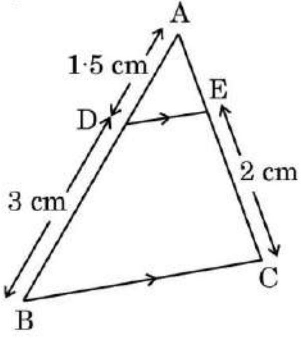
Set 430/2/1

SOLUTIONS
MATHEMATICS (BASIC)

SECTION A *This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each. 20×1=20* **1.** The value of (HCF – LCM) for the two numbers 3 and 5 is : (A) 2 (B) 4 (C) 14 (D) – 14	
Ans: (D) -14	1
2. The number 2^n, where n is a natural number, cannot end with the digit : (A) 4 (B) 6 (C) 2 (D) 0	
Ans: (D) 0	1
3. If (0, 0) is the solution of the equation $x + y = c - 1$, then the value of c is : (A) 0 (B) 1 (C) –1 (D) any real number	
Ans: (B) 1	1
4. The value of 'a' for which $ax^2 + 3x + 1 = 0$ has real and equal roots is : (A) $\frac{4}{9}$ (B) $\frac{9}{4}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$	
Ans: (B) $\frac{9}{4}$	1
5. Which of the following equations is a quadratic equation ? (A) $x^2 = (x + 1)^2$ (B) $(x - 1)(x + 2) = 2x + 1$ (C) $(x + 2)^3 = 2x(x^2 - 1)$ (D) $\sqrt{x} = x^2$	
Ans: (B) $(x-1)(x+2) = 2x+1$	1
6. The distance of the point (2, 3) from the origin is : (A) 2 (B) 3 (C) 5 (D) $\sqrt{13}$	

Ans: (D) $\sqrt{13}$	1
7. The mid-point of the line segment joining points (1, 3) and (1, – 3) lies : (A) at the origin (B) in the second quadrant (C) on x-axis (D) on y-axis	
Ans: (C) on x-axis	1
8. In the given figure, if DE || BC, AD = 1·5 cm, DB = 3 cm and EC = 2 cm, the length of AC is :  (A) 1·5 cm (B) 3 cm (C) 3·5 cm (D) 4·5 cm	
Ans: (B) 3 cm	1
9. In two concentric circles, a tangent to the smaller circle will intersect the larger circle at : (A) zero point (B) one point (C) two points (D) three points	
Ans: (C) two points	1
10. The value of $\dfrac{2\tan 60^\circ}{1-\tan^2 60^\circ}$ is : (A) -3 (B) $\sqrt{3}$ (C) $-\dfrac{1}{\sqrt{3}}$ (D) $-\sqrt{3}$	
Ans: (D) $-\sqrt{3}$	1
11. The value of $(\sin^2 A + \cos^2 A) + (\sec^2 A - \tan^2 A) - (\cot^2 A - \operatorname{cosec}^2 A)$ is : (A) 1 (B) –1 (C) 3 (D) – 2	

Ans: (C) 3	1

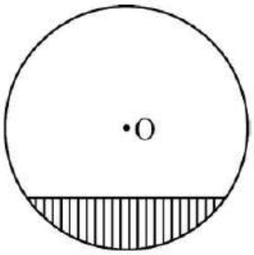
12. The length of the shadow of a tower when the sun's altitude changes from 30° to 60° will :

(A) become shorter (B) become longer

(C) remain same (D) be doubled

Ans: (A) become shorter	1

13. In the given figure, the shaded region represents :



(A) minor sector (B) major sector

(C) minor segment (D) major segment

Ans: (C) minor segment	1

14. The perimeter of a quadrant of a circle of radius r is :

(A) $\frac{1}{4}\pi r^2$ (B) $\frac{1}{4}\pi r^2 + 2r$

(C) $\frac{\pi r}{2}$ (D) $\frac{\pi r}{2} + 2r$

Ans: (D) $\frac{\pi r}{2}+2r$	1

15. A cone and a cylinder have the same base radius and volume.

The (height of cone : height of cylinder) is :

(A) 1 : 1 (B) 3 : 1

(C) 1 : 3 (D) 3 : 2

Ans: (B) 3:1	1

16. In the formula of mode given by mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$,

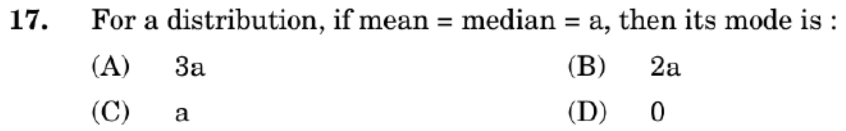
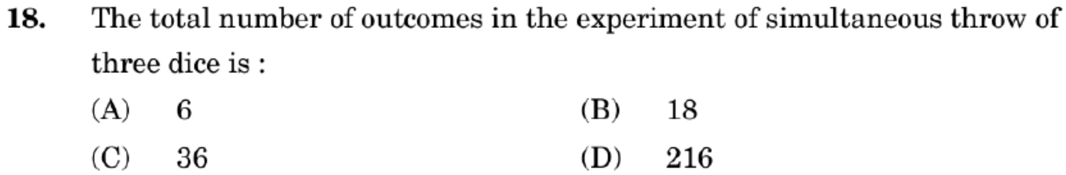
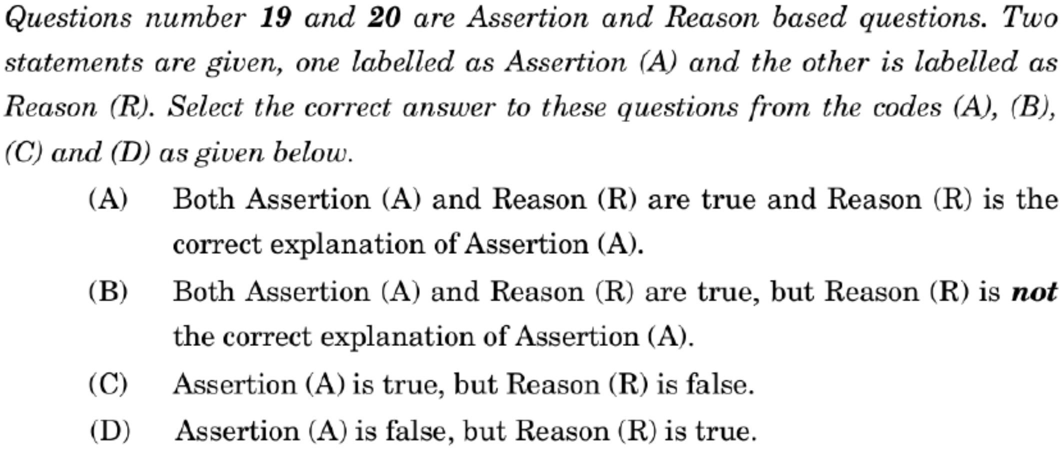
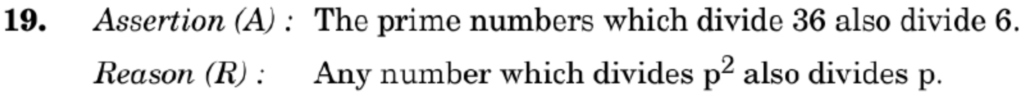
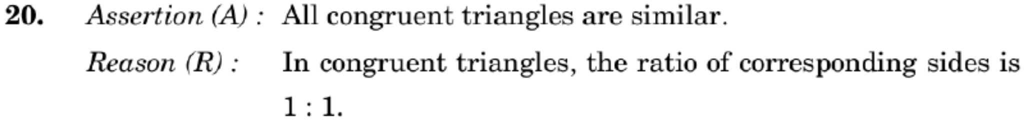
f_1 denotes the :

(A) frequency of the modal class

(B) frequency of class preceding modal class

(C) frequency of class succeeding modal class

(D) cumulative frequency of modal class

Ans: (A) frequency of the modal class	1
**17.** For a distribution, if mean = median = a, then its mode is : (A) 3a (B) 2a (C) a (D) 0	
Ans: (C) a	1
**18.** The total number of outcomes in the experiment of simultaneous throw of three dice is : (A) 6 (B) 18 (C) 36 (D) 216	
Ans: (D) 216	1
*Questions number* ***19*** *and* ***20*** *are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
**19.** *Assertion (A) :* The prime numbers which divide 36 also divide 6. *Reason (R) :* Any number which divides p^2 also divides p.	
Ans: (C) Assertion (A) is true, but Reason (R) is false.	1
**20.** *Assertion (A) :* All congruent triangles are similar. *Reason (R) :* In congruent triangles, the ratio of corresponding sides is 1 : 1.	
Ans: (A)Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	1

SECTION B

*This section has **5** Very Short Answer (VSA) type questions carrying **2** marks each.* *5×2=10*

21. (a) Find the value of c for which the following pair of linear equations has infinitely many solutions :

$$cx + 3y = c - 3$$
$$12x + cy = c$$

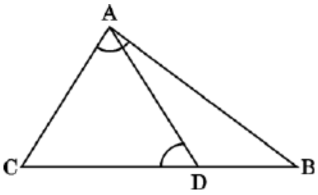
OR

(b) Solve for x and y :

$$3x + 2y = 65$$
$$2x + 3y = 60$$

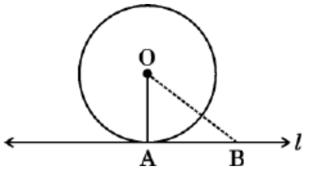
Solution: (a) $\frac{c}{12} = \frac{3}{c} = \frac{c-3}{c}$	1
$\Rightarrow c^2 = 36$ and $c^2 - 6c = 0$	
$\Rightarrow c = \pm 6$ and $c = 0,6$	½
$\therefore c = 6$	½
OR	
(b) Solving the given equations to get x = 15	1
and y =10	1

22. D is a point on side BC of Δ ABC such that ∠ ADC = ∠ BAC. Prove that $(CA)^2 = CB . CD$.

Solution:	
In ΔADC and ΔBAC, 	Correct figure: ½
∠ADC = ∠BAC (given)	
∠C = ∠C (common)	½
ΔADC ~ ΔBAC (By AA similarity criterion)	½
$\Rightarrow \frac{CA}{CB} = \frac{CD}{CA} \Rightarrow (CA)^2 = CB.CD$	½

23. Prove that the tangent drawn at any point of the circle is perpendicular to the radius through the point of contact.

Solution:



Correct figure: ½

Given: OA is the radius of circle with centre O and ℓ is the tangent at A

To prove: OA $\perp$ ℓ

Proof:

Let B be any point on ℓ other than A ½

B lies outside the circle ½

OB > radius

$\therefore$ OB > OA

OA $\perp$ ℓ (As perpendicular distance is the shortest distance) ½

24. Evaluate :

$$\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

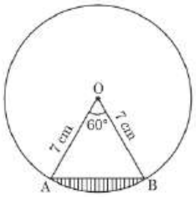
Solution: $\dfrac{\cos 45^0}{\sec 30^0 + \cos ec\, 30^0} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$ 1½

$= \dfrac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}}$ or $\dfrac{3\sqrt{2} - \sqrt{6}}{8}$ ½

25. (a) The area of a smaller circle is equal to the area of a sector of a larger circle with central angle 120°. The radii of the smaller and larger circles are ‘r’ and ‘R’ respectively. Find **r : R**.

OR

(b) In the given figure, O is the centre of a circle of radius 7 cm. AB is a chord of the circle. Find the perimeter of the shaded region.



Solution:(a) A.T.Q.

$\pi r^2 = \dfrac{120}{360}\ \pi R^2$ 1

$\dfrac{r^2}{R^2} = \dfrac{1}{3}$ ½

$r : R = 1 : \sqrt{3}$ ½

OR	
(b) Δ AOB is an equilateral triangle as ∠AOB = 60^0	
∴ AB = 7 cm	½
Length of minor arc AB = $\frac{60}{360} \times 2 \times \frac{22}{7} \times 7 = \frac{22}{3}$	½
∴ Perimeter of the shaded region = $\frac{22}{3} + 7$	½
$= \frac{43}{3}$ cm or 14.33 cm	½

SECTION C

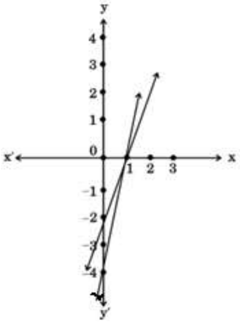
This section has ***6*** *Short Answer (SA) type questions carrying* ***3*** *marks each.* *6×3=18*

26. (a) Prove that $\sqrt{5}$ is an irrational number.

OR

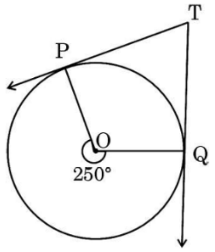
(b) **State the "Fundamental Theorem of Arithmetic" and use it to find LCM of 36 and 54.**

Solution: (a)Let $\sqrt{5}$ be a rational number such that $\sqrt{5} = \frac{p}{q}$ (p and q are co-prime numbers,	
q ≠ 0)	½
$\sqrt{5}$ q = p ⇒ $5q^2 = p^2$	
5 divides p^2 ⇒ 5 divides p as well	1
p = 5m (for some integer m)	
$5q^2 = 25m^2$ ⇒ $q^2 = 5m^2$	
5 divides q^2 ⇒ 5 divides q as well	
p and q have a common factor 5 which is a contradiction as p and q are co-prime.	1
∴ our assumption is wrong	½
Hence, $\sqrt{5}$ is an irrational number	
OR	
(b)Statement: "Every composite number can be factorized as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur."	1
$36 = 2^2 \times 3^2$	½
$54 = 2 \times 3^3$	½
$LCM(36, 54) = 2^2 \times 3^3$ or 108	1

27. Find the zeroes of the polynomial $p(x) = 3x^2 - 2x - 1$ and verify the relationship between the zeroes of p(x) and the coefficients of p(x).	
Solution:	
$p(x) = 3x^2 - 2x - 1$ $= (3x+1)(x-1)$	
$Zeroes\ are\ x = -\frac{1}{3}, 1$	1
$Sum\,of\ zeroes = -\frac{1}{3} + 1 = \frac{2}{3} = \frac{-coefficient\,of\ x}{coefficient\,of\ x^2}$	1
$Product\,of\ zeroes = -\frac{1}{3} \times 1 = \frac{-1}{3} = \frac{constant\ term}{coefficient\,of\ x^2}$	1
28. (a) Solve graphically the following pair of linear equations : $2x - y = 2$ and $4x - y = 4$ Also, write the coordinates of the points where the lines represented by these equations cut the y-axis. **OR** (b) An academy offering cricket coaching bought 10 bats and 5 balls for ₹ 32,500. Later, the academy bought 2 bats and 8 balls for ₹ 10,000. If there is no change in the cost of the bat and of the ball, find the cost of 1 bat and 1 ball.	
Solution: (a) Correct graph of each equation	1+1
	
Solution is x=1, y=0 or (1, 0)	½
Lines cut y axis at (0, – 2) and (0, – 4)	½
OR	
(b) Let the cost of 1 bat be ₹x and the cost of 1 ball be ₹y	
A.T.Q.	

10x + 5y = 32500 or 2x + y = 6500 -------(i)	1
2x + 8y = 10000 or 2x + 8y = 10000 ----(ii)	1
Solving (i) and (ii) to get x = 3000 and y = 500	½+½
Cost of 1 bat = ₹3,000	
Cost of 1 ball = ₹500	

29. In the given figure, TP and TQ are tangents at points P and Q of the circle respectively. If reflex ∠ POQ = 250°, find the measure of each angle of quadrilateral POQT.



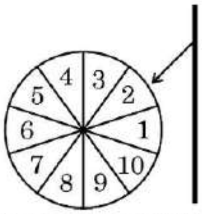
Solution: ∠POQ = 360° – 250° = 110°	1
As tangent is perpendicular to the radius through the point of contact	
∠OPT = ∠OQT = 90°	1
In Quadrilateral OPTQ	
∠OPT + ∠OQT + ∠POQ + ∠PTQ = 360°	
∠PTQ = 70°	1

30. Prove the following trigonometric identity :

$$\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2\sec\theta$$

Solution: LHS: $\dfrac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)\cos\theta}$	1
$= \dfrac{\cos^2\theta + 1 + \sin^2\theta + 2\sin\theta}{(1+\sin\theta)\cos\theta}$	½
$= \dfrac{2\ (1+\sin\theta)}{(1+\sin\theta)\cos\theta}$	1
$= 2\sec\theta$ = RHS	½

31. A game of chance consists of spinning a wheel which comes to rest at one of the numbers from 1 to 10 (as shown in the given figure) with equal probabilities.



What is the probability that the wheel stops at
(i) a prime number greater than 2 ?
(ii) an odd number less than 9 ?
(iii) a multiple of 4 ?

Solution:(i) P (a prime number greater than 2) = $\frac{3}{10}$ 1

(ii) P (an odd number less than 9) = $\frac{4}{10}$ or $\frac{2}{5}$ 1

(iii) P (a multiple of 4) = $\frac{2}{10}$ or $\frac{1}{5}$ 1

SECTION D

*This section has **4** Long Answer (LA) type questions carrying **5** marks each.* 4×5=20

32. (a) The sum of areas of two squares is 2650 cm^2. If the sum of their perimeters is 280 cm, find the sides of the two given squares.

OR

(b) Express the equation $\frac{1}{x} - \frac{1}{x-2} = 3$, $(x \neq 0, 2)$ as a quadratic equation in standard form. Hence, find the roots of the quadratic equation so obtained.

Solution:

(a) Let the sides of squares be x and y

A.T.Q.

$x^2 + y^2 = 2650$ ---------(i) 1

$4x + 4y = 280 \Rightarrow x + y = 70$ ---(ii) 1

getting $2x^2 - 140x + 2250 = 0$ *or* $x^2 - 70x + 1125 = 0$ 1

$\Rightarrow (x-25)(x-45) = 0$

$\Rightarrow x = 25$ *and* $x = 45$ 1

$\therefore\ y = 45$ *and* $y = 25$ 1

sides of square are 25*cm and* 45*cm.*

OR

(b)

$$\frac{x-2-x}{x(x-2)} = 3$$ 1

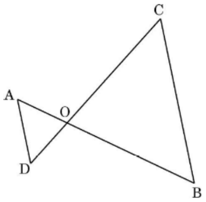
$\Rightarrow 3x^2 - 6x + 2 = 0$ 1

Discriminant = 36 – 24 = 12 1

Roots are $\frac{6+\sqrt{12}}{6}$ *and* $\frac{6-\sqrt{12}}{6}$ 1+1

or $1+\frac{\sqrt{3}}{3}$ *and* $1-\frac{\sqrt{3}}{3}$

33. State the SAS criteria of similarity of two triangles. In the given figure, it is given that OA . OC = OB . OD. Use the SAS criteria to prove that AD || CB.

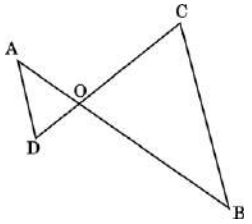


Solution:

Statement: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. 1

Given: OA·OC = OB·OD

To prove: AD││CB



Proof: In ΔOAD and ΔOBC,

OA·OC = OB·OD (given)

$\Rightarrow \frac{OA}{OB} = \frac{OD}{OC}$ 1

∠ AOD = ∠ BOC (vertically opposite angles) ½

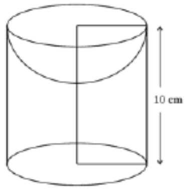
ΔOAD ~ ΔOBC (by SAS similarity criterion) 1

$\Rightarrow \angle A = \angle B$ 1

AD││CB (alternate interior angles are equal) ½

34. A wooden article was made by scooping out a hemisphere (of same diameter) from one end of a solid cylinder as shown in the given figure. If the height of the cylinder is 10 cm and the diameter of the cylinder is 14 cm, find the total surface area of the remaining wooden article.

(Use $\pi = \frac{22}{7}$)



Solution: Radius of hemisphere = radius of cylinder = 7 cm

Surface area of toy = $2\pi rh + \pi r^2 + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 10 + \frac{22}{7} \times 7 \times 7 + 2 \times \frac{22}{7} \times 7 \times 7$ 3

$= 440 + 154 + 308$ 1½

$= 902$ sq.cm. ½

35. (a) The lengths of 40 leaves of a plant are measured, correct to the nearest millimetre and data obtained is represented in the following table :

Length in (mm)	*Number of leaves*
100 – 120	8
120 – 140	9
140 – 160	12
160 – 180	5
180 – 200	6

Find the median length (in mm) of the leaves.

OR

(b) A class teacher has the following absentees record of 30 students of a class.

Number of days	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24
Number of Absent students	1	8	x	6	5	y

If the mean number of days a student was absent is 12, find the values of x and y.

Solution:(a)

C.I.	f	CF
100 – 120	8	8
120 – 140	9	17
140 – 160	12	29
160 – 180	5	34
180 – 200	6	40
	40	

Correct table: 2

median class: 140 – 160

$$\text{Median} = \ell + \frac{\frac{N}{2} - cf}{f} \times h$$

$= 140 + \frac{20 - 17}{12} \times 20$ 2

$= 145$ 1

∴ The median length of the leaves is 145 mm

OR

(b)

C.I.	f_i	x_i	$f_i x_i$
0 – 4	1	2	2
4 – 8	8	6	48
8 – 12	x	10	10x
12 – 16	6	14	84
16 – 20	5	18	90
20 – 24	y	22	22y
	20 + x + y		224 + 10x + 22y

Correct table: 2

$x + y + 20 = 30 \quad \Rightarrow \quad x + y = 10 \qquad – (i)$ 1

$12 = \frac{10x + 22y + 224}{30} \quad \Rightarrow \quad 5x + 11y = 68 \qquad – (ii)$ 1

Solving (i) and (ii) we get

$x = 7$ ½

$y = 3$ ½

SECTION E

*This section has **3** case study based questions carrying **4** marks each.* *3×4=12*

Case Study – 1

36. Kite festival is a popular festival in India which takes place during Makar Sankranti. The festival is celebrated by people flying kites from their rooftops. Reena and Ravi are also flying kites to enjoy the festival. **The height of Reena's kite is 60 m above the ground. The string attached** to the kite is temporarily tied to a point on the ground, and the inclination of the string with the ground is 30°. Ravi is flying a kite from a 10 m high building. His kite is also flying 60 m above the ground and **the length of the string used by Ravi is same as that of Reena's.** θ is the **angle of elevation of Ravi's kite from a point on the rooftop.**

Based on the above information, answer the following questions :

(i) Find the length of string used by Reena. *1*

(ii) Find the value of sin θ. *1*

(iii) (a) If θ changes to 60°, without changing the length of the string, what will be the height of Ravi's kite above the ground ? (Use $\sqrt{3} = 1{\cdot}7$) *2*

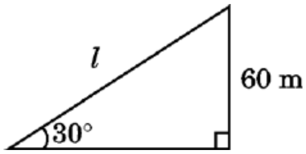
OR

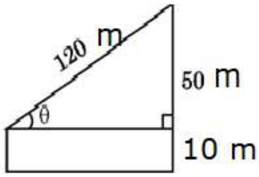
(b) What would have been the height of Ravi's kite above the ground, if the string had an inclination of 30° with the ground, assuming that the length of the string does not change ? *2*

Solution: (i)

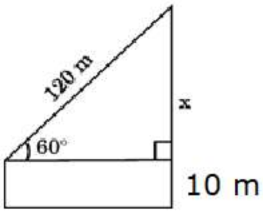
$$\frac{60}{l} = \sin 30°$$

$$l = 120 \text{ m}$$ 1





(ii) $\sin\theta = \frac{50}{120}$ or $\frac{5}{12}$ 1



(iii) (a) $\frac{x}{120} = \sin 60^0$

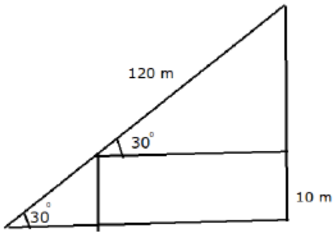
$\Rightarrow x = 60\sqrt{3}$ 1

Height of Ravi's kite = $60\sqrt{3}$ + 10 ½

= 102 + 10

= 112 m ½

OR



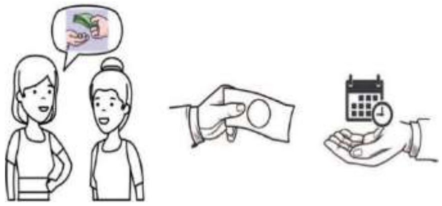
(iii) (b) $\frac{x}{120} = \sin 30^0$

x = 60 m 1

Height of Ravi's kite = 60 + 10 = 70 m 1

Case Study – 2

37. A woman borrowed ₹ 10,00,000 from her friend and promised to return the borrowed money in monthly instalments beginning from the next month. After one month, she returned ₹ 10,000, the next month she returned ₹ 15,000, the third month she returned ₹ 20,000 and so on, thereby increasing the monthly instalment uniformly.



Based on the above information, answer the following questions :

(i) Find the amount of instalment paid in the tenth month. *1*

(ii) In which instalment did she pay ₹ 40,000 ? *1*

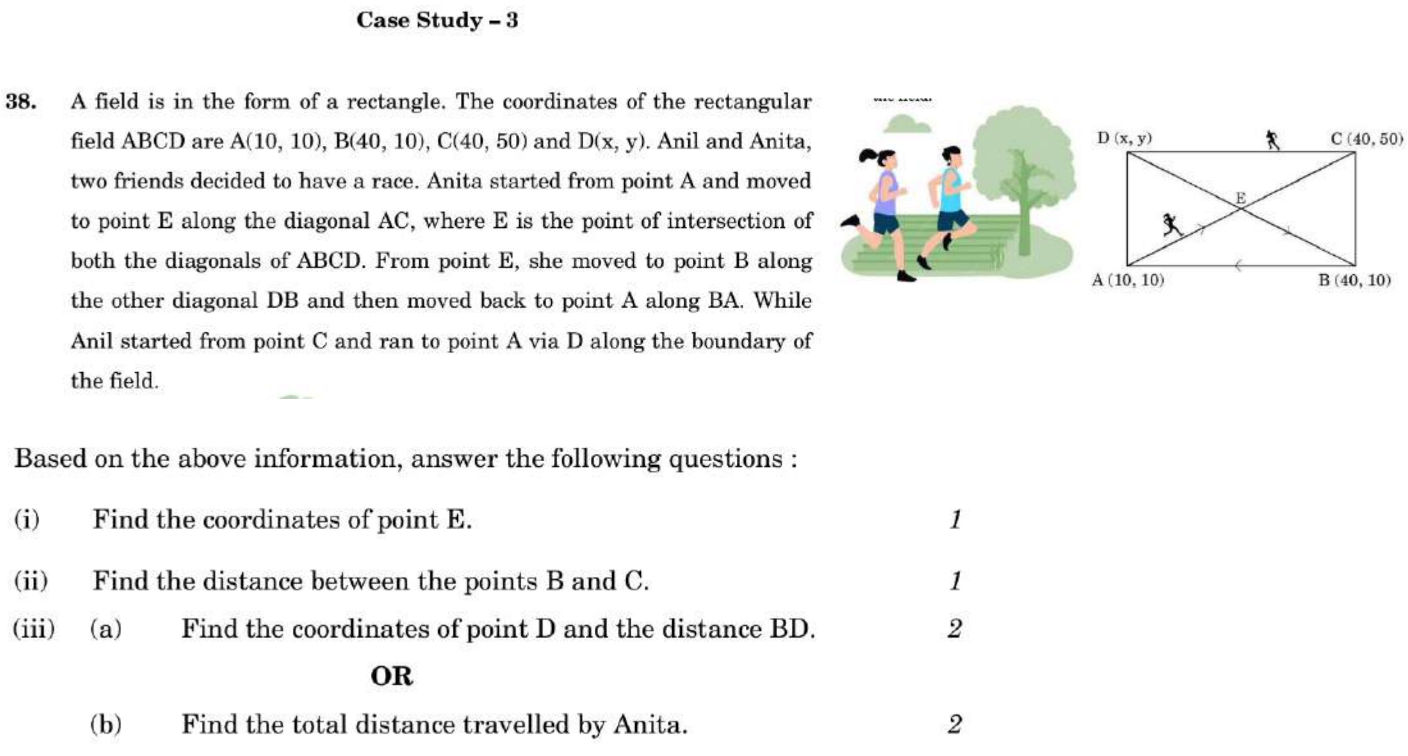
(iii) (a) If she returned ₹ 11,50,000 in all, how many instalments did she pay ? *2*

OR

(b) By which instalment has she returned a total amount of ₹ 3,25,000 ? *2*

Solution: (i) $a_{10} = 10000 + 5000 \times 9$	
$= 55000$	1
$\Rightarrow$ Amount of instalment paid in the tenth month =₹55,000	
(ii) $40000 = 10000 + (n-1)\,5000$	
$\Rightarrow n = 7$	1
$\Rightarrow$ In the 7th instalment she paid ₹40,000	
(iii) (a) $1150000 = \frac{n}{2}[20000 + (n-1)\,5000]$	1
$5n^2 + 15n - 2300 = 0$ or $n^2 + 3n - 460 = 0$	½
$(n-20)(n+23) = 0$	
$n = 20, n = -23$ (rejected)	½
$\Rightarrow$ She paid 20 instalments in order to return ₹11,50,000	
OR	
(iii) (b) $325000 = \frac{n}{2}[20000 + (n-1)\,5000]$	1
$5n^2 + 15n - 650 = 0$ or $n^2 + 3n - 130 = 0$	½
$(n-10)(n+13) = 0$	

n = 10 , n = – 13 (rejected) ⇒ She returned ₹3,25,000 by the 10th instalment.	½



Case Study – 3

38. A field is in the form of a rectangle. The coordinates of the rectangular field ABCD are A(10, 10), B(40, 10), C(40, 50) and D(x, y). Anil and Anita, two friends decided to have a race. Anita started from point A and moved to point E along the diagonal AC, where E is the point of intersection of both the diagonals of ABCD. From point E, she moved to point B along the other diagonal DB and then moved back to point A along BA. While Anil started from point C and ran to point A via D along the boundary of the field.

Based on the above information, answer the following questions :

(i) Find the coordinates of point E. *1*

(ii) Find the distance between the points B and C. *1*

(iii) (a) Find the coordinates of point D and the distance BD. *2*

OR

(b) Find the total distance travelled by Anita. *2*

Solution: (i) Coordinates of E are (25, 30)	1
(ii) Distance BC = $\sqrt{(40-40)^2+(50-10)^2}$ = 40	1
(iii) (a) Co-ordinates of D $\left(\frac{x+40}{2},\frac{y+10}{2}\right)=(25,30)$ By comparing we get, x = 10 and y = 50 Co-ordinates of D are (10, 50)	½+½
BD = $\sqrt{(30)^2+(-40)^2}$ = 50	1
OR	
(iii) (b) Distance travelled by Anita = AE + EB + BA = $\sqrt{(15)^2+(20)^2}+\sqrt{(-15)^2+(20)^2}+\sqrt{(30)^2+(0)^2}$	1½
= 25 + 25 + 30 = 80	½