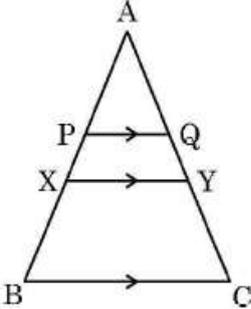
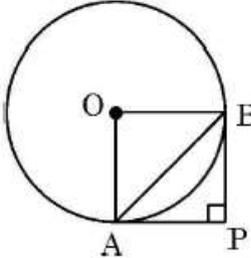
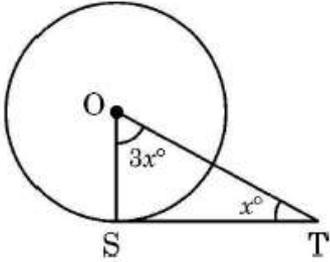


SOLUTIONS
MATHEMATICS (Subject Code–
041) (PAPER CODE: 30/6/1)

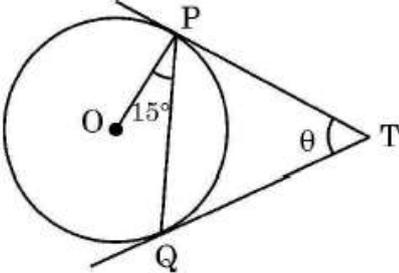
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
SECTION A		
This section consists of 20 multiple choice questions of 1 mark each.		
1.	$\sqrt{0.4}$ is a/an (A) natural number (B) integer (C) rational number (D) irrational number	
Sol.	(D) irrational number	1
2.	Which of the following cannot be the unit digit of 8^n , where n is a natural number ? (A) 4 (B) 2 (C) 0 (D) 6	
Sol.	(C) 0	1
3.	Which of the following quadratic equations has real and equal roots ? (A) $(x + 1)^2 = 2x + 1$ (B) $x^2 + x = 0$ (C) $x^2 - 4 = 0$ (D) $x^2 + x + 1 = 0$	
Sol.	(A) $(x + 1)^2 = 2x + 1$	1
4.	If the zeroes of the polynomial $ax^2 + bx + \frac{2a}{b}$ are reciprocal of each other, then the value of b is (A) 2 (B) $\frac{1}{2}$ (C) -2 (D) $-\frac{1}{2}$	
Sol.	(A) 2	1
5.	The distance of the point A(-3, -4) from x-axis is (A) 3 (B) 4 (C) 5 (D) 7	
Sol.	(B) 4	1

6.	<p>In the adjoining figure, $PQ \parallel XY \parallel BC$, $AP = 2$ cm, $PX = 1.5$ cm and $BX = 4$ cm. If $QY = 0.75$ cm, then $AQ + CY =$</p>  <p>(A) 6 cm (B) 4.5 cm (C) 3 cm (D) 5.25 cm</p>	
Sol.	(C) 3 cm	1
7.	<p>Given $\triangle ABC \sim \triangle PQR$, $\angle A = 30^\circ$ and $\angle Q = 90^\circ$. The value of $(\angle R + \angle B)$ is</p> <p>(A) 90° (B) 120° (C) 150° (D) 180°</p>	
Sol.	(C) 150°	1
8.	<p>Two coins are tossed simultaneously. The probability of getting atleast one head is</p> <p>(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1</p>	
Sol.	(C) $\frac{3}{4}$	1
9.	<p>In the adjoining figure, PA and PB are tangents to a circle with centre O such that $\angle P = 90^\circ$. If $AB = 3\sqrt{2}$ cm, then the diameter of the circle is</p>  <p>(A) $3\sqrt{2}$ cm (B) $6\sqrt{2}$ cm (C) 3 cm (D) 6 cm</p>	
Sol.	(D) 6 cm	1

10.	<p>For a circle with centre O and radius 5 cm, which of the following statements is true ?</p> <p>P : Distance between every pair of parallel tangents is 5 cm.</p> <p>Q : Distance between every pair of parallel tangents is 10 cm.</p> <p>R : Distance between every pair of parallel tangents must be between 5 cm and 10 cm.</p> <p>S : There does not exist a point outside the circle from where length of tangent is 5 cm.</p> <p>(A) P (B) Q (C) R (D) S</p>	
Sol.	(B) Q	1
11.	<p>In the adjoining figure, TS is a tangent to a circle with centre O. The value of $2x^\circ$ is</p>  <p>(A) 22.5 (B) 45 (C) 67.5 (D) 90</p>	
Sol.	(B) 45	1
12.	<p>If $x\left(\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}\right) = y\left(\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}\right)$, then $x : y =$</p> <p>(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1</p>	
Sol.	(C) 2:1	1
13.	<p>A peacock sitting on the top of a tree of height 10 m observes a snake moving on the ground. If the snake is $10\sqrt{3}$ m away from the base of the tree, then angle of depression of the snake from the eye of the peacock is</p> <p>(A) 30° (B) 45° (C) 60° (D) 90°</p>	
Sol.	(A) 30°	1

	<p>Directions : In Question Numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).</p> <p>Choose the correct option from the following :</p> <p>(A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).</p> <p>(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).</p> <p>(C) Assertion (A) is true, but Reason (R) is false.</p> <p>(D) Assertion (A) is false, but Reason (R) is true.</p>	
19.	<p>Assertion (A) : For any two prime numbers p and q, their HCF is 1 and LCM is $p + q$.</p> <p>Reason (R) : For any two natural numbers, $\text{HCF} \times \text{LCM} = \text{product of numbers}$.</p>	
Sol.	(D) Assertion (A) is false, but Reason (R) is true.	1
20.	<p>In an experiment of throwing a die,</p> <p>Assertion (A) : Event E_1 : getting a number less than 3 and Event E_2 : getting a number greater than 3 are complementary events.</p> <p>Reason (R) : If two events E and F are complementary events, then $P(E) + P(F) = 1$.</p>	
Sol.	(D) Assertion (A) is false, but Reason (R) is true.	1
SECTION B		
This section has 5 very short answer type questions of 2 marks each.		
21. (a)	<p>Solve the following pair of equations algebraically :</p> $101x + 102y = 304$ $102x + 101y = 305$	
Sol.	<p>Adding equations we get</p> $x + y = 3$ <p>Subtracting equations we get</p> $-x + y = -1$ <p>Solving to get</p> $x = 2 \text{ and } y = 1$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2} + \frac{1}{2}$</p>
	OR	

21. (b)	In a pair of supplementary angles, the greater angle exceeds the smaller by 50° . Express the given situation as a system of linear equations in two variables and hence obtain the measure of each angle.	
Sol.	Let smaller angle be x and greater angle be y ATQ, $x + y = 180$ Also $y = x + 50$ Solving we get $x = 65^\circ$ and $y = 115^\circ$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$
22. (a)	If $a \sec \theta + b \tan \theta = m$ and $b \sec \theta + a \tan \theta = n$, prove that $a^2 + n^2 = b^2 + m^2$	
Sol.	$m^2 = a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta$ $n^2 = b^2 \sec^2 \theta + a^2 \tan^2 \theta + 2ab \sec \theta \tan \theta$ $m^2 - n^2 = a^2(\sec^2 \theta - \tan^2 \theta) + b^2(\tan^2 \theta - \sec^2 \theta)$ $\Rightarrow m^2 - n^2 = a^2 - b^2$ or $a^2 + n^2 = m^2 + b^2$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
OR		
22. (b)	Use the identity : $\sin^2 A + \cos^2 A = 1$ to prove that $\tan^2 A + 1 = \sec^2 A$. Hence, find the value of $\tan A$, when $\sec A = \frac{5}{3}$, where A is an acute angle.	
Sol.	$\sin^2 A + \cos^2 A = 1$ Dividing both sides by $\cos^2 A$, we get $\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$ $\tan^2 A + 1 = \sec^2 A$ $\tan^2 A + 1 = \left(\frac{5}{3}\right)^2$ $\tan A = \frac{4}{3}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
23.	Prove that abscissa of a point P which is equidistant from points with coordinates $A(7, 1)$ and $B(3, 5)$ is 2 more than its ordinate.	
Sol.	Let $P(x, y)$ be equidistant from $A(7, 1)$ and $B(3, 5)$ $PA = PB \Rightarrow PA^2 = PB^2$ $(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$ $x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

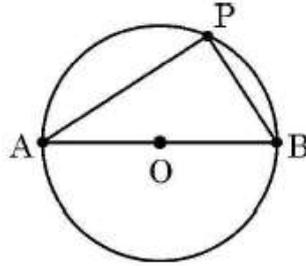
OR		
30. (b)	Given that $\sin \theta + \cos \theta = x$, prove that $\sin^4 \theta + \cos^4 \theta = \frac{2 - (x^2 - 1)^2}{2}$.	
Sol.	<p>Given: $\sin \theta + \cos \theta = x$ Squaring both sides $\sin^2 \theta + \cos^2 \theta + 2 \cos \theta \sin \theta = x^2$ $2 \sin \theta \cos \theta = x^2 - 1$</p> <p>RHS = $\frac{2 - (2 \sin \theta \cos \theta)^2}{2}$ $= \frac{2 - 4 \sin^2 \theta \cos^2 \theta}{2}$ $= 1 - 2 \sin^2 \theta \cos^2 \theta$ $= (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$ $= (\sin^4 \theta + \cos^4 \theta) = \text{LHS}$</p>	<p>1</p> <p>½</p> <p>½</p> <p>½</p> <p>½</p>
31.	<p>In the adjoining figure, TP and TQ are tangents drawn to a circle with centre O. If $\angle OPQ = 15^\circ$ and $\angle PTQ = \theta$, then find the value of $\sin 2\theta$.</p> 	
Sol.	<p>$\angle QPT = 75^\circ$ $\angle PQT = 75^\circ$ $\theta = 30^\circ$ $\sin 2\theta = \sin 2(30^\circ)$ $= \sin 60^\circ = \frac{\sqrt{3}}{2}$</p>	<p>½</p> <p>½</p> <p>1</p> <p>½</p> <p>½</p>

SECTION D

This section has 4 long answer questions of 5 marks each.

32.
(a)

There is a circular park of diameter 65 m as shown in the following figure, where AB is a diameter.



An entry gate is to be constructed at a point P on the boundary of the park such that distance of P from A is 35 m more than the distance of P from B. Find distance of point P from A and B respectively.

Sol.

Let distance of gate at P from point B is x m
 Then distance of gate at P from point A is $(35 + x)$ m
 In right ΔAPB
 $(x + 35)^2 + x^2 = (65)^2$
 $x^2 + 35x - 1500 = 0$
 $(x + 60)(x - 25) = 0$
 $x = 25$
 Hence, $x + 35 = 60$
 Distance of P from A = 60 m
 Distance of P from B = 25 m

$\frac{1}{2}$
1
2
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$

OR

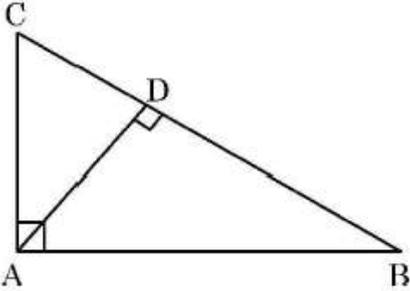
32.
(b)

Find the smallest value of p for which the quadratic equation $x^2 - 2(p + 1)x + p^2 = 0$ has real roots. Hence, find the roots of the equation so obtained.

Sol.

For real roots, $D \geq 0$
 $[-2(p + 1)]^2 - 4p^2 \geq 0$
 $\Rightarrow p \geq -\frac{1}{2}$
 \therefore smallest value of $p = -\frac{1}{2}$
 At $p = -\frac{1}{2}$ given equation becomes
 $x^2 - 2\left(\frac{-1}{2} + 1\right)x + \left(\frac{-1}{2}\right)^2 = 0$
 $x^2 - x + \frac{1}{4} = 0$ or $4x^2 - 4x + 1 = 0$
 $(2x - 1)(2x - 1) = 0$
 \therefore roots are $\frac{1}{2}, \frac{1}{2}$

$\frac{1}{2}$
 $\frac{1}{2}$
1
 $\frac{1}{2}$
 $\frac{1}{2}$
1
 $\frac{1}{2}$
 $\frac{1}{2}$

33. (a)	<p>If a line drawn parallel to one side of triangle intersecting the other two sides in distinct points divides the two sides in the same ratio, then it is parallel to third side.</p> <p>State and prove the converse of the above statement.</p>	
Sol.	<p>Correct Statement of BPT Correct figure, Given, To Prove, Construction Correct Proof of BPT</p> <p>NOTE* Given statement in English version is not a correct statement. Full marks may be awarded to any attempt in English medium.</p>	<p>1 2 2</p>
OR		
33. (b)	<p>In the adjoining figure, $\triangle CAB$ is a right triangle, right angled at A and $AD \perp BC$. Prove that $\triangle ADB \sim \triangle CDA$. Further, if $BC = 10$ cm and $CD = 2$ cm, find the length of AD.</p> 	
Sol.	<p>$\triangle ABC \sim \triangle DAC$ ----- ① Similarly, $\triangle ABC \sim \triangle DBA$ ----- ② From equations ① and ② $\triangle DAC \sim \triangle DBA$ or $\triangle ADB \sim \triangle CDA$ $\frac{AD}{CD} = \frac{BD}{AD}$ $AD^2 = BD \times CD$ $= 8 \times 2$ $\therefore AD = 4$ cm.</p>	<p>1 $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$</p>
34.	<p>From one face of a solid cube of side 14 cm, the largest possible cone is carved out. Find the volume and surface area of the remaining solid. (Use $\pi = \frac{22}{7}$, $\sqrt{5} = 2.2$)</p>	
Sol.	<p>Diameter of cone = 14 cm Radius = 7 cm Height of cone = 14 cm Slant height $l = \sqrt{14^2 + 7^2} = 7\sqrt{5} = 15.4$ cm Volume of remaining solid = Volume of cube – Volume of cone $= (14)^3 - \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 14$ $= \frac{6076}{3} \text{ cm}^3$</p>	<p>$\frac{1}{2}$ $\frac{1}{2}$ 1 1 $\frac{1}{2}$</p>

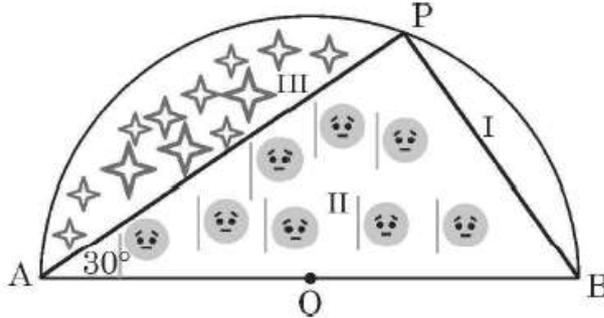
	<p>Surface area of remaining solid = Surface area of cube – Area of circle + Curved surface area of cone</p> $= 6 \times 14 \times 14 - \frac{22}{7} \times 7 \times 7 + \frac{22}{7} \times 7 \times 15.4$ $= 1360.8 \text{ cm}^2$	1 $\frac{1}{2}$																											
35.	<p>Following distribution shows the marks of 230 students in a particular subject. If the median marks are 46, then find the values of x and y.</p> <table border="1" data-bbox="215 427 876 844" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Marks</th> <th>Number of Students</th> </tr> </thead> <tbody> <tr> <td>10 – 20</td> <td>12</td> </tr> <tr> <td>20 – 30</td> <td>30</td> </tr> <tr> <td>30 – 40</td> <td>x</td> </tr> <tr> <td>40 – 50</td> <td>65</td> </tr> <tr> <td>50 – 60</td> <td>y</td> </tr> <tr> <td>60 – 70</td> <td>25</td> </tr> <tr> <td>70 – 80</td> <td>18</td> </tr> </tbody> </table>	Marks	Number of Students	10 – 20	12	20 – 30	30	30 – 40	x	40 – 50	65	50 – 60	y	60 – 70	25	70 – 80	18												
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Marks	Number of Students	Cf																											
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50 – 60	y	$107 + x + y$																											
60 – 70	25	$132 + x + y$																											
70 – 80	18	$150 + x + y$																											
	230																												

SECTION E

This section has 3 case study-based questions of 4 marks each.

36.

Anurag purchased a farmhouse which is in the form of a semicircle of diameter 70 m. He divides it into three parts by taking a point P on the semicircle in such a way that $\angle PAB = 30^\circ$ as shown in the following figure, where O is the centre of semicircle.



In part I, he planted saplings of Mango tree, in part II, he grew tomatoes and in part III, he grew oranges. Based on given information, answer the following questions.

- (i) What is the measure of $\angle POA$?
- (ii) Find the length of wire needed to fence entire piece of land.
- (iii) (a) Find the area of region in which saplings of Mango tree are planted.

OR

- (iii) (b) Find the length of wire needed to fence the region III.

Sol.

(i) $\angle POA = 120^\circ$

(ii) Length of wire needed to fence entire piece of land = $\frac{22}{7} \times 35 + 70 = 180$ m

(iii) Required area = $\frac{60}{360} \times \frac{22}{7} \times (35)^2 - \frac{\sqrt{3}}{4} \times (35)^2$
 $= \left(\frac{1925}{3} - \frac{1225\sqrt{3}}{4} \right) \text{ m}^2$ or 111.89 m² (approx.)

OR

(iii) In ΔAPB , $\frac{AP}{AB} = \cos 30^\circ$

$AP = 35\sqrt{3}$ m

Required length of wire = $\frac{120}{360} \times 2 \times \frac{22}{7} \times 35 + 35\sqrt{3}$

$= \left(\frac{220}{3} + 35\sqrt{3} \right) \text{ m}$ or 133.8 m (approx.)

1

1

1

1

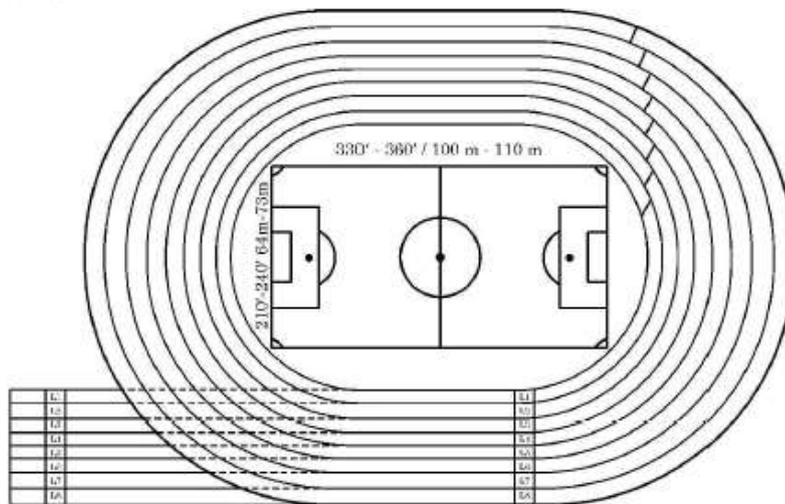
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$\frac{1}{2}$

$\frac{1}{2}$

37.

In order to organise, Annual Sports Day, a school prepared an eight lane running track with an integrated football field inside the track area as shown below :



The length of innermost lane of the track is 400 m and each subsequent lane is 7.6 m longer than the preceding lane.

Based on given information, answer the following questions, using concept of Arithmetic Progression.

- (i) What is the length of the 6th lane ?
- (ii) How long is the 8th lane than that of 4th lane ?
- (iii) (a) While practicing for a race, a student took one round each in first six lanes. Find the total distance covered by the student.

OR

- (iii) (b) A student took one round each in lane 4 to lane 8. Find the total distance covered by the student.

Sol. Here AP is 400, 407.6, 415.2, ...

(i) $a_6 = 400 + 5(7.6) = 438$ m

(ii) $a_8 - a_4 = 30.4$ m

(iii) $S_6 = \frac{6}{2} (2 \times 400 + 5 \times 7.6)$
 $= 2514$ m

OR

(iii) Total distance covered = $S_8 - S_3$
 $= \frac{8}{2} (2 \times 400 + 7 \times 7.6) - \frac{3}{2} (2 \times 400 + 2 \times 7.6)$
 $= 2190$ m

1
1
1
1

1
1

38.

The Statue of Unity situated in Gujarat is the world's largest Statue which stands over a 58 m high base. As part of the project, a student constructed an inclinometer and wishes to find the height of Statue of Unity using it.

He noted following observations from two places :

Situation – I :

The angle of elevation of the top of Statue from Place A which is $80\sqrt{3}$ m away from the base of the Statue is found to be 60° .

Situation – II :

The angle of elevation of the top of Statue from a Place B which is 40 m above the ground is found to be 30° and entire height of the Statue including the base is found to be 240 m.



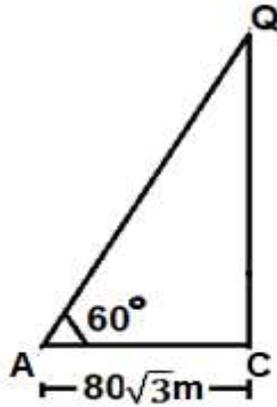
Based on given information, answer the following questions :

- (i) Represent the Situation – I with the help of a diagram.
- (ii) Represent the Situation – II with the help of a diagram.
- (iii) (a) Calculate the height of Statue excluding the base and also find the height including the base with the help of Situation – I.

OR

- (iii) (b) Find the horizontal distance of point B (Situation – II) from the Statue and the value of $\tan \alpha$, where α is the angle of elevation of top of base of the Statue from point B.

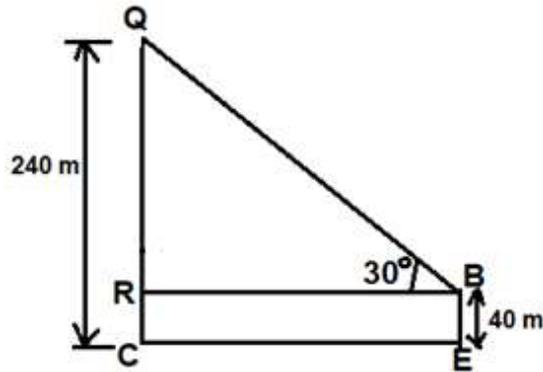
Sol. (i)



Correct figure

1

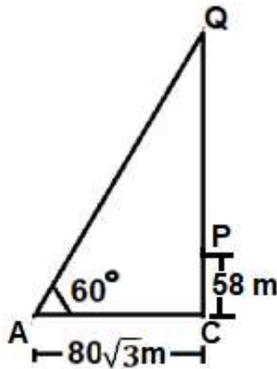
(ii)



Correct figure

1

(iii) (a)



In ΔACQ

$$\frac{QC}{AC} = \tan 60^\circ = \sqrt{3}$$

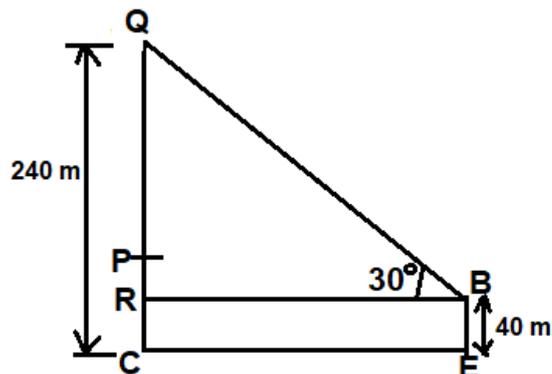
$$QC = 240 \text{ m}$$

Height of statue including base = 240 m

Height of statue excluding base = $240 - 58 = 182 \text{ m}$

OR

(iii) (b)



1

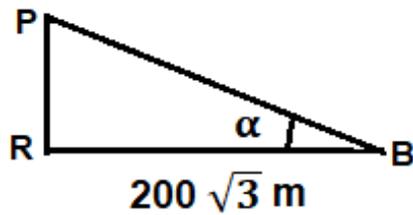
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$$QR = 240 - 40 = 200 \text{ m}$$

In $\triangle QRB$

$$\frac{QR}{RB} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{Horizontal distance } RB = 200\sqrt{3} \text{ m}$$



In $\triangle PRB$

$$\tan \alpha = \frac{PR}{BR}$$

$$= \frac{18}{200\sqrt{3}} \text{ or } \frac{3\sqrt{3}}{100}$$

$\frac{1}{2}$

$\frac{1}{2}$

Correct figure

$\frac{1}{2}$

$\frac{1}{2}$