SOLUTIONS

MATHEMATICS (BASIC)

SECTION A

*This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each. 20×1=20*

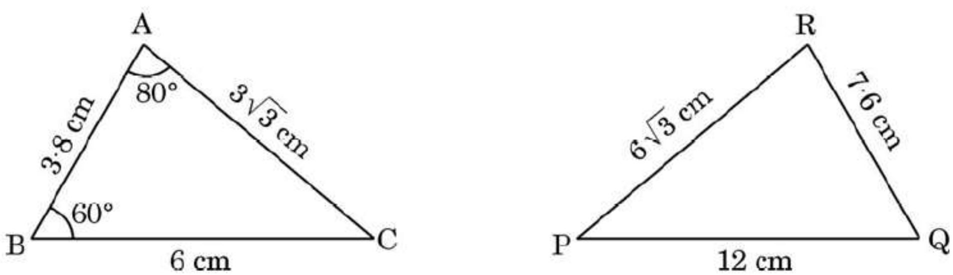
1. Which of the following is ***not*** the criterion for similarity of triangles ?

(A) AAA (B) SSS

(C) SAS (D) RHS

Answer : None of the given options is correct. Note – One mark to be given to all students who have attempted this question.	1

2. From the figures given below, which of the following is true about the measure of $\angle P$?



(A) $\angle P = 60°$

(B) $\angle P = 80°$

(C) $\angle P = 40°$

(D) The measure of $\angle P$ cannot be determined

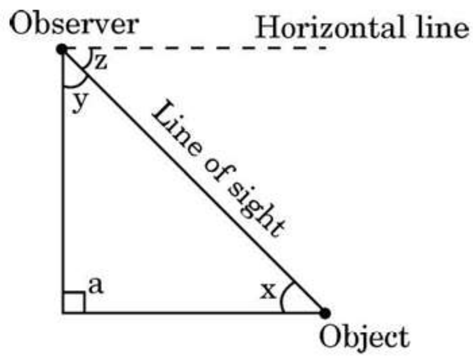
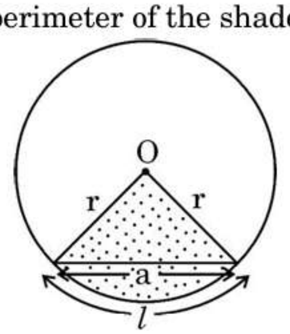
Answer : (C) $\angle P = 40°$	1

3. If the distance of a tangent to a circle from its centre is 4 cm, then the length of diameter of the circle is :

(A) 2 cm (B) 4 cm

(C) 8 cm (D) 16 cm

Answer : (C) 8 cm	1

4.	Which of the following statements is ***false*** ? (A) tan 45° = cot 45° (B) sin 90° = tan 45° (C) sin 30° = cos 30° (D) sin 45° = cos 45°	
Answer : (C) sin 30° = cos 30°		1
5.	The value of $\left(\cot^2 A - \frac{1}{\sin^2 A}\right)$ is : (A) more than 1 (B) 1 (C) 0 (D) – 1	
Answer : (D) – 1		1
6.	In the given figure, which of the following angles represents the angle of depression ?  (A) x (B) y (C) z (D) a	
Answer : (C) z		1
7.	The perimeter of the shaded region in the given figure is : 	

(A) l

(B) $l + a$

(C) $l + 2r$

(D) $l + 2r + a$

Answer : (C) $l + 2r$ — 1

8. The ratio of the area of a quadrant of a circle to the area of the same circle is :

(A) 1 : 2

(B) 2 : 1

(C) 1 : 4

(D) 4 : 1

Answer : (C) 1 : 4 — 1

9. For which of the following solids is the lateral/curved surface area and total surface area the same ?

(A) Cube

(B) Cuboid

(C) Hemisphere

(D) Sphere

Answer : (D) Sphere — 1

10. The class mark of the median class of the following data is :

Class Interval	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
Frequency	2	3	7	6	6	6

(A) 40

(B) 55

(C) 47·5

(D) 62·5

Answer : (D) 62·5 — 1

11. The following distribution shows the number of runs scored by some batsmen in test matches :

Runs Scored	3000 – 4000	4000 – 5000	5000 – 6000	6000 – 7000
Number of Batsmen	5	10	9	8

The lower limit of the modal class is :

(A) 3000

(B) 4000

(C) 5000

(D) 6000

Answer : (B) 4000 — 1

12. A bag contains 3 red, 4 white and 7 green balls. A ball is drawn at random. The probability that the ball drawn is ***not*** of red colour is :

(A) $\frac{1}{11}$ (B) $\frac{3}{14}$

(C) $\frac{11}{14}$ (D) $\frac{3}{11}$

Answer : (C) $\frac{11}{14}$ — 1

13. If the HCF of two positive integers a and b is 1, then their LCM is :

(A) a + b (B) a

(C) b (D) ab

Answer : (D) ab — 1

14. $\frac{\sqrt{3}-3}{\sqrt{3}}$ is :

(A) a rational number (B) an irrational number

(C) an integer (D) a natural number

Answer : (B) an irrational number — 1

15. The discriminant of the quadratic equation $-x^2 - 5x + 6 = 0$ is :

(A) 1 (B) – 1

(C) 49 (D) 7

Answer : (C) 49 — 1

16. The equation $x + \frac{1}{x} = 3$ $(x \neq 0)$ is expressed as a quadratic equation in the form of $ax^2 + bx + c = 0$. The value of $a - b + c$ is :

(A) 5 (B) 2

(C) 1 (D) – 1

Answer : (A) 5 — 1

17. The distance of a point P(3, – 7) from y-axis is :

(A) 3 (B) 7

(C) – 7 (D) $\sqrt{58}$

Answer : (A) 3	1

18. The mid-point of a line segment divides the line segment in the ratio :

(A) 1 : 2 (B) 2 : 1

(C) 1 : 1 (D) $\frac{1}{2}$: 2

Answer : (C) 1 : 1	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The value of p for which the system of equations $4x + py + 8 = 0$ and $2x + 2y + 2 = 0$ is consistent is 4.

Reason (R) : The system of equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ is consistent with infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Answer : (D) Assertion (A) is false, but Reason (R) is true.	1

20. *Assertion (A) :* For any two natural numbers a and b, the HCF of a and b is a factor of the LCM of a and b.

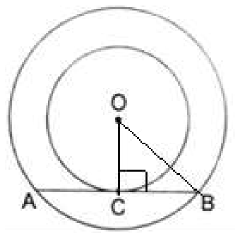
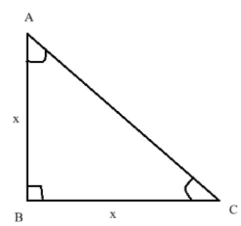
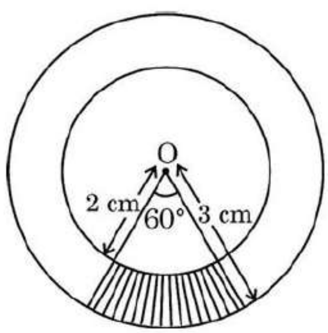
Reason (R) : HCF of any two natural numbers divides both the numbers.

Answer : (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	1

SECTION B

*This section has **5** Very Short Answer (VSA) type questions carrying **2** marks each.* *5×2=10*

21. Two concentric circles are of radii 6 cm and 10 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:  $BC^2 = OB^2 – OC^2$ $BC^2 = 10^2 – 6^2 = 64$ BC = 8 cm AB = 8 × 2 = 16 cm	Correct figure ½ 1 ½
22. (a) Find the values of A and B $(0 \leq A < 90°,\ 0 \leq B < 90°)$, if $\tan(A + B) = 1$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$. **OR** (b) Prove that $\tan 45° = 1$ geometrically.	
Solution: (a) A + B = 45° A – B = 30° Solving and getting A = 37·5° and B = 7·5° **OR** (b) Consider an isosceles right Δ ABC Using angle sum property ∠A = ∠ C = 45° Clearly, $\tan 45° = \frac{AB}{BC} = \frac{x}{x} = 1$  	½ ½ 1 ½ ½ 1
23. In the given figure, two concentric circles with centre O and radii 2 cm and 3 cm are shown. Find the perimeter of the shaded region.  	
Solution : Perimeter of the shaded region $= \frac{60}{360} \times 2\pi \times 3 + \frac{60}{360} \times 2\pi \times 2 + 2$ $= \frac{152}{21}$ cm or 7.24 cm	1½ ½

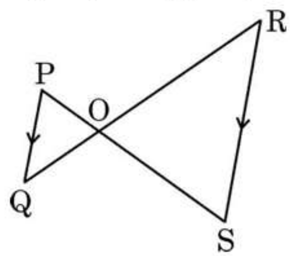
24. Solve for x and y :

$$0{\cdot}1x + 0{\cdot}3y = 1$$

$$0{\cdot}2x - 0{\cdot}1y = -\,0{\cdot}1$$

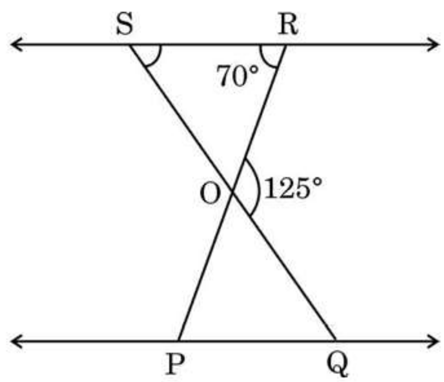
Solution : Solving the two equations to get x = 1 and y = 3	1+1

25. (a) In the given figure, if PQ ∥ RS, then prove that Δ POQ ~ Δ SOR.



OR

(b) In the given figure, Δ OSR ~ Δ OQP, $\angle$ ROQ = 125° and $\angle$ ORS = 70°. Find the measures of $\angle$ OSR and $\angle$ OQP.



Solution : (a) As PQ ∥ RS	
$\angle$P = $\angle$S } Alternate interior angles	1
$\angle$ Q = $\angle$ R	
∴ΔPOQ ~ ΔSOR (by AA similarity criterion)	1
OR	
(b) $\angle$ OSR = 125° – 70° = 55° [by exterior angle property]	1
As Δ OSR ~ Δ OQP	
$\angle$ OSR = $\angle$ OQP (Corresponding angles of similar triangles)	½
$\angle$ OQP = 55°	½

SECTION C

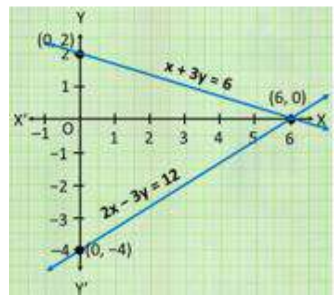
*This section has **6** Short Answer (SA) type questions carrying **3** marks each.* *6×3=18*

26. (a) Solve the following system of equations graphically :

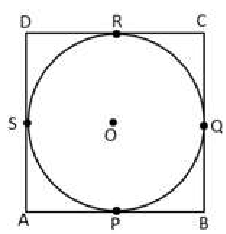
$$x + 3y = 6; \; 2x - 3y = 12$$

OR

(b) x and y are complementary angles such that x : y = 1 : 2. Express the given information as a system of linear equations in two variables and hence solve it.

Solution: (a) Correct graph of each equation	1+1
	
Solution is x = 6, y = 0 or (6, 0)	1
OR	
(b) $x + y = 90°$	1
$2x = y$	1
Solving to get $x = 30°$, $y = 60°$	½ + ½

27. Prove that a rectangle circumscribing a circle is a square.

Solution: 	Correct figure ½
As the length of tangents from an external point to a circle are equal Thus, AP = AS BP = BQ DR = DS CR = CQ	1
Adding the above equations, AB+CD=BC+AD As AB=CD & BC = AD (opp. sides of rectangle) $\Rightarrow$ AB = AD	1
$\therefore$ ABCD is a square	½

28. Prove the following trigonometric identity :

$$\sqrt{\frac{\text{cosec A} - 1}{\text{cosec A} + 1}} = \sec A - \tan A$$

Solution : LHS

$$= \sqrt{\frac{\text{cosec A} - 1}{\text{cosec A} + 1} \times \frac{\text{cosec A} - 1}{\text{cosec A} - 1}}$$ 1

$$= \sqrt{\frac{(\text{cosec A} - 1)^2}{\cot^2 A}} = \frac{\text{cosec A} - 1}{\cot A}$$ 1

$$= \frac{\text{cosec A}}{\cot A} - \frac{1}{\cot A} = \sec A - \tan A = \text{RHS}$$ 1

29. A lot consists of 200 pens of which 180 are good and the rest are defective. A customer will buy a pen if it is not defective. The shopkeeper draws a pen at random and gives it to the customer. What is the probability that the customer will not buy it ? Another lot of 100 pens containing 80 good pens is mixed with the previous lot of 200 pens. The shopkeeper now draws one pen at random from the entire lot and gives it to the customer. What is the probability that the customer will buy the pen ?

Solution: P (customer will not buy the pen) = $\frac{20}{200} = \frac{1}{10}$ 1

After mixing the two lots

Total pens = 200 + 100 = 300

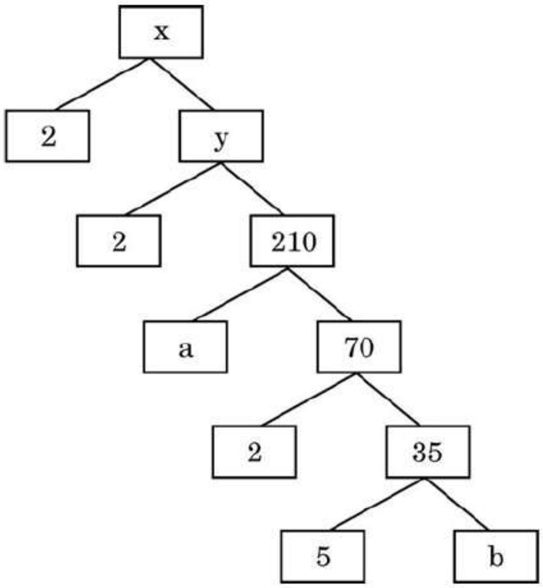
Number of good pens = 180 + 80 = 260 1

P (customer will buy the pen) = $\frac{260}{300}$ or $\frac{13}{15}$ 1

30. (a) Prove that $\sqrt{3}$ is an irrational number.

OR

(b) The factor tree of a number x is shown below :



Find the values of x, y, a and b. Hence, write the product of the prime factors of the number x so obtained.

Solution: (a) Let $\sqrt{3}$ be a rational number such that $\sqrt{3} = \frac{p}{q}$ (p and q are co-prime numbers, $q \neq 0$)	½
$\sqrt{3}q = p \Rightarrow 3q^2 = p^2$	
3 divides p^2 $\Rightarrow$ 3 divides p as well	1
Let, p = 3m (for some integer m)	
$3q^2 = 9m^2 \Rightarrow q^2 = 3m^2$	
3 divides q^2 $\Rightarrow$ 3 divides q as well	1
p and q have a common factor 3, which is a contradiction as p and q are co-prime.	
$\therefore$ our assumption is wrong	½
Hence, $\sqrt{3}$ is an irrational number	
OR	
(b) b = 7	½
a = 3	½
y = 420	½
x = 840	½
$x = 840 = 2^3 \times 3 \times 5 \times 7$	1

31. Find a quadratic polynomial, sum and product of whose zeroes are 5 and – 6, respectively. Also, find the zeroes of the polynomial so obtained.

Solution: Required polynomial is $x^2 - 5x - 6$	1
For zeroes; $x^2 - 5x - 6 = (x - 6)(x + 1)$	1
Zeroes are $x = 6, -1$	1

SECTION D

*This section has **4** Long Answer (LA) type questions carrying **5** marks each.* 4×5=20

32. **State "Basic Proportionality Theorem" and use it to prove the following :**

A line through the mid-point of one side of a triangle, parallel to another side, bisects the third side.

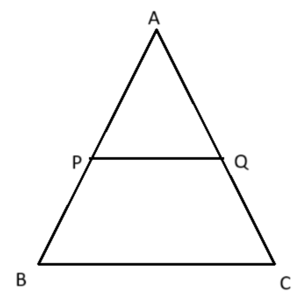
Solution : Statement : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. — 1

correct figure, given, to prove 1

Given : In Δ ABC, P is mid-point of AB and PQ ∥ BC

To prove : Q is the mid-point of AC

Proof : As PQ ∥ BC



$\frac{AP}{PB} = \frac{AQ}{QC}$ (by BPT) — 1

AP = PB (given) $\Rightarrow \frac{AP}{PB} = 1$ — 1

$\therefore \frac{AQ}{QC} = 1 \Rightarrow AQ = QC$ — 1

$\Rightarrow$ Q is the mid-point of AC

33. (a) **A toy is in the form of a cone surmounted on a hemisphere. The cone and hemisphere have the same radii. The height of the conical part of the toy is equal to the diameter of its base. If the radius of the conical part is 5 cm, find the volume of the toy.**

OR

(b) **A cubical block is surmounted by a hemisphere of radius 3·5 cm. What is the smallest possible length of the edge of the cube so that the hemisphere can totally lie on the cube ? Find the total surface area of the solid so formed.**

Solution: (a) Radius = r = 5 cm

Height of cone = h = 10 cm

Volume of toy = volume of hemisphere + volume of cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times 5 \times 5 \times 5 + \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 10$ — 2+2

$= \frac{5500}{21} + \frac{5500}{21}$

$= \frac{11000}{21}$ cu. cm or 523.81 cu. cm — 1

OR

	(b) Edge of cube = a = 3·5 × 2 = 7 cm Total surface area of solid $= 6\,a^2 + 2\pi r^2 - \pi r^2$ $= 6\,a^2 + \pi r^2$ $= 6 \times 7 \times 7 + \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5$ $= \frac{665}{2}$ sq. cm or 332·5 sq. cm	1 1½ + 1½ 1

34. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the monthly mean consumption from the data.

Monthly Consumption (in units)	*Number of Consumers*
50 – 100	4
100 – 150	5
150 – 200	13
200 – 250	20
250 – 300	14
300 – 350	8
350 – 400	4

34.

CI	x_i	f_i	u_i	$f_i u_i$
50 – 100	75	4	– 3	– 12
100 – 150	125	5	– 2	– 10
150 – 200	175	13	– 1	– 13
200 – 250	225 = a	20	0	0
250 – 300	275	14	1	14
300 – 350	325	8	2	16
350 – 400	375	4	3	12
Total		68		7

correct table 2

Mean $= 225 + \frac{7}{68} \times 50$ — 2

Mean = 230·15 — 1

Thus, the monthly mean consumption from the data is 230.15

35. (a) The difference of the squares of two positive numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers.

OR

(b) Find the value(s) of k for which the equation $2x^2 + kx + 3 = 0$ has real and equal roots. Hence, find the roots of the equations so obtained.

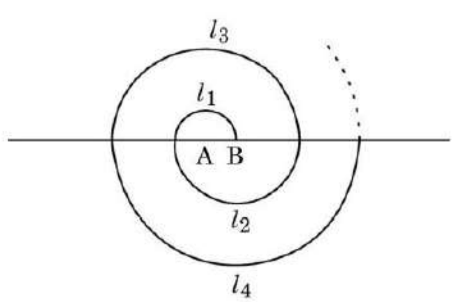
Solution: (a) Let the smaller number be y and greater number be x.	
A.T.Q.	
$x^2 - y^2 = 180$	1
$y^2 = 8x$	1
$\Rightarrow x^2 - 8x = 180$	
$x^2 - 8x - 180 = 0$	1
$(x - 18)(x + 10) = 0$	
$x = 18, x = -10$ (rejected)	1
∴ The numbers are 18 and 12	1
OR	
(b) For equal roots; $b^2 - 4ac = 0$	
$k^2 - 24 = 0$	1
$\Rightarrow k = \pm 2\sqrt{6}$	1
Equations are	
$2x^2 + 2\sqrt{6}x + 3 = 0$; $2x^2 - 2\sqrt{6}x + 3 = 0$	½ + ½
Roots are $x = -\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$; $x = \sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}$	1 + 1

SECTION E

*This section has **3** case study based questions carrying **4** marks each.* *3×4=12*

Case Study – 1

36. In a garden, saplings of rose flowers were planted at equal intervals to form a spiral pattern. The spiral is made up of successive semicircles, with centres alternatively at A and B, starting with centre at A, of radii 50 cm, 100 cm, 150 cm, ... as shown in the figure given below. Spiral 1 has 10 flowers, Spiral 2 has 20 flowers, Spiral 3 has 30 flowers and so on.



Based on the above information, answer the following questions :

(i) What is the radius of the 13^{th} spiral ? *1*

(ii) If the radius of the n^{th} spiral is 500 cm, find the value of n. *1*

(iii) (a) Find the total number of saplings till the 11^{th} spiral. *2*

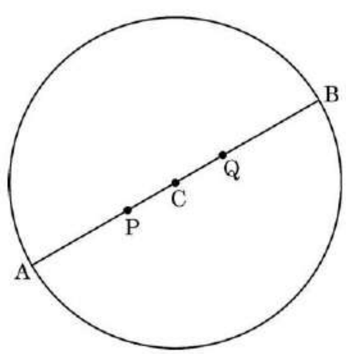
OR

(b) Till which spiral, will there be a total of 450 saplings ? *2*

Solution: (i) $a_{13} = 650$ cm	1
(ii) $a_n = 500$	
$50 + (n - 1)50 = 500$	
$n = 10$	1
(iii) (a) $a = 10, d = 10$	
$S_{11} = \frac{11}{2}[20 + 10 \times 10]$	1½
$= 660$	½
OR	
(b) $a = 10, d = 10$	
$450 = \frac{n}{2}[20 + (n - 1)10]$	1
$n^2 + n - 90 = 0$	½
$n = 9$	½

Case Study – 2

37. In a society, there is a circular park having two gates. The gates are placed at points A(10, 20) and B(50, 50), as shown in the figure below. Two fountains are installed at points P and Q on AB such that AP = PQ = QB.



Based on the above information, answer the following questions :

(i) Find the coordinates of the centre C. *1*

(ii) Find the radius of the circular park. *1*

(iii) (a) Find the coordinates of the point P. *2*

OR

(b) Find the distance of the fountain at Q from gate A. *2*

Solution:		Marks
(i)	Co-ordinates of C are $\left(\frac{10+50}{2}, \frac{20+50}{2}\right)$ = C(30, 35)	1
(ii)	Radius = $\sqrt{(30-10)^2 + (35-20)^2}$ = 25	1
(iii) (a)	P divides AB in the ratio 1 : 2,	½
	co-ordinates of P are $\left(\frac{1\times 50+2\times 10}{3}, \frac{1\times 50+2\times 20}{3}\right)$	½ + ½
	i.e. $\left(\frac{70}{3}, 30\right)$	½
	OR	
(b)	Distance AB = 2 × 25 = 50	½
	AQ = $\frac{2}{3}$AB = $\frac{2}{3}$× 50	1
	AQ = $\frac{100}{3}$	½

Case Study – 3

38. An injured bird was found on the roof of a building. The building is 15 m high. A fireman was called to rescue the bird. The fireman used an adjustable ladder to reach the roof. He placed the ladder in such a way that the ladder makes an angle of 60° with the ground in order to reach the roof.

Based on the above information, answer the following questions :

(i) Find the length of the ladder used by the fireman to reach the roof. *1*

(ii) Find the distance of the point on the ground at which the ladder was fixed from the bottom of the building. *1*

(iii) In order to avoid skidding, the fireman placed the ladder in such a way that the bottom of the ladder touches the base of the wall which is opposite to the building, making an angle of 30° with the ground.

(a) Draw a neat diagram to represent the above situation and hence find the width of the road between the building and the wall. *2*

OR

(b) Find the length of the ladder used by the fireman in this case. *2*

Solution: (i) Let the length of the ladder be 'a'

$\frac{15}{a} = \sin 60°$ ½

$a = \frac{30}{\sqrt{3}}$ or $10\sqrt{3}$ ½

Thus the length of the ladder is $\frac{30}{\sqrt{3}}$ m or $10\sqrt{3}$ m

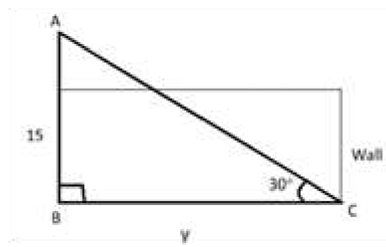
(ii) Let the distance of the point on the ground be 'x'

$\frac{15}{x} = \tan 60°$ ½

$x = \frac{15}{\sqrt{3}}$ or $5\sqrt{3}$ ½

Thus, the distance of the point on the ground is $\frac{15}{\sqrt{3}}$ m or $5\sqrt{3}$ m

(iii) (a) Let the width of the road be y.



Correct figure 1

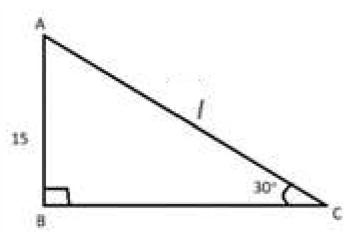
$\frac{15}{y} = \tan 30°$ ½

$y = 15\sqrt{3}$ ½

Thus, the width of the road is $15\sqrt{3}$ m.

OR

(b) Let the length of the ladder be *l*.



$\frac{15}{l} = \sin 30°$ 1

$l = 30$ 1

Thus, the length of the ladder is 30 m.