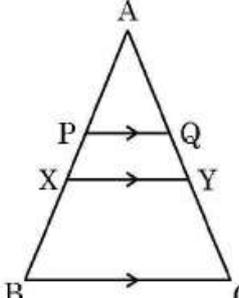


**SOLUTIONS**  
**MATHEMATICS (Subject Code–**  
**041) (PAPER CODE: 30/6/2)**

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
<b>SECTION A</b>		
<b>This section consists of 20 multiple choice questions of 1 mark each.</b>		
<b>1.</b>	The system of equations $x + 5 = 0$ and $2x - 1 = 0$ , has (A) No solution (B) Unique solution (C) Two solutions (D) Infinite solutions	
<b>Sol.</b>	(A) No solution	<b>1</b>
<b>2.</b>	In a right triangle ABC, right-angled at A, if $\sin B = \frac{1}{4}$ , then the value of $\sec B$ is (A) 4 (B) $\frac{\sqrt{15}}{4}$ (C) $\sqrt{15}$ (D) $\frac{4}{\sqrt{15}}$	
<b>Sol.</b>	(D) $\frac{4}{\sqrt{15}}$	<b>1</b>
<b>3.</b>	$\sqrt{0.4}$ is a/an (A) natural number (B) integer (C) rational number (D) irrational number	
<b>Sol.</b>	(D) irrational number	<b>1</b>
<b>4.</b>	Which of the following cannot be the unit digit of $8^n$ , where n is a natural number ? (A) 4 (B) 2 (C) 0 (D) 6	
<b>Sol.</b>	(C) 0	<b>1</b>
<b>5.</b>	Which of the following quadratic equations has real and distinct roots ? (A) $x^2 + 2x = 0$ (B) $x^2 + x + 1 = 0$ (C) $(x - 1)^2 = 1 - 2x$ (D) $2x^2 + x + 1 = 0$	
<b>Sol.</b>	(A) $x^2 + 2x = 0$	<b>1</b>

6.	<p>If the zeroes of the polynomial <math>ax^2 + bx + \frac{2a}{b}</math> are reciprocal of each other, then the value of b is</p> <p>(A) 2 (B) <math>\frac{1}{2}</math> (C) -2 (D) <math>-\frac{1}{2}</math></p>	
Sol.	(A) 2	1
7.	<p>The distance of point (a, -b) from x-axis is</p> <p>(A) a (B) -a (C) b (D) -b</p>	
Sol.	(C) b	1
8.	<p>In the adjoining figure, <math>PQ \parallel XY \parallel BC</math>, <math>AP = 2</math> cm, <math>PX = 1.5</math> cm and <math>BX = 4</math> cm. If <math>QY = 0.75</math> cm, then <math>AQ + CY =</math></p> <div style="text-align: center;">  </div> <p>(A) 6 cm (B) 4.5 cm (C) 3 cm (D) 5.25 cm</p>	
Sol.	(C) 3 cm	1
9.	<p>Given <math>\triangle ABC \sim \triangle PQR</math>, <math>\angle A = 30^\circ</math> and <math>\angle Q = 90^\circ</math>. The value of <math>(\angle R + \angle B)</math> is</p> <p>(A) <math>90^\circ</math> (B) <math>120^\circ</math> (C) <math>150^\circ</math> (D) <math>180^\circ</math></p>	
Sol.	(C) $150^\circ$	1
10.	<p>Two coins are tossed simultaneously. The probability of getting atleast one head is</p> <p>(A) <math>\frac{1}{4}</math> (B) <math>\frac{1}{2}</math> (C) <math>\frac{3}{4}</math> (D) 1</p>	
Sol.	(C) $\frac{3}{4}$	1



15.	A peacock sitting on the top of a tree of height 10 m observes a snake moving on the ground. If the snake is $10\sqrt{3}$ m away from the base of the tree, then angle of depression of the snake from the eye of the peacock is (A) $30^\circ$ (B) $45^\circ$ (C) $60^\circ$ (D) $90^\circ$	
Sol.	(A) $30^\circ$	1
16.	If a cone of greatest possible volume is hollowed out from a solid wooden cylinder, then the ratio of the volume of remaining wood to the volume of cone hollowed out is (A) 1 : 1 (B) 1 : 3 (C) 2 : 1 (D) 3 : 1	
Sol.	(C) 2:1	1
17.	If the mode of some observations is 10 and sum of mean and median is 25, then the mean and median respectively are (A) 12 and 13 (B) 13 and 12 (C) 10 and 15 (D) 15 and 10	
Sol.	(B) 13 and 12	1
18.	If the maximum number of students has obtained 52 marks out of 80, then (A) 52 is the mean of the data. (B) 52 is the median of the data. (C) 52 is the mode of the data. (D) 52 is the range of the data.	
Sol.	(C) 52 is the mode of the data.	1
	<b>Directions :</b> In Question Numbers 19 and 20, a statement of <b>Assertion (A)</b> is followed by a statement of <b>Reason (R)</b> . Choose the correct option from the following : (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	<b>Assertion (A) :</b> For two prime numbers $x$ and $y$ ( $x < y$ ), $HCF(x, y) = x$ and $LCM(x, y) = y$ . <b>Reason (R) :</b> $HCF(x, y) \leq LCM(x, y)$ , where $x, y$ are any two natural numbers.	
Sol.	(D) Assertion (A) is false, but Reason (R) is true.	1



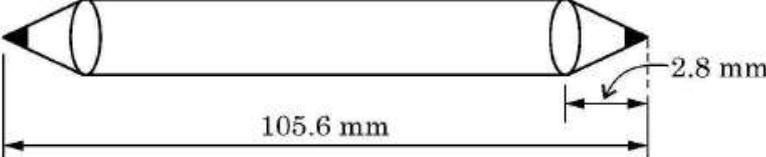
<b>Sol.</b>	Adding equations we get $x + y = 3$ Subtracting equations we get $-x + y = -1$ Solving to get $x = 2$ and $y = 1$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$
<b>OR</b>		
<b>23.</b> <b>(b)</b>	In a pair of supplementary angles, the greater angle exceeds the smaller by $50^\circ$ . Express the given situation as a system of linear equations in two variables and hence obtain the measure of each angle.	
<b>Sol.</b>	Let smaller angle be $x$ and greater angle be $y$ ATQ, $x + y = 180$ Also $y = x + 50$ Solving we get $x = 65^\circ$ and $y = 115^\circ$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$
<b>24.</b> <b>(a)</b>	If $a \sec \theta + b \tan \theta = m$ and $b \sec \theta + a \tan \theta = n$ , prove that $a^2 + n^2 = b^2 + m^2$	
<b>Sol.</b>	$m^2 = a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta$ $n^2 = b^2 \sec^2 \theta + a^2 \tan^2 \theta + 2ab \sec \theta \tan \theta$ $m^2 - n^2 = a^2(\sec^2 \theta - \tan^2 \theta) + b^2(\tan^2 \theta - \sec^2 \theta)$ $\Rightarrow m^2 - n^2 = a^2 - b^2$ or $a^2 + n^2 = m^2 + b^2$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
<b>OR</b>		
<b>24.</b> <b>(b)</b>	Use the identity : $\sin^2 A + \cos^2 A = 1$ to prove that $\tan^2 A + 1 = \sec^2 A$ . Hence, find the value of $\tan A$ , when $\sec A = \frac{5}{3}$ , where $A$ is an acute angle.	
<b>Sol.</b>	$\sin^2 A + \cos^2 A = 1$ Dividing both sides by $\cos^2 A$ , we get $\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$ $\tan^2 A + 1 = \sec^2 A$ $\tan^2 A + 1 = \left(\frac{5}{3}\right)^2$ $\tan A = \frac{4}{3}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

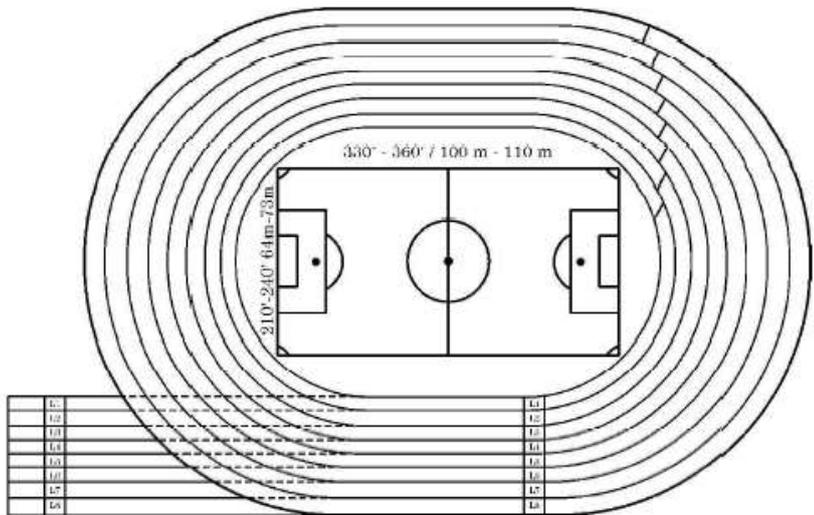
25.	Prove that abscissa of a point P which is equidistant from points with coordinates A(7, 1) and B(3, 5) is 2 more than its ordinate.	
Sol.	<p>Let P (x, y) be equidistant from A(7, 1) and B(3, 5)</p> $PA = PB \Rightarrow PA^2 = PB^2$ $(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$ $x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$ $x = 2 + y$ <p>Thus, abscissa of the point P is 2 more than its ordinate.</p>	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
<b>SECTION C</b> <b>This section has 6 short answer type questions of 3 marks each.</b>		
26. (a)	Prove that : $\frac{\cos \theta - 2 \cos^3 \theta}{\sin \theta - 2 \sin^3 \theta} + \cot \theta = 0.$	
Sol.	$\text{LHS} = \frac{\cos \theta - 2 \cos^3 \theta}{\sin \theta - 2 \sin^3 \theta} + \cot \theta$ $= \frac{\cos \theta (1 - 2 \cos^2 \theta)}{\sin \theta (1 - 2 \sin^2 \theta)} + \cot \theta$ $= \frac{\cos \theta}{\sin \theta} \left[ \frac{\sin^2 \theta + \cos^2 \theta - 2 \cos^2 \theta}{\sin^2 \theta + \cos^2 \theta - 2 \sin^2 \theta} \right] + \cot \theta$ $= \frac{\cot \theta (\sin^2 \theta - \cos^2 \theta)}{(\cos^2 \theta - \sin^2 \theta)} + \cot \theta$ $= -\cot \theta + \cot \theta$ $= 0 = \text{RHS}$	<p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p>
<b>OR</b>		
26. (b)	Given that $\sin \theta + \cos \theta = x$ , prove that $\sin^4 \theta + \cos^4 \theta = \frac{2 - (x^2 - 1)^2}{2}.$	
Sol.	<p>Given: <math>\sin \theta + \cos \theta = x</math></p> <p>Squaring both sides</p> $\sin^2 \theta + \cos^2 \theta + 2 \cos \theta \sin \theta = x^2$ $2 \sin \theta \cos \theta = x^2 - 1$ $\text{RHS} = \frac{2 - (2 \sin \theta \cos \theta)^2}{2}$ $= \frac{2 - 4 \sin^2 \theta \cos^2 \theta}{2}$ $= 1 - 2 \sin^2 \theta \cos^2 \theta$ $= (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$ $= (\sin^4 \theta + \cos^4 \theta) = \text{LHS}$	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>



	$pqr + 1 = 2.3.5 + 1 = 31$ is a prime number or any other correct example.	1
29.	Find the zeroes of the polynomial $q(x) = 8x^2 - 2x - 3$ . Hence, find a polynomial whose zeroes are 2 less than the zeroes of $q(x)$ .	
Sol.	$p(x) = 8x^2 - 2x - 3$ Zeroes are $-\frac{1}{2}$ and $\frac{3}{4}$ New zeroes are $-\frac{5}{2}$ and $-\frac{5}{4}$ Sum of new zeroes = $\frac{-5}{2} + \frac{-5}{4} = \frac{-15}{4}$ Product of new zeroes = $\left(-\frac{5}{2}\right) \times \left(-\frac{5}{4}\right) = \frac{25}{8}$ Required polynomial is $x^2 + \frac{15}{4}x + \frac{25}{8}$ or $8x^2 + 30x + 25$	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
30.	Check whether the following system of equations is consistent or not. If consistent, solve graphically $x - 2y + 4 = 0$ , $2x - y - 4 = 0$	
Sol.	$\frac{a_1}{a_2} = \frac{1}{2}$ ; $\frac{b_1}{b_2} = \frac{-2}{-1} = 2$ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ $\therefore$ System of equations is consistent.	$\frac{1}{2}$ $\frac{1}{2}$
		Correct graph $1\frac{1}{2}$ $\frac{1}{2}$
31.	If the points A(6, 1), B(p, 2), C(9, 4) and D(7, q) are the vertices of a parallelogram ABCD, then find the values of p and q. Hence, check whether ABCD is a rectangle or not.	
Sol.	Diagonals of a parallelogram bisect each other $\therefore$ Co-ordinates of mid point of diagonal AC = Co-ordinates of mid-point of diagonal BD. $\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{p+7}{2}, \frac{2+q}{2}\right)$	1



<b>Sol.</b>	<p>Let distance of gate at P from point B is <math>x</math> m  Then distance of gate at P from point A is <math>(35 + x)</math> m  In right <math>\Delta</math> APB  <math>(x + 35)^2 + x^2 = (65)^2</math>  <math>x^2 + 35x - 1500 = 0</math>  <math>(x + 60)(x - 25) = 0</math>  <math>x = 25</math>  Hence, <math>x + 35 = 60</math>  Distance of P from A = 60 m  Distance of P from B = 25 m</p>	<p><math>\frac{1}{2}</math>  <b>1</b> <b>2</b> <math>\frac{1}{2}</math> <math>\frac{1}{2}</math> <math>\frac{1}{2}</math></p>
<b>OR</b>		
<b>33.</b> (b)	<p>Find the smallest value of <math>p</math> for which the quadratic equation <math>x^2 - 2(p + 1)x + p^2 = 0</math> has real roots. Hence, find the roots of the equation so obtained.</p>	
<b>Sol.</b>	<p>For real roots, <math>D \geq 0</math>  <math>[-2(p + 1)]^2 - 4p^2 \geq 0</math>  <math>\Rightarrow p \geq -\frac{1}{2}</math>  <math>\therefore</math> smallest value of <math>p = -\frac{1}{2}</math>  At <math>p = -\frac{1}{2}</math> given equation becomes  <math>x^2 - 2\left(\frac{-1}{2} + 1\right)x + \left(\frac{-1}{2}\right)^2 = 0</math>  <math>x^2 - x + \frac{1}{4} = 0</math> or <math>4x^2 - 4x + 1 = 0</math>  <math>(2x - 1)(2x - 1) = 0</math>  <math>\therefore</math> roots are <math>\frac{1}{2}, \frac{1}{2}</math></p>	<p><math>\frac{1}{2}</math> <math>\frac{1}{2}</math> <b>1</b> <math>\frac{1}{2}</math> <math>\frac{1}{2}</math> <b>1</b> <math>\frac{1}{2}</math> <math>\frac{1}{2}</math></p>
<b>34.</b>	<p>On the day of her examination, Riya sharpened her pencil from both ends as shown below :</p>  <p>The diameter of the cylindrical and conical part of the pencil is 4.2 mm. If the height of each conical part is 2.8 mm and length of entire pencil is 105.6 mm, find the total surface area of the pencil.</p>	
<b>Sol.</b>	<p><math>d = 4.2</math> mm  <math>r = 2.1</math> mm  Height of cylindrical part (<math>h</math>) = 100 mm  Height of conical part = 2.8mm  Slant height (<math>l</math>) = <math>\sqrt{(2.8)^2 + (2.1)^2} = 3.5</math> mm  Curved Surface Area of Cylinder = <math>2 \times \frac{22}{7} \times 2.1 \times 100 = 1320</math> mm<sup>2</sup></p>	<p><math>\frac{1}{2}</math> <math>\frac{1}{2}</math> <b>1</b> <b>1</b></p>

	<p>Curved Surface Area of Cone = <math>\frac{22}{7} \times 2.1 \times 3.5 = 23.1 \text{ mm}^2</math></p> <p>Total Surface Area of pencil = Curved Surface Area of cylinder + Curved Surface Area of two cones</p> <p style="text-align: center;">= <math>1366.2 \text{ mm}^2</math></p>	<p><b>1</b></p> <p><b>1</b></p>
35.	<p>From one face of a solid cube of side 14 cm, the largest possible cone is carved out. Find the volume and surface area of the remaining solid.</p> <p>(Use <math>\pi = \frac{22}{7}</math>, <math>\sqrt{5} = 2.2</math>)</p>	
Sol.	<p>Diameter of cone = 14 cm</p> <p>Radius = 7 cm</p> <p>Height of cone = 14 cm</p> <p>Slant height <math>l = \sqrt{14^2 + 7^2} = 7\sqrt{5} = 15.4 \text{ cm}</math></p> <p>Volume of remaining solid = Volume of cube – Volume of cone</p> $= (14)^3 - \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 14$ $= \frac{6076}{3} \text{ cm}^3$ <p>Surface area of remaining solid = Surface area of cube – Area of circle + Curved surface area of cone</p> $= 6 \times 14 \times 14 - \frac{22}{7} \times 7 \times 7 + \frac{22}{7} \times 7 \times 15.4$ $= 1360.8 \text{ cm}^2$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><b>1</b></p> <p><b>1</b></p> <p><math>\frac{1}{2}</math></p> <p><b>1</b></p> <p><math>\frac{1}{2}</math></p>
<p><b>SECTION E</b></p> <p><b>This section has 3 case study based questions of 4 marks each.</b></p>		
36.	<p>In order to organise, Annual Sports Day, a school prepared an eight lane running track with an integrated football field inside the track area as shown below :</p>  <p>The length of innermost lane of the track is 400 m and each subsequent lane is 7.6 m longer than the preceding lane.</p>	

	<p>Based on given information, answer the following questions, using concept of Arithmetic Progression.</p> <p>(i) What is the length of the 6<sup>th</sup> lane ?</p> <p>(ii) How long is the 8<sup>th</sup> lane than that of 4<sup>th</sup> lane ?</p> <p>(iii) (a) While practicing for a race, a student took one round each in first six lanes. Find the total distance covered by the student.</p> <p style="text-align: center;"><b>OR</b></p> <p>(iii) (b) A student took one round each in lane 4 to lane 8. Find the total distance covered by the student.</p>	
<p><b>Sol.</b></p>	<p>Here AP is 400, 407.6, 415.2, ...</p> <p>(i) <math>a_6 = 400 + 5(7.6) = 438</math> m</p> <p>(ii) <math>a_8 - a_4 = 30.4</math> m</p> <p>(iii) <math>S_6 = \frac{6}{2}(2 \times 400 + 5 \times 7.6)</math>  <math>= 2514</math> m</p> <p style="text-align: center;"><b>OR</b></p> <p>(iii) Total distance covered = <math>S_8 - S_3</math>  <math>= \frac{8}{2}(2 \times 400 + 7 \times 7.6) - \frac{3}{2}(2 \times 400 + 2 \times 7.6)</math>  <math>= 2190</math> m</p>	<p style="text-align: center;"><b>1</b></p>
<p><b>37.</b></p>	<p>The Statue of Unity situated in Gujarat is the world's largest Statue which stands over a 58 m high base. As part of the project, a student constructed an inclinometer and wishes to find the height of Statue of Unity using it.</p> <p>He noted following observations from two places :</p> <p><b>Situation – I :</b></p> <p>The angle of elevation of the top of Statue from Place A which is <math>80\sqrt{3}</math> m away from the base of the Statue is found to be <math>60^\circ</math>.</p> <p><b>Situation – II :</b></p> <p>The angle of elevation of the top of Statue from a Place B which is 40 m above the ground is found to be <math>30^\circ</math> and entire height of the Statue including the base is found to be 240 m.</p>	



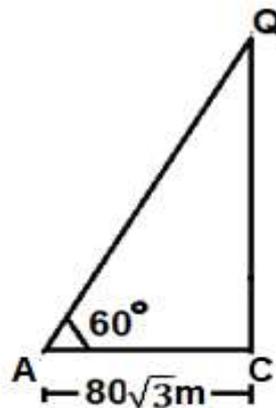
Based on given information, answer the following questions :

- (i) Represent the Situation – I with the help of a diagram.
- (ii) Represent the Situation – II with the help of a diagram.
- (iii) (a) Calculate the height of Statue excluding the base and also find the height including the base with the help of Situation – I.

OR

- (iii) (b) Find the horizontal distance of point B (Situation – II) from the Statue and the value of  $\tan \alpha$ , where  $\alpha$  is the angle of elevation of top of base of the Statue from point B.

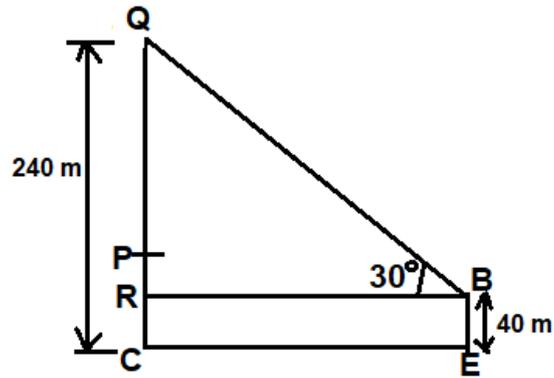
Sol. (i)



Correct figure

1

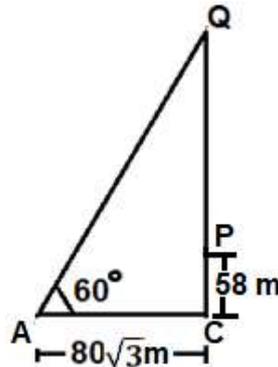
(ii)



Correct figure

1

(iii) (a)



In  $\Delta ACQ$

$$\frac{QC}{AC} = \tan 60^\circ = \sqrt{3}$$

$$QC = 240 \text{ m}$$

Height of statue including base = 240 m

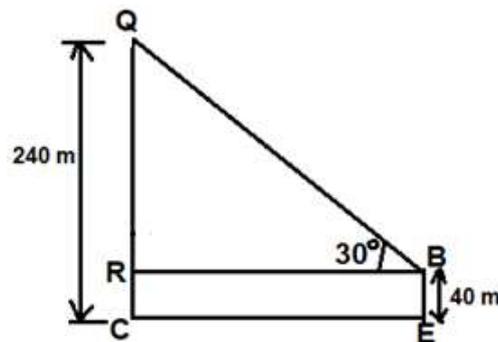
Height of statue excluding base =  $240 - 58 = 182 \text{ m}$

1

1

OR

(iii) (b)



$$QR = 240 - 40 = 200 \text{ m}$$

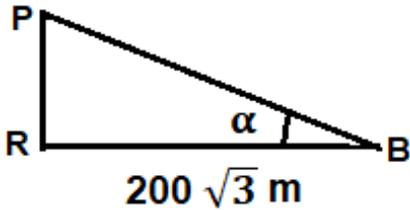
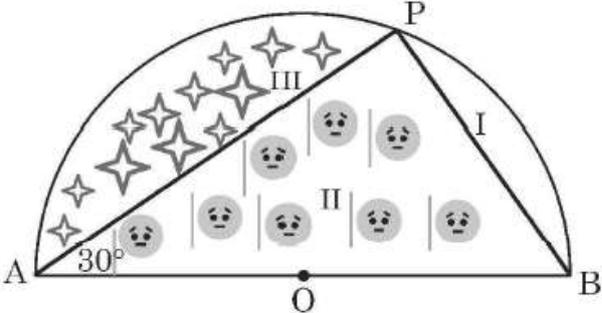
In  $\Delta QRB$

$$\frac{QR}{RB} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{Horizontal distance } RB = 200 \sqrt{3} \text{ m}$$

$\frac{1}{2}$

$\frac{1}{2}$

	<div style="text-align: center;">  <p style="text-align: right;">Correct figure</p> </div> <p>In <math>\Delta PRB</math>  <math>\tan \alpha = \frac{PR}{BR}</math>  <math>= \frac{18}{200\sqrt{3}}</math> or <math>\frac{3\sqrt{3}}{100}</math></p>	$\frac{1}{2}$  $\frac{1}{2}$
<p><b>38.</b></p>	<p>Anurag purchased a farmhouse which is in the form of a semicircle of diameter 70 m. He divides it into three parts by taking a point P on the semicircle in such a way that <math>\angle PAB = 30^\circ</math> as shown in the following figure, where O is the centre of semicircle.</p> <div style="text-align: center;">  </div> <p>In part I, he planted saplings of Mango tree, in part II, he grew tomatoes and in part III, he grew oranges. Based on given information, answer the following questions.</p> <p>(i) What is the measure of <math>\angle POA</math> ?</p> <p>(ii) Find the length of wire needed to fence entire piece of land.</p> <p>(iii) (a) Find the area of region in which saplings of Mango tree are planted.</p> <p style="text-align: center;"><b>OR</b></p> <p>(iii) (b) Find the length of wire needed to fence the region III.</p>	
<p><b>Sol.</b></p>	<p>(i) <math>\angle POA = 120^\circ</math></p> <p>(ii) Length of wire needed to fence entire piece of land <math>= \frac{22}{7} \times 35 + 70 = 180</math> m</p> <p>(iii) Required area <math>= \frac{60}{360} \times \frac{22}{7} \times (35)^2 - \frac{\sqrt{3}}{4} \times (35)^2</math>  <math>= \left( \frac{1925}{3} - \frac{1225\sqrt{3}}{4} \right) \text{ m}^2</math> or 111.89 m<sup>2</sup> (approx.)</p> <p style="text-align: center;"><b>OR</b></p>	<p><b>1</b></p> <p><b>1</b></p> <p><b>1</b></p> <p><b>1</b></p>

	<p>(iii) In <math>\Delta APB</math>, <math>\frac{AP}{AB} = \cos 30^\circ</math>  <math>AP = 35\sqrt{3}</math> m</p> <p>Required length of wire = <math>\frac{120}{360} \times 2 \times \frac{22}{7} \times 35 + 35\sqrt{3}</math>  <math>= \left(\frac{220}{3} + 35\sqrt{3}\right)</math> m or 133.8 m (approx.)</p>	<p><b>1</b></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
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