Set 430/2/2

SOLUTIONS
MATHEMATICS (BASIC)

SECTION A

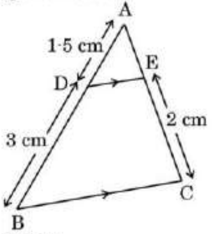
*This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each. 20×1=20*

1. The mid-point of the line segment joining points (1, 3) and (1, – 3) lies :

(A) at the origin (B) in the second quadrant
(C) on x-axis (D) on y-axis

Ans: (C) on x-axis	1

2. In the given figure, if DE || BC, AD = 1·5 cm, DB = 3 cm and EC = 2 cm, the length of AC is :



(A) 1·5 cm (B) 3 cm
(C) 3·5 cm (D) 4·5 cm

Ans: (B) 3 cm	1

3. In two concentric circles, a tangent to the smaller circle will intersect the larger circle at :

(A) zero point (B) one point
(C) two points (D) three points

Ans: (C) two points	1

4. The value of $\frac{2 \tan 60^\circ}{1 - \tan^2 60^\circ}$ is :

(A) -3 (B) $\sqrt{3}$
(C) $-\frac{1}{\sqrt{3}}$ (D) $-\sqrt{3}$

Ans: (D) $-\sqrt{3}$	1

5. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 =$

(A) 1 (B) 2
(C) $2 + 2 \sin\theta \cos\theta$ (D) $2 + 4 \sin\theta \cos\theta$

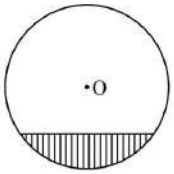
Answer: (B) 2	1

6. The length of the shadow of a tower when the sun's altitude changes from 30° to 60° will :

(A) become shorter (B) become longer
(C) remain same (D) be doubled

Ans: (A) become shorter	1

7. In the given figure, the shaded region represents :



(A) minor sector (B) major sector

(C) minor segment (D) major segment

Ans: (C) minor segment	1

8. **The area of a quadrant of a circle of radius '2r' is :**

(A) $\frac{1}{4}\pi r^2$ (B) $\frac{1}{2}\pi r^2$

(C) πr^2 (D) $2\pi r^2$

Ans: (C) πr^2	1

9. A cone of height 'h' and radius 'r' is surmounted on a solid cylinder of same dimensions. The total surface area of the entire solid will be :

(A) $2\pi rh + \pi r\sqrt{h^2 + r^2}$

(B) $2\pi rh + \pi r^2 + \pi r\sqrt{h^2 + r^2}$

(C) $2\pi rh + 2\pi r^2 + \pi r\sqrt{h^2 + r^2}$

(D) $2\pi rh + \pi r\sqrt{h^2 + r^2} - \pi r^2$

Ans: (B) $2\pi rh + \pi r^2 + \pi r\sqrt{h^2 + r^2}$	1

10. In the formula of mode given by mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$,

f_1 denotes the :

(A) frequency of the modal class

(B) frequency of class preceding modal class

(C) frequency of class succeeding modal class

(D) cumulative frequency of modal class

Ans: (A) frequency of the modal class	1

11. For a distribution, if mean = median = a, then its mode is :

(A) 3a (B) 2a

(C) a (D) 0

Ans: (C) a	1

12. The total number of outcomes in the experiment of simultaneous throw of three dice is :

(A) 6 (B) 18

(C) 36 (D) 216

Ans: (D) 216	1
13. The value of (HCF – LCM) for the two numbers 3 and 5 is : (A) 2 (B) 4 (C) 14 (D) – 14	
Ans: (D) -14	1
14. The number 3^n, where n is a natural number, cannot end with the digit : (A) 1 (B) 3 (C) 5 (D) 7	
Ans: (C) 5	1
15. The number of solutions of the system of equations x = 3, y = –1 is : (A) 0 (B) 1 (C) 2 (D) Infinite	
Ans: (B) 1	1
16. The value of 'a' for which the equation $x^2 + 3x + a = 0$ has real and equal roots is : (A) $\frac{4}{9}$ (B) $\frac{9}{4}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$	
Ans: (B) $\frac{9}{4}$	1
17. Which of the following equations is a quadratic equation ? (A) $x^2 = (x + 1)^2$ (B) $(x - 1)(x + 2) = 2x + 1$ (C) $(x + 2)^3 = 2x(x^2 - 1)$ (D) $\sqrt{x} = x^2$	
Ans: (B) $(x-1)(x+2) = 2x+1$	1
18. The distance of the point (2, 3) from the origin is : (A) 2 (B) 3 (C) 5 (D) $\sqrt{13}$	
Ans: (D) $\sqrt{13}$	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* All congruent triangles are similar.

Reason (R) : In congruent triangles, the ratio of corresponding sides is 1 : 1.

Ans: (A)Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). 1

20. *Assertion (A) :* The prime numbers which divide 36 also divide 6.

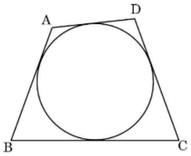
Reason (R) : Any number which divides p^2 also divides p.

Ans: (C) Assertion (A) is true, but Reason (R) is false. 1

SECTION B

*This section has **5** Very Short Answer (VSA) type questions carrying **2** marks each.* 5×2=10

21. A quadrilateral circumscribes the circle as shown in the given figure. If AB = 5 cm, BC = 7 cm and CD = 6 cm, then find the length of AD.



Solution: Proving AB + CD = BC + AD

5 + 6 = 7 + AD 1½

$\Rightarrow$ AD = 4 cm ½

22. Evaluate :

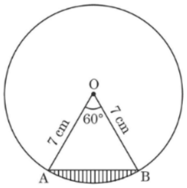
$$\frac{\sin 45^\circ}{\sec 30^\circ - \tan 30^\circ}$$

Solution: $\frac{\sin 45^0}{\sec 30^0 - \tan 30^0} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}}$ 1½

$= \frac{\sqrt{3}}{\sqrt{2}}$ or $\frac{\sqrt{6}}{2}$ ½

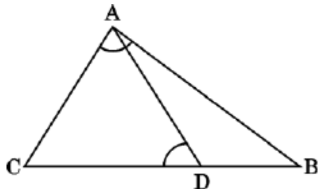
23. (a) The area of a smaller circle is equal to the area of a sector of a larger circle with central angle 120°. The radii of the smaller and larger circles are 'r' and 'R' respectively. Find r : R.

OR

(b) In the given figure, O is the centre of a circle of radius 7 cm. AB is a chord of the circle. Find the perimeter of the shaded region. 	
Solution:(a) A.T.Q.	
$\pi r^2 = \frac{120}{360}\pi R^2$	1
$\frac{r^2}{R^2} = \frac{1}{3}$	½
$r : R = 1 : \sqrt{3}$	½
OR	
(b) Δ AOB is an equilateral triangle as $\angle AOB = 60^0$	
$\therefore$ AB = 7 cm	½
Length of minor arc AB $= \frac{60}{360} \times 2 \times \frac{22}{7} \times 7 = \frac{22}{3}$	½
$\therefore$ Perimeter of the shaded region $= \frac{22}{3} + 7$	½
$= \frac{43}{3}$ cm or 14.33 cm	½
24. (a) Find the value of c for which the following pair of linear equations has infinitely many solutions : $cx + 3y = c - 3$ $12x + cy = c$ **OR** (b) Solve for x and y : $3x + 2y = 65$ $2x + 3y = 60$	
Solution: (a) $\frac{c}{12} = \frac{3}{c} = \frac{c-3}{c}$	1
$\Rightarrow c^2 = 36$ and $c^2 - 6c = 0$	
$\Rightarrow c = \pm 6$ and $c = 0,6$	½
$\therefore c = 6$	½
OR	
(b) Solving the given equations to get x = 15	1
and y =10	1

25. D is a point on side BC of Δ ABC such that ∠ ADC = ∠ BAC. Prove that $(CA)^2 = CB . CD$.

Solution:



Correct figure: ½

In ΔADC and ΔBAC,

∠ADC = ∠BAC (given)

∠C = ∠C (common) ½

ΔADC ~ ΔBAC (By AA similarity criterion) ½

$$\Rightarrow \frac{CA}{CB} = \frac{CD}{CA} \Rightarrow (CA)^2 = CB.CD$$ ½

SECTION C

*This section has **6** Short Answer (SA) type questions carrying **3** marks each. 6×3=18*

26. (a) Solve graphically the following pair of linear equations :

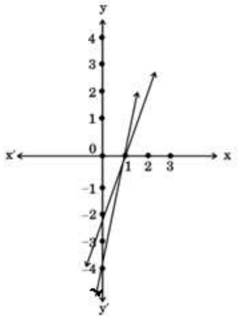
$2x - y = 2$ and $4x - y = 4$

Also, write the coordinates of the points where the lines represented by these equations cut the y-axis.

OR

(b) An academy offering cricket coaching bought 10 bats and 5 balls for ₹ 32,500. Later, the academy bought 2 bats and 8 balls for ₹ 10,000. If there is no change in the cost of the bat and of the ball, find the cost of 1 bat and 1 ball.

Solution:(a) Correct graph of each equation 1+1



Solution is x=1, y=0 or (1, 0) ½

Lines cut y axis at (0, – 2) and (0, – 4) ½

OR

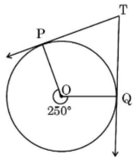
(b) Let the cost of 1 bat be ₹x and the cost of 1 ball be ₹y

A.T.Q.

$10x + 5y = 32500$ or $2x + y = 6500$ -------(i) 1

$2x + 8y = 10000$ or $2x + 8y = 10000$ ----(ii)	1
Solving (i) and (ii) to get $x = 3000$ and $y = 500$	½+½
Cost of 1 bat = ₹3,000	
Cost of 1 ball = ₹500	

27. In the given figure, TP and TQ are tangents at points P and Q of the circle respectively. If reflex ∠ POQ = 250°, find the measure of each angle of quadrilateral POQT.



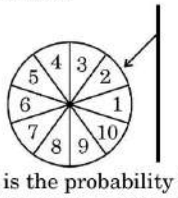
Solution: $\angle POQ = 360° - 250° = 110°$	1
As tangent is perpendicular to the radius through the point of contact	
$\angle OPT = \angle OQT = 90°$	1
In Quadrilateral OPTQ	
$\angle OPT + \angle OQT + \angle POQ + \angle PTQ = 360°$	
$\angle PTQ = 70°$	1

28. Prove the following trigonometric identity :

$$\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$$

Solution: LHS = $\dfrac{(1-\sin\theta)^2 + \cos^2\theta}{\cos\theta\,(1-\sin\theta)}$	½
$= \dfrac{1 - 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta\,(1-\sin\theta)}$	1
$= \dfrac{2 - 2\sin\theta}{\cos\theta\,(1-\sin\theta)}$	1
$= 2\sec\theta$ = RHS	½

29. A game of chance consists of spinning a wheel which comes to rest at one of the numbers from 1 to 10 (as shown in the given figure) with equal probabilities.



What is the probability that the wheel stops at

(i) a prime number greater than 2 ?

(ii) an odd number less than 9 ?

(iii) a multiple of 4 ?

Solution:(i) P (a prime number greater than 2) = $\frac{3}{10}$	1
(ii) P (an odd number less than 9) = $\frac{4}{10}$ or $\frac{2}{5}$	1
(iii) P (a multiple of 4) = $\frac{2}{10}$ or $\frac{1}{5}$	1
30. (a) Prove that $\sqrt{5}$ is an irrational number. **OR** (b) State the "Fundamental Theorem of Arithmetic" and use it to find LCM of 36 and 54.	
Solution:	
(a) Let $\sqrt{5}$ be a rational number such that $\sqrt{5} = \frac{p}{q}$ (p and q are co-prime numbers, q $\neq 0$)	½
$\sqrt{5}$ q = p $\Rightarrow$ 5q^2 = p^2 5 divides p^2 $\Rightarrow$ 5 divides p as well	1
p = 5m (for some integer m) 5q^2 = 25m^2 $\Rightarrow$ q^2 = 5m^2 5 divides q^2 $\Rightarrow$ 5 divides q as well p and q have a common factor 5 which is a contradiction as p and q are coprime.	1
$\therefore$ our assumption is wrong Hence, $\sqrt{5}$ is an irrational number	½
OR	
(b)Statement: "Every composite number can be factorized as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur."	1
$36 = 2^2 \times 3^2$	½
$54 = 2 \times 3^3$	½
$LCM\,(36, 54) = 2^2 \times 3^3 \quad or \quad 108$	1
31. Find the zeroes of the polynomial p(x) = 2x^2 + 5x + 2 and verify the relationship between zeroes of p(x) and its coefficients.	
Solution: p(x) = 2x^2 + 5x + 2 = (2x + 1) (x + 2)	

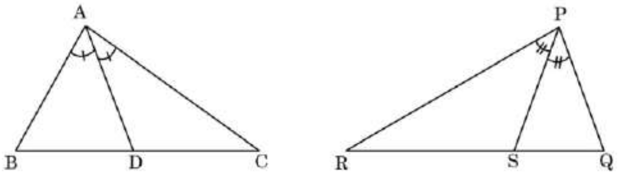
Zeroes are $-\frac{1}{2}, -2$	1
Sum of zeroes $= -\frac{1}{2} + (-2) = -\frac{5}{2} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$	1
Product of zeroes = $= -\frac{1}{2} \times (-2) = \frac{2}{2} = \frac{constant\ term}{coefficient\ of\ x^2}$	1

SECTION D

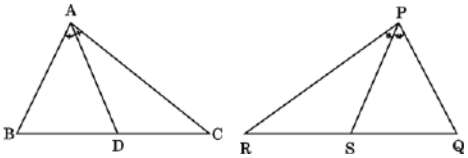
*This section has **4** Long Answer (LA) type questions carrying **5** marks each.* *4×5=20*

32. State AA criterion of similarity of two triangles and use it to prove the following.

In the given figures of Δ ABC and Δ PQR, AD and PS are angle bisectors of ∠ BAC and ∠ RPQ respectively.

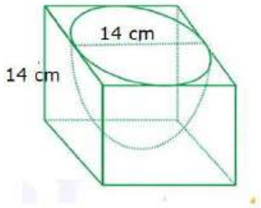


If Δ ABC ~ Δ PQR, prove that Δ ACD ~ Δ PRS.

Solution:	
Statement: If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.	1
	
Proof: ΔABC ~ ΔPQR (given)	
As corresponding angles of similar triangles are equal.	
∴ ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R -------(i)	1
In ΔACD and ΔPRS	
$\frac{1}{2}$∠BAC = $\frac{1}{2}$∠RPQ (As AD and PS are angle bisectors)	1
∠CAD = ∠RPS	1
∠C = ∠R (from (i))	½
∴ ΔACD ~ ΔPRS (by AA similarity criterion)	½

33. A hemispherical depression is scooped out from the top face of a wooden cubical block of side 14 cm. If the diameter of the hemisphere is equal to the side of the cube, find the total surface area of the remaining solid block. (Use $\pi = \frac{22}{7}$)

Solution:



Radius of hemisphere = r = $\frac{14}{2}$ = 7 cm

Total surface area of the remaining solid = $6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$

$= 6 \times 14 \times 14 + \frac{22}{7} \times 7 \times 7$ 2 +2

= 1330 sq. cm 1

34. (a) The lengths of 40 leaves of a plant are measured, correct to the nearest millimetre and data obtained is represented in the following table :

Length in (mm)	*Number of leaves*
100 – 120	8
120 – 140	9
140 – 160	12
160 – 180	5
180 – 200	6

Find the median length (in mm) of the leaves.

OR

(b) A class teacher has the following absentees record of 30 students of a class.

Number of days	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24
Number of Absent students	1	8	x	6	5	y

If the mean number of days a student was absent is 12, find the values of x and y.

Solution:(a)

C.I.	f	CF
100 – 120	8	8
120 – 140	9	17
140 – 160	12	29
160 – 180	5	34
180 – 200	6	40
	40	

Correct table: 2

median class: 140 – 160

$$\text{Median} = \ell + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 140 + \frac{20 - 17}{12} \times 20$$ 2

$$= 145$$ 1

∴ The median length of the leaves is 145 mm

OR

(b)

C.I.	f_i	x_i	$f_i\,x_i$
0 – 4	1	2	2
4 – 8	8	6	48
8 – 12	x	10	10x
12 – 16	6	14	84
16 – 20	5	18	90
20 – 24	y	22	22y
	20 + x + y		224 + 10x + 22y

Correct Table: 2

$x + y + 20 = 30 \Rightarrow x + y = 10$ – (i) 1

$12 = \frac{10x + 22y + 224}{30} \Rightarrow 5x + 11y = 68$ – (ii) 1

Solving (i) and (ii) we get

$x = 7$ ½

$y = 3$ ½

35. (a) The sum of areas of two squares is 2650 cm^2. If the sum of their perimeters is 280 cm, find the sides of the two given squares.

OR

(b) Express the equation $\frac{1}{x} - \frac{1}{x-2} = 3$, $(x \neq 0, 2)$ as a quadratic equation in standard form. Hence, find the roots of the quadratic equation so obtained.

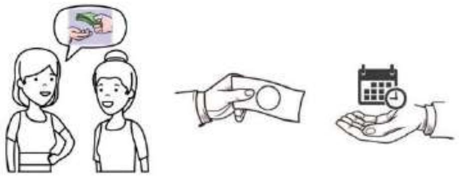
Solution:	
(a) Let the sides of squares be x and y	
A.T.Q.	
$x^2 + y^2 = 2650 \quad ---------(i)$	1
$4x+4y=280 \Rightarrow x+y=70 \; ---(ii)$	1
getting $2x^2 - 140x + 2250 = 0$ or $x^2 - 70x + 1125 = 0$	1
$\Rightarrow (x-25)(x-45)=0$	
$\Rightarrow x = 25$ and $x = 45$	1
$\therefore y = 45$ and $y = 25$	1
sides of square are 25cm and 45cm.	
OR	
(b)	
$\frac{x-2-x}{x(x-2)} = 3$	1
$\Rightarrow 3x^2 - 6x + 2 = 0$	1
D = 36 – 24 = 12	1
Roots are $\frac{6+\sqrt{12}}{6}$ and $\frac{6-\sqrt{12}}{6}$	1+1
or $1+\frac{\sqrt{3}}{3}$ and $1-\frac{\sqrt{3}}{3}$	

SECTION E

*This section has **3** case study based questions carrying **4** marks each.* 3×4=12

Case Study – 1

36. A woman borrowed ₹ 10,00,000 from her friend and promised to return the borrowed money in monthly instalments beginning from the next month. After one month, she returned ₹ 10,000, the next month she returned ₹ 15,000, the third month she returned ₹ 20,000 and so on, thereby increasing the monthly instalment uniformly.



Based on the above information, answer the following questions :

(i) Find the amount of instalment paid in the tenth month. *1*

(ii) In which instalment did she pay ₹ 40,000 ? *1*

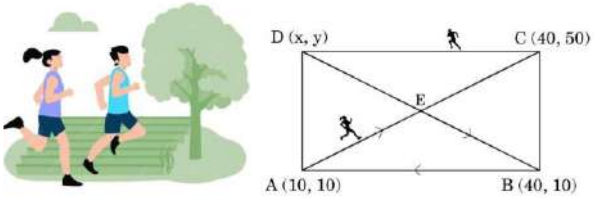
(iii) (a) If she returned ₹ 11,50,000 in all, how many instalments did she pay ? *2*

OR

(b) By which instalment has she returned a total amount of ₹ 3,25,000 ? *2*

Solution: (i) $a_{10} = 10000 + 5000 \times 9$	
$= 55000$	1
$\Rightarrow$ Amount of instalment paid in the tenth month =₹55,000	
(ii) $40000 = 10000 + (n - 1)\,5000$	
$\Rightarrow n = 7$	1
$\Rightarrow$ In the 7th instalment she paid ₹40,000	
(iii) (a) $1150000 = \frac{n}{2}[20000 + (n - 1)\,5000]$	1
$5n^2 + 15n - 2300 = 0$ or $n^2 + 3n - 460 = 0$	½
$(n - 20)(n + 23) = 0$	
$n = 20, n = -23$ (rejected)	½
$\Rightarrow$ She paid 20 instalments in order to return ₹11,50,000	
OR	
(iii) (b) $325000 = \frac{n}{2}[20000 + (n - 1)\,5000]$	1
$5n^2 + 15n - 650 = 0$ or $n^2 + 3n - 130 = 0$	½
$(n - 10)(n + 13) = 0$	
$n = 10, n = -13$ (rejected)	½
$\Rightarrow$ She returned ₹3,25,000 by the 10th instalment.	

37. A field is in the form of a rectangle. The coordinates of the rectangular field ABCD are A(10, 10), B(40, 10), C(40, 50) and D(x, y). Anil and Anita, two friends decided to have a race. Anita started from point A and moved to point E along the diagonal AC, where E is the point of intersection of both the diagonals of ABCD. From point E, she moved to point B along the other diagonal DB and then moved back to point A along BA. While Anil started from point C and ran to point A via D along the boundary of the field.



Based on the above information, answer the following questions :

(i) Find the coordinates of point E. *1*

(ii) Find the distance between the points B and C. *1*

(iii) (a) Find the coordinates of point D and the distance BD. *2*

OR

(b) Find the total distance travelled by Anita. *2*

Solution: (i) Coordinates of E are (25, 30)	1
(ii) Distance BC = $\sqrt{(40-40)^2+(50-10)^2}$ = 40	1
(iii) (a) Co-ordinates of D	
$\left(\frac{x+40}{2},\frac{y+10}{2}\right)=(25,30)$	
By comparing we get, x = 10 and y = 50	½+½
Co-ordinates of D are (10, 50)	
BD = $\sqrt{(30)^2+(-40)^2}$ = 50	1
OR	
(iii) (b)Distance travelled by Anita	
= AE + EB + BA	
= $\sqrt{(15)^2+(20)^2}+\sqrt{(-15)^2+(20)^2}+\sqrt{(30)^2+(0)^2}$	1½
= 25 + 25 + 30	
= 80	½

Case Study – 3

38. Kite festival is a popular festival in India which takes place duri Makar Sankranti. The festival is celebrated by people flying kites frc their rooftops. Reena and Ravi are also flying kites to enjoy the festiv The height of Reena's kite is 60 m above the ground. The string attach to the kite is temporarily tied to a point on the ground, and t inclination of the string with the ground is 30°. Ravi is flying a kite frc a 10 m high building. His kite is also flying 60 m above the ground a the length of the string used by Ravi is same as that of Reena's. θ is t angle of elevation of Ravi's kite from a point on the rooftop.

Based on the above information, answer the following questions :

(i) Find the length of string used by Reena. *1*

(ii) Find the value of sin θ. *1*

(iii) (a) If θ changes to 60°, without changing the length of the string, what will be the height of Ravi's kite above the ground ? (Use $\sqrt{3} = 1{\cdot}7$) *2*

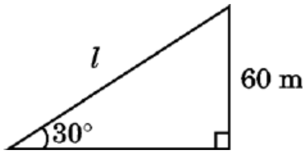
OR

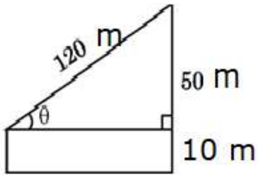
(b) What would have been the height of Ravi's kite above the ground, if the string had an inclination of 30° with the ground, assuming that the length of the string does not change ? *2*

Solution: (i)

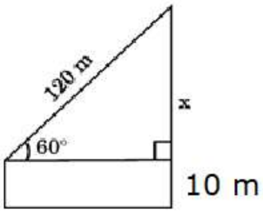
$$\frac{60}{l} = \sin 30°$$

$$l = 120 \text{ m}$$ 1





(ii) $\sin\theta = \frac{50}{120}$ or $\frac{5}{12}$ 1



(iii) (a) $\frac{x}{120} = \sin 60^0$

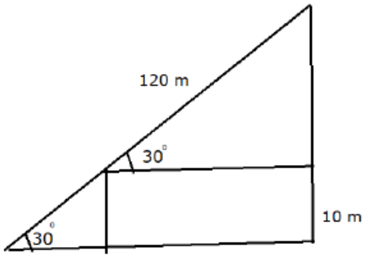
$\Rightarrow x = 60\sqrt{3}$ 1

Height of Ravi's kite = $60\sqrt{3}$ + 10 ½

= 102 + 10

= 112 m ½

OR

(iii) (b) $\frac{x}{120} = \sin 30^0$ 	
x = 60 m	1
Height of Ravi’s kite = 60 + 10 = 70 m	1