times5 + 2\times\frac{22}{7}\times14\times14$$ 1½ +1½+1½

$$= 880 + 220 + 1232$$

$$= 2332 \text{ sq. cm}$$ ½



34. The following distribution shows the weekly pocket allowance (in ₹) of some children of a locality. The mean pocket allowance is ₹ 180.

Weekly Pocket Allowance (in ₹)	Number of Children
110 – 130	7
130 – 150	6
150 – 170	9
170 – 190	13
190 – 210	f
210 – 230	5
230 – 250	4

Find the value of f. Hence find the mode of given data.

Solution:

C. I,	x_i	f_i	$f_i\ x_i$
110 – 130	120	7	840
130 – 150	140	6	840
150 – 170	160	9	1440
170 – 190	180	13	2340
190 – 210	200	f	200 f
210 – 230	220	5	1100
230 – 250	240	4	960
		44+f	7520 + 200 f

Correct Table: 1½ marks

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i}$$

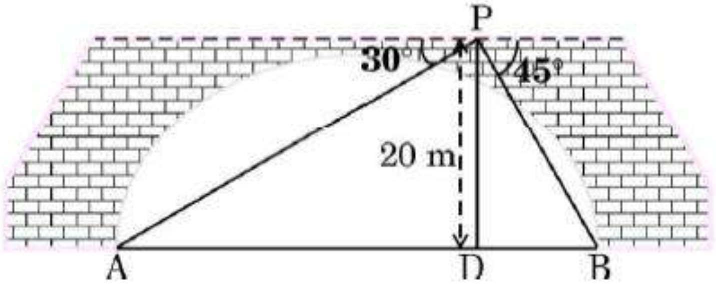
$180 = \frac{7520+200\,f}{44 + f}$	1
$f = 20$	½
$\text{Mode} = 190 + \frac{20-13}{20(2)-13-5}\times 20$	1½
$= 196.36$	½
35. (a) Find two consecutive odd integers, sum of whose squares is 290. **OR** (b) A charity trust decides to build a rectangular hall having an area of 300 m^2. The length of the hall is one metre more than twice its width. Find the length and breadth of the hall.	
Solution: (a) Let the two consecutive odd integers be x and x + 2	½
$x^2 + (x+2)^2 = 2$	1½
$2x^2 + 4x - 286 = 0$ or $x^2 + 2x - 143 = 0$	1½
$(x - 11)(x + 13) = 0$ $x = 11$	1
Required odd integers are 11 and 13	½
OR	
(b) Let width be x m and length be (2x + 1) m	½
A.T.Q. $(2x + 1)x = 300$	1½
$2x^2 + x - 300 = 0$	1½
$(x - 12)(2x + 25) = 0$ $x = 12$ (Rejecting $x = \frac{-25}{2}$)	1
length = 25 m and width = 12 m	½

SECTION E

*This section has **3** case study based questions carrying **4** marks each.* *3×4=12*

Case Study – 1

36. Two motorboats A and B are waiting at the opposite banks of a river in order to reach the opposite side. From a point P on the bridge, 20 m above the river, the angles of depression of the boats are 30° and 45° respectively, as shown in the figure given below. Both the boats leave at the same time at the speed of 10 m/s and 5 m/s, respectively



Based on the above information, answer the following questions :

(i) Find the distance travelled by boat A to reach point D in the river, vertically below the point P. (Use $\sqrt{3}$ =1·73) *1*

(ii) What is the width of the river ? *1*

(iii) (a) Which boat will reach point D first, and how much earlier, than the other boat ? *2*

OR

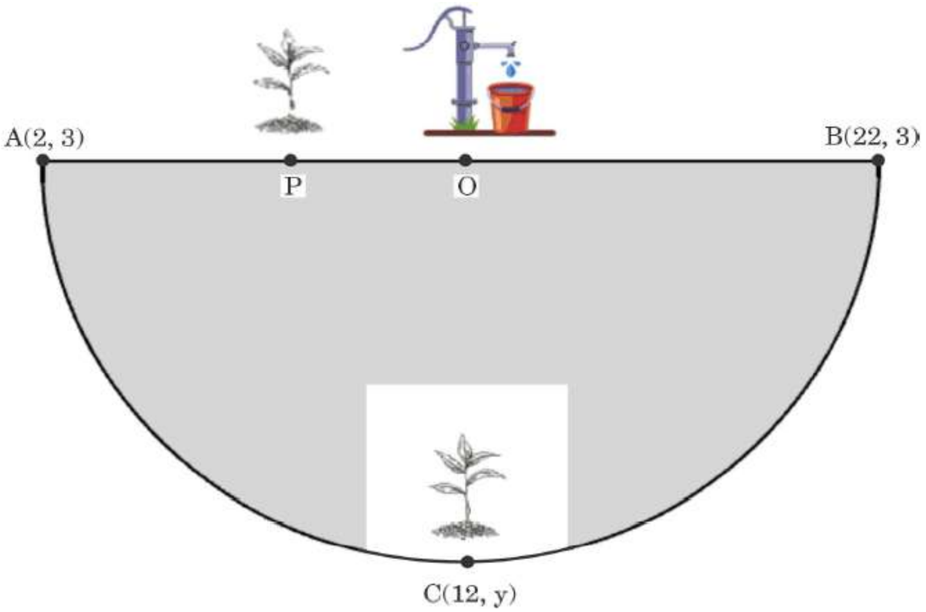
(b) What is the distance between the two boats after 3 seconds ? *2*

Solution:

(i) $\frac{20}{AD} = \tan 30°$	½
$AD = 20\sqrt{3} = 34.6$ m	½
(ii) $\frac{20}{DB} = \tan 45° \Rightarrow DB = 20$ m	½
Width of river = 34.6 + 20 = 54·6 m	½
(iii) (a) Time taken by boat A = $\frac{34 \cdot 6}{10}$ = 3.46 seconds	1
Time taken by boat B = $\frac{20}{5}$ = 4 seconds	½
Boat A will reach earlier by 0·54 seconds	½
OR	
(iii) (b) Distance covered by boat A in 3 seconds = 3 × 10 = 30 m	½
Distance covered by boat B in 3 seconds = 3 × 5 = 15 m	½
Distance between them after 3 seconds = 54.6 – (30+15) = 9.6 m	1

Case Study – 2

37. There is a semicircular park in Aman's society. He wishes to plant saplings along the boundary of the park. There is a borewell at the centre O of the park along the diameter AB as shown in the figure below.



Based on the above information, answer the following questions :

(i) Find the coordinates of point O. *1*

(ii) Find the radius of the semicircular park. *1*

(iii) (a) One sapling is kept at point C(12, y). Find the coordinates of C. *2*

OR

(b) One sapling is kept at point P along AB so that PA = $\frac{1}{3}$ PB. Find the coordinates of P. *2*

Solution: (i) Coordinates of O are (12, 3) 1

(ii) Radius = 10 1

(iii) (a) OC = radius = 10 ½

y = 13, y = – 7 1

Coordinates of the point 'C' are (12, 13) or (12, –7) ½

OR

(iii) (b) P divides AB in the ratio 1 : 3 ½

Coordinates of P are $\left(\frac{1\times 22 + 3\times 2}{4}, \frac{1\times 3 + 3\times 3}{4}\right)$ 1

i.e. (7, 3) ½

Case Study – 3

38. In a society, a yoga instructor was hired to train the people of the society to live a healthy lifestyle. Yoga sessions were held daily from 5 p.m. to 7 p.m. in the society park. On day one, 5 people joined the yoga session, on day two, 3 more people joined, on day three, another 3 people joined and in this manner every next day, 3 more people kept on joining.

Based on the given information, answer the following questions :

(i) On which day did 59 people join the yoga session ? *1*

(ii) How many people joined the yoga session on the 31st day ? *1*

(iii) (a) The yoga instructor was paid ₹ 100 for each person attending the yoga session. On which day would he earn ₹ 5,000 ? *2*

OR

(b) What was the total amount earned by the yoga instructor in 16 days ? *2*

Solution:	
(i) $5 + (n - 1)\,3 = 59$ $n = 19$	1
(ii) $a_{31} = 95$	1
(iii) (a) Number of persons = $\frac{5000}{100} = 50$	1
$5 + (n - 1)\,3 = 50$ $n = 16$	1
OR	
(iii) (b) $S_{16} = \frac{16}{2}\,[\,10 + 15\,(3)]$	1
$= 440$	½
Total amount earned in 16 days = 440 × 100	
= ₹ 44000	½