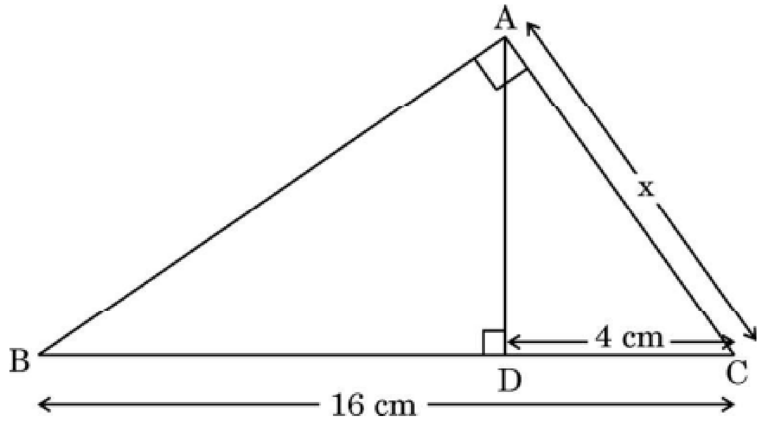
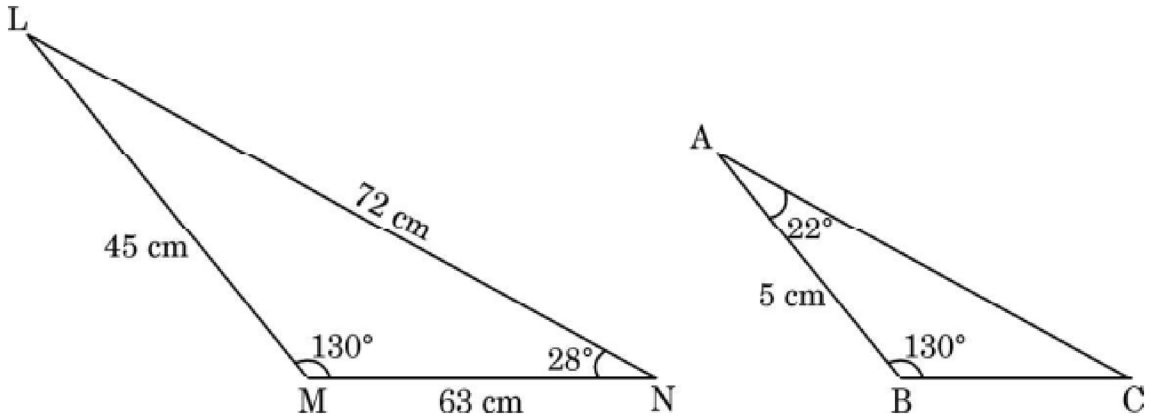
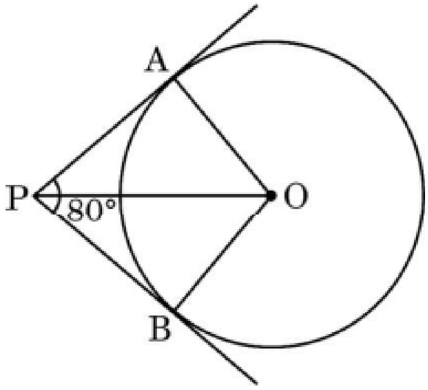
SOLUTIONS MATHEMATICS
(Subject Code–041) (PAPER CODE: 30/3/3)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **This section has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.**	
1.	The diameter of a wheel is 63 cm. The distance travelled by the wheel in 100 revolutions is : (A) 99 m (B) 198 m (C) 63 m (D) 136 m	
Sol.	(B)198 m	**1**
2.	The mean of seven observations is 17. If the mean of the first four observations is 15 and that of the last four observations is 18, then the fourth observation is : (A) 14 (B) 13 (C) 12 (D) 10	
Sol.	(B)13	**1**
3.	The distance of the point (4, 0) from x-axis is : (A) 4 units (B) 16 units (C) 0 units (D) $4\sqrt{2}$ units	
Sol.	(C) 0 units	**1**
4.	If α and β are zeroes of the polynomial $p(x) = kx^2 - 30x + 45k$ and $\alpha + \beta = \alpha\beta$, then the value of 'k' is : (A) $-\frac{2}{3}$ (B) $-\frac{3}{2}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$	
Sol.	(D) $\frac{2}{3}$	**1**
5.	The radii 'r' of a sphere and that of the base of a cone are same. If their volumes are also same, then the height of the cone is : (A) r (B) 2r (C) 3r (D) 4r	
Sol.	(D) 4r	**1**

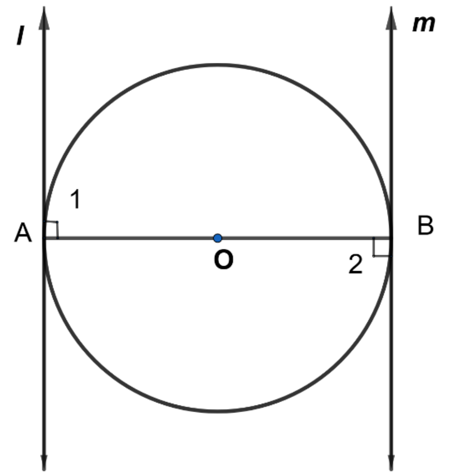
6.	If in a lottery, there are 10 prizes and 30 blanks, then the probability of winning a prize is : (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$	
Sol.	(A) $\frac{1}{4}$	**1**
7.	If $\tan 3\theta = \sqrt{3}$, then $\frac{\theta}{2}$ equals : (A) 60° (B) 30° (C) 20° (D) 10°	
Sol.	(D) 10°	**1**
8.	If in two triangles Δ DEF and Δ PQR, $\angle$ D = $\angle$ Q and $\angle$ R = $\angle$ E, then which of the following is ***not*** true ? (A) $\frac{DE}{QR} = \frac{DF}{PQ}$ (B) $\frac{EF}{PR} = \frac{DF}{PQ}$ (C) $\frac{EF}{RP} = \frac{DE}{QR}$ (D) $\frac{DE}{PQ} = \frac{EF}{RP}$	
Sol.	(D) $\frac{DE}{PQ} = \frac{EF}{RP}$	**1**
9.	In the given figure, in Δ ABC, AD $\perp$ BC and $\angle$ BAC = 90°. If BC = 16 cm and DC = 4 cm, then the value of x is :  (A) 4 cm (B) 5 cm (C) 8 cm (D) 3 cm	
Sol.	(C) 8 cm	**1**

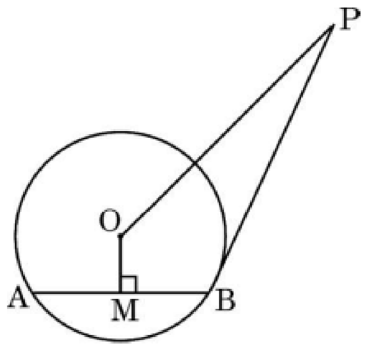
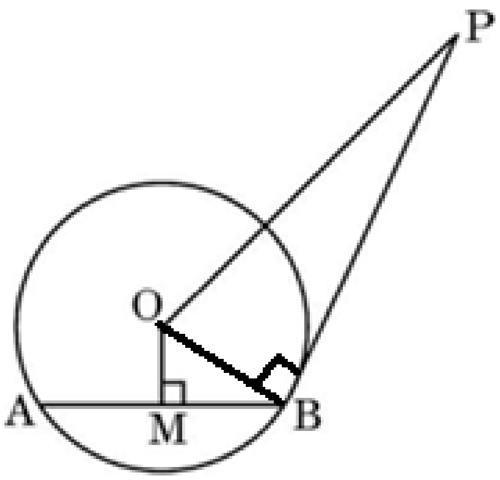
10.	Two of the vertices of Δ PQR are P(−1, 5) and Q(5, 2). The coordinates of a point which divides PQ in the ratio 2 : 1 are : (A) (3, −3) (B) (5, 5) (C) (3, 3) (D) (5, 1)	
Sol.	(C) (3, 3)	**1**
11.	$(\cot\theta + \tan\theta)$ equals : (A) $\operatorname{cosec}\theta \sec\theta$ (B) $\sin\theta \sec\theta$ (C) $\cos\theta \tan\theta$ (D) $\sin\theta \cos\theta$	
Sol.	(A) $\operatorname{cosec}\theta \sec\theta$	**1**
12.	Zeroes of the polynomial $p(x) = x^2 - 3\sqrt{2}x + 4$ are : (A) $2, \sqrt{2}$ (B) $2\sqrt{2}, \sqrt{2}$ (C) $4\sqrt{2}, -\sqrt{2}$ (D) $\sqrt{2}, 2$	
Sol.	(B) $2\sqrt{2}, \sqrt{2}$	**1**
13.	The value of 'k' for which the system of linear equations $6x + y = 3k$ and $36x + 6y = 3$ have infinitely many solutions is : (A) 6 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$	
Sol.	(B) $\frac{1}{6}$	**1**

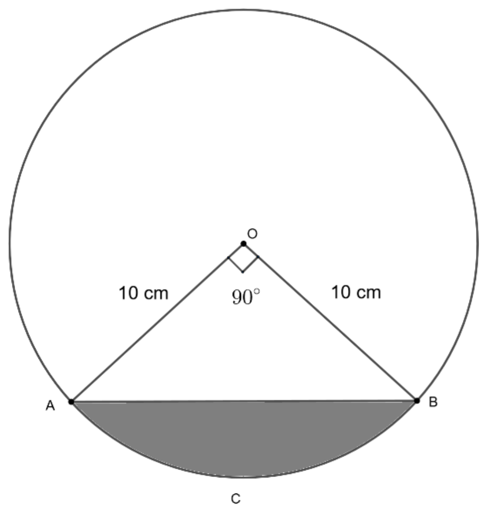
14.	The measurements of Δ LMN and Δ ABC are shown in the figure given below. The length of side AC is :  (A) 16 cm (B) 7 cm (C) 8 cm (D) 4 cm	
Sol.	(C) 8 cm	**1**
15.	The line represented by $\frac{x}{4} + \frac{y}{6} = 1$, intersects x-axis and y-axis respectively at P and Q. The coordinates of the mid-point of line segment PQ are : (A) (2, 3) (B) (3, 2) (C) (2, 0) (D) (0, 3)	
Sol.	(A) (2, 3)	**1**
16.	If x is the LCM of 4, 6, 8 and y is the LCM of 3, 5, 7 and p is the LCM of x and y, then which of the following is true ? (A) p = 35x (B) p = 4y (C) p = 8x (D) p = 16y	
Sol.	(A) p = 35x	**1**
17.	If $\frac{x}{12} - \frac{3}{x} = 0$, then the values of x are : (A) ± 6 (B) ± 4 (C) ± 12 (D) ± 3	
Sol.	(A) ± 6	**1**

18.	If tangents PA and PB drawn from an external point P to the circle with centre O are inclined to each other at an angle of 80° as shown in the given figure, then the measure of $\angle$ POA is :  (A) 40° (B) 50° (C) 60° (D) 80°	
Sol.	(B) 50°	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre of the circle. *Reason (R):* A parallelogram circumscribing a circle is a rhombus.	
Sol.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).	**1**
20.	*Assertion (A) :* A ladder leaning against a wall, stands at a horizontal distance of 6 m from the wall. If the height of the wall up to which the ladder reaches is 8 m, then the length of the ladder is 10 m. *Reason (R):* The ladder makes an angle of 60° with the ground.	
Sol.	(C) Assertion (A) is true, but Reason (R) is false.	**1**

SECTION B This section has 5 Very Short Answer (VSA) type questions carrying 2 marks each.		
21.	If $\tan A + \cot A = 6$, then find the value of $\tan^2 A + \cot^2 A - 4$.	
Sol.	$(\tan A + \cot A)^2 = 36$ $\tan^2 A + \cot^2 A + 2\tan A \cot A = 36$ $\tan^2 A + \cot^2 A = 34$ $\therefore \tan^2 A + \cot^2 A - 4 = 30$	½ ½ ½ ½
22. (a)	Find the value(s) of 'k' so that the quadratic equation $4x^2 + kx + 1 = 0$ has real and equal roots.	
Sol.	For real and equal roots, $D = 0$ $k^2 - 16 = 0$ $k = \pm 4$	½ **1** ½
OR		
22. (b)	If 'α' and 'β' are the zeroes of the polynomial $p(y) = y^2 - 5y + 3$, then find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.	
Sol.	$\alpha + \beta = 5$ $\alpha\beta = 3$ $\alpha^4\beta^3 + \alpha^3\beta^4 = (\alpha\beta)^3(\alpha + \beta)$ $= 27 \times 5 = 135$	½ ½ ½ ½
23.	The probability of guessing the correct answer of a certain test question is $\frac{x}{12}$. If the probability of not guessing the correct answer is $\frac{5}{6}$, then find the value of x.	
Sol.	$\frac{x}{12} + \frac{5}{6} = 1$ $x = 2$	**1** **1**
24.	Prove that the tangents drawn at the ends of a diameter of a circle are parallel.	

Sol.	 Tangents l and m are drawn at the end points A and B of the diameter AB of the circle $\angle 1 = 90°, \angle 2 = 90°$ $\therefore\ \angle 1 = \angle 2$ But these are alternate interior angles. $\therefore l \parallel m$	Correct figure ½ Mark ½ ½ ½
25. (a)	Find the smallest number which is divisible by both 644 and 462.	
Sol.	$462 = 2 \times 3 \times 7 \times 11$ $644 = 2^2 \times 7 \times 23$ $\text{LCM}(462, 644) = 2^2 \times 3 \times 7 \times 11 \times 23 = 21252$ $\therefore$ Smallest number which is divisible by both 462 and 644 is 21252	½ ½ **1**
	OR	
25. (b)	Two numbers are in the ratio 4 : 5 and their HCF is 11. Find the LCM of these numbers.	
Sol.	Let the two numbers be 4x and 5x where x is common factor Now HCF $= 11$ $\therefore$ x $= 11$ Numbers are 44 and 55 $\text{LCM}(44, 55) = \dfrac{44 \times 55}{11} = 220$	 ½ **1** ½

SECTION C This section has 6 Short Answer (SA) type questions carrying 3 marks each.		
26.	All face cards of spades are removed from a pack of 52 playing cards and the remaining pack is shuffled well. A card is then drawn at random from the remaining pack. Find the probability of getting : (a) a face card (b) an ace or a jack	
Sol.	After removing face cards of spades, total number of cards $= 52 - 3 = 49$ (a) P(a face card) $= \frac{9}{49}$ (b) P(an ace or a jack) $= \frac{7}{49}$ *or* $\frac{1}{7}$	**1** **1** **1**
27.	In the given figure, PB is a tangent to the circle with centre O at B. AB is a chord of the circle of length 24 cm and at a distance of 5 cm from the centre of the circle. If the length PB of the tangent is 20 cm, find the length of OP.  	
Sol.	 Join OB	½

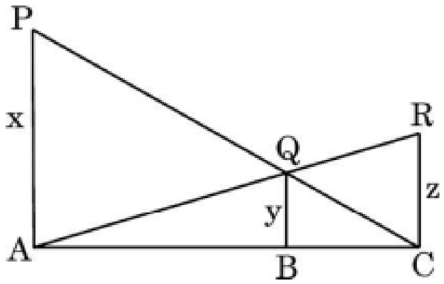
	$AB = 24$ cm, $OM = 5$ cm, $PB = 20$ cm $AM = MB = 12$ cm In ΔOMB, $OB = \sqrt{5^2 + 12^2} = 13$ cm As PB is tangent $\Rightarrow PB \perp OB$ In rt ΔOBP, $OP = \sqrt{13^2 + 20^2} = \sqrt{569}$ cm	½ 1 1
28.	A chord of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding minor segment. [Use $\pi = 3{\cdot}14$]	
Sol.	 Area of minor segment ACB = Area of sector OACB − Area of right ΔOAB Area of sector OACB $= \frac{90}{360} \times 3.14 \times 10 \times 10$ $= 78.5$ cm^2 Area of right $\Delta OAB = \frac{1}{2} \times 10 \times 10$ $= 50$ cm^2 Area of minor segment ACB $= (78.5 - 50)$ $= 28.5$ cm^2	 ½ ½ ½ ½ ½ ½

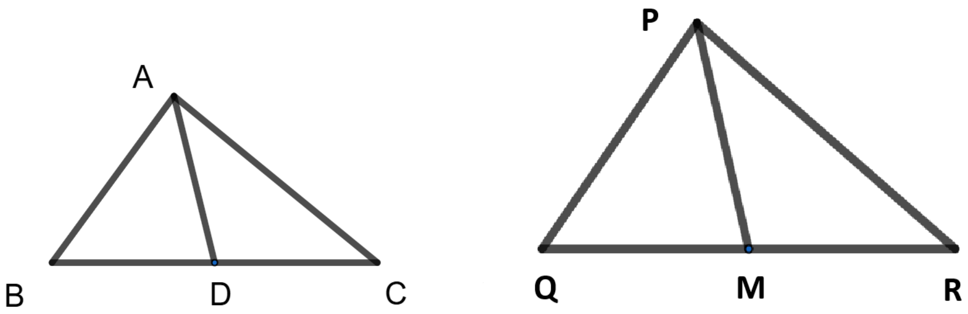
29.	Prove that $\left(5\sqrt{3}+\frac{2}{3}\right)$ is an irrational number given that $\sqrt{3}$ is an irrational number.	
Sol.	Let $5\sqrt{3}+\frac{2}{3}$ be a rational number. $\therefore 5\sqrt{3}+\frac{2}{3}=\frac{a}{b}$ where a and b are integers and $b \neq 0$. $5\sqrt{3}=\frac{a}{b}-\frac{2}{3}$ $\sqrt{3}=\frac{3a-2b}{15b}$ $3a-2b$ and $15b$ are integers. $\therefore$ RHS is rational. But LHS $=\sqrt{3}$ is an irrational number which is contradiction to our supposition. Hence $5\sqrt{3}+\frac{2}{3}$ is an irrational number.	**1** **1** ½ ½
30. (a)	Prove that : $\sqrt{\frac{\sec A-1}{\sec A+1}}+\sqrt{\frac{\sec A+1}{\sec A-1}}=2\operatorname{cosec} A$	
Sol.	$\text{LHS}=\frac{\sec A-1+\sec A+1}{\sqrt{\sec^2 A-1}}$ $=\frac{2\sec A}{\tan A}$ $=2\operatorname{cosec} A=\text{RHS}$	**1** **1** **1**
OR		
30. (b)	Prove that : $\left(\frac{1}{\cos A}-\cos A\right)\left(\frac{1}{\sin A}-\sin A\right)=\frac{1}{\tan A+\cot A}$	
Sol.	$\text{LHS}=\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{1-\sin^2 A}{\sin A}\right)$ $=\frac{\sin^2 A}{\cos A}\cdot\frac{\cos^2 A}{\sin A}$	**1** ½

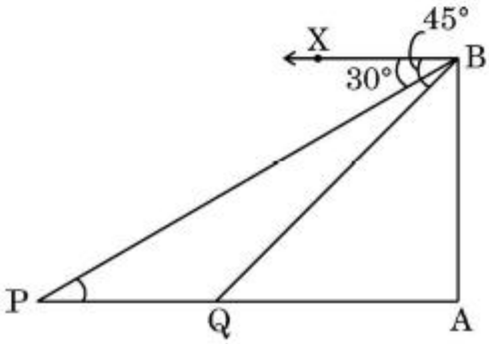
	$= \sin A . \cos A$ $\text{RHS} = \dfrac{\sin A . \cos A}{\sin^2 A + \cos^2 A}$ $= \sin A . \cos A$ $\therefore \text{LHS} = \text{RHS}$	½ ½ ½
31. (a)	If the mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y) and x + y – 10 = 0, find the value of k.	
Sol.	$P(x, y)$ is the mid – point $\therefore (x, y) = (\dfrac{3+k}{2}, \dfrac{4+6}{2})$ $x = \dfrac{3+k}{2}, y = 5$ $x + y - 10 = 0$ $\dfrac{3+k}{2} + 5 - 10 = 0$ $k = 7$	**1** **1** **1**
OR		
31. (b)	Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.	
Sol.	**1 : 1 : 1 : 1** A (−2, 2) P Q R B (2, 8) Coordinates of mid – point Q of AB $= (0, 5)$ Coordinates of mid – point P of AQ $= \left(-1, \dfrac{7}{2}\right)$ Coordinates of mid – point R of BQ $= \left(1, \dfrac{13}{2}\right)$	**1** **1** **1**

SECTION D This section has 4 Long Answer (LA) type questions carrying 5 marks each.		
32.	A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. Also, find the surface area of the toy. (Take $\pi = 3{\cdot}14$)	
Sol.	Height of the cone (h) =2 cm Radius of the hemisphere = radius of the base of the cone $= r = 2$ cm Volume of the toy $= \frac{1}{3} \times 3.14 \times (2)^2 \times 2 + \frac{2}{3} \times 3.14 \times (2)^3$ $= 25.12\ \text{cm}^3$ Slant heightof the cone $(l) = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ cm Surface area of the toy $= 3.14 \times 2 \times 2\sqrt{2} + 2 \times 3.14 \times (2)^2$ $= 12.56\left(2 + \sqrt{2}\right)\ \text{cm}^2$	½ 1 1 ½ 1 1
33.	The students of a class are made to stand equally in rows. If 3 students are extra in each row, there would be 1 row less. If 3 students are less in a row, there would be 2 more rows. Find the number of students in the class.	
Sol.	Let number of students in each row be x and the number of rows be y $\therefore$ Total number of students $= xy$ ATQ, $(x + 3)(y - 1) = xy$ $\Rightarrow x - 3y + 3 = 0$ Also, $(x - 3)(y + 2) = xy$ $\Rightarrow 2x - 3y - 6 = 0$ On solving these equations, we get $x = 9$ and $y = 4$ $\therefore$ Number of students in the class $= xy = 9 \times 4 = 36$	½ 1 ½ 1 ½ 1 ½
34. (a)	The sum of the third term and the seventh term of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP.	

Sol.	Let first term = a and common difference = d ATQ, $(a+2d)+(a+6d)=6$ $a+4d=3$ $a=3-4d$ Also, $(a+2d)(a+6d)=8$ $(3-4d+2d)(3-4d+6d)=8$ $9-4d^2=8$ $d=\pm\frac{1}{2}$ When $d=\frac{1}{2} \Rightarrow a=1$ $S_{16}=\frac{16}{2}\left[2\times 1+15\times\frac{1}{2}\right]$ $=76$ When $d=-\frac{1}{2} \Rightarrow a=5$ $S_{16}=\frac{16}{2}\left[2\times 5+15\times(-\frac{1}{2})\right]$ $=20$	½ ½ **1** ½ ½ ½ ½ ½ ½
OR		
34. (b)	The minimum age of children eligible to participate in a painting competition is 8 years. It is observed that the age of the youngest boy was 8 years and the ages of the participants, when seated in order of age, have a common difference of 4 months. If the sum of the ages of all the participants is 168 years, find the age of the eldest participant in the painting competition.	
Sol.	The ages of the participants form the following AP $8, 8\frac{1}{3}, 8\frac{2}{3}, 9, \cdots$ where first term = 8 and common difference $=\frac{1}{3}$ Let the number of participants be n $S_n=\frac{n}{2}\left[2\times 8+(n-1)\frac{1}{3}\right]=168$	**1** **1**

	$n^2 + 47n - 1008 = 0$	1
	$\Rightarrow n = 16$	1
	$\therefore$ the age of the eldest participant $= 8 + 15 \times \frac{1}{3} = 13$ years	1
35. (a)	In the given figure, PA, QB and RC are perpendicular to AC. If PA = x units, QB = y units and RC = z units, prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$. 	
Sol.	$\Delta ABQ \sim \Delta ACR$	1
	$\frac{AB}{AC} = \frac{QB}{RC} = \frac{y}{z}$ ……………………………… (i)	½
	Similarly, $\Delta CBQ \sim \Delta CAP$	½
	$\frac{BC}{AC} = \frac{QB}{PA} = \frac{y}{x}$ ……………………………… (ii)	½
	On adding (i) & (ii), we get	
	$\frac{AB}{AC} + \frac{BC}{AC} = \frac{y}{z} + \frac{y}{x}$	1
	$\frac{AB + BC}{AC} = y\left(\frac{1}{z} + \frac{1}{x}\right)$	
	$\frac{AC}{AC} = y\left(\frac{1}{z} + \frac{1}{x}\right)$	1
	$\therefore \frac{1}{x} + \frac{1}{z} = \frac{1}{y}$	½
	OR	
35. (b)	Sides AB and BC and median AD of triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR. Show that $\Delta ABC \sim \Delta PQR$.	

Sol.		**Correct figure 1 mark**
	In ΔABD and ΔPQM	
	$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ (given)	
	$\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$	**1**
	$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$	
	$\therefore \Delta ABD \sim \Delta PQM$	**1**
	$\therefore \angle B = \angle Q$	½
	In ΔABC and ΔPQR	
	$\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$	**1**
	$\Delta ABC \sim \Delta PQR$	½

SECTION E This section has 3 Case Study based questions carrying 4 marks each.		
36.	**Case Study – 1** A lighthouse stands tall on a cliff by the sea, watching over ships that pass by. One day a ship is seen approaching the shore and from the top of the lighthouse, the angles of depression of the ship are observed to be 30° and 45° as it moves from point P to point Q. The height of the lighthouse is 50 metres.   Based on the information given above, answer the following questions : (i) Find the distance of the ship from the base of the lighthouse when it is at point Q, where the angle of depression is 45°. (ii) Find the measures of ∠ PBA and ∠ QBA. (iii) (a) Find the distance travelled by the ship between points P and Q. **OR** (b) If the ship continues moving towards the shore and takes 10 minutes to travel from Q to A, calculate the speed of the ship in km/h, from Q to A.	
Sol.	(i) $\angle AQB = \angle QBX = 45°$ and $\angle APB = \angle PBX = 30°$ In $\Delta AQB, \tan 45° = \frac{50}{AQ}$ $AQ = 50 \text{ m}$ (ii) $\angle PBA = 60°$	 ½ ½ ½

	$\angle QBA = 45°$ $(iii)(a)$ In $\Delta APB, \tan 30° = \dfrac{50}{AP}$ $AP = 50\sqrt{3}$ m Distance travelled by the ship $= PQ = 50\sqrt{3} - 50 = 50(\sqrt{3} - 1)$ m *or* 36.5 m OR (iii)(b) Speed of the ship $= \dfrac{50 \text{ metres}}{10 \text{ minutes}}$ $= 0.3$ km/h	½ ½ ½ **1** **1** **1**
37.	**Case Study – 2** The India Meteorological Department observes seasonal and annual rainfall every year in different sub-divisions of our country. It helps them to compare and analyse the results.	

	The table below shows sub-divisions wise seasonal (monsoon) rainfall (in mm) in 2023.	

Rainfall (mm)	*No. of Sub-divisions*
200 – 400	3
400 – 600	4
600 – 800	7
800 – 1000	4
1000 – 1200	3
1200 – 1400	3

Based on the information given above, answer the following questions :

(i) Write the modal class.

(ii) (a) Find the median of the given data.

OR

(b) Find the mean rainfall in the season.

(iii) If a sub-division having at least 800 mm rainfall during monsoon season is considered a good rainfall sub-division, then how many sub-divisions had good rainfall ?

Sol. (i) Modal Class $= 600 - 800$ — **1**

(ii)(a)

Rainfall (mm)	No. of Sub-divisions (f_i)	cf
200−400	**3**	**3**
400−600	**4**	**7**
600−800	**7**	**14**
800−1000	**4**	**18**
1000−1200	**3**	**21**
1200−1400	**3**	**24**

Correct table ½ mark

$\mathrm{N} = 24$

Median Class $= 600 - 800$ — **½**

$$\text{Median} = 600 + \frac{12-7}{7} \times 200$$ **½**

$= \frac{5200}{7}$ or 742.8 mm $(approx.)$ ½

OR

(ii)(b)

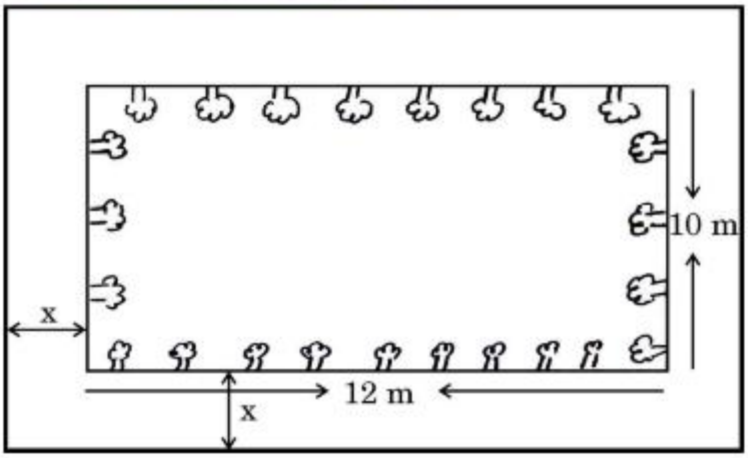
Rainfall (mm)	No. of Sub-divisions (f_i)	x_i	$f_i x_i$
200−400	3	300	900
400−600	4	500	2000
600−800	7	700	4900
800−1000	4	900	3600
1000−1200	3	1100	3300
1200−1400	3	1300	3900
	$\sum f_i = 24$		$\sum f_i x_i = 18600$

Correct table 1 Mark

Mean $= \frac{18600}{24} = 775$ **1**

$\therefore$ Mean rainfall $= 775$ mm

(iii) Required number of sub − divisions $= 4 + 3 + 3 = 10$ **1**

38.	**Case Study – 3** A garden designer is planning a rectangular lawn that is to be surrounded by a uniform walkway.   The total area of the lawn and the walkway is 360 square metres. The width of the walkway is same on all sides. The dimensions of the lawn itself are 12 metres by 10 metres. Based on the information given above, answer the following questions : (i) Formulate the quadratic equation representing the total area of the lawn and the walkway, taking width of walkway = x m. (ii) (a) Solve the quadratic equation to find the width of the walkway 'x'. **OR** (b) If the cost of paving the walkway at the rate of ₹ 50 per square metre is ₹ 12,000, calculate the area of the walkway. (iii) Find the perimeter of the lawn.	
Sol.	$(i)\ (12 + 2x)(10 + 2x) = 360$	½
	$4x^2 + 44x - 240 = 0$ *or* $x^2 + 11x - 60 = 0$	½
	$(ii)(a)\ (x + 15)(x - 4) = 0$	**1**
	$x = 4$	
	$\therefore$ width of the walkway $= 4$ m	**1**
	OR	

	(ii)(b) Area of the walkway $= \frac{12000}{50}$ $= 240\ \text{m}^2$ (iii) Perimeter of the lawn $= 2(12 + 10) = 44$ m	**1** **1** **1**