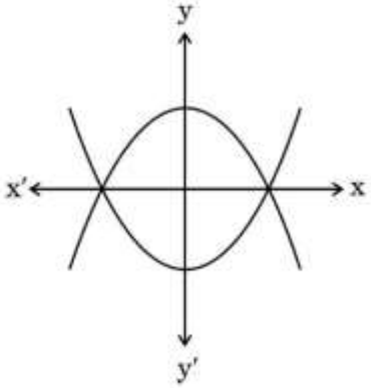
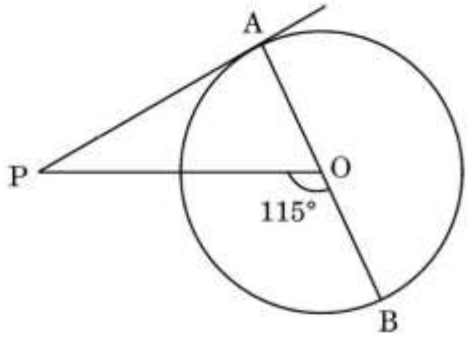
SOLUTIONS
MATHEMATICS (Subject Code–041) (PAPER CODE: 30/1/3)

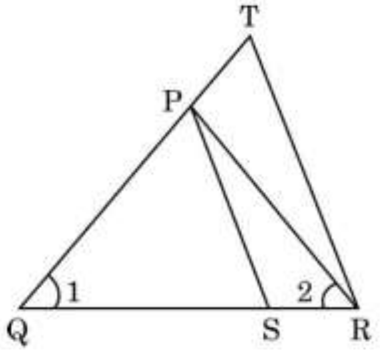
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A This section has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.	
1.	What is the mode of a data if median and mean of the same data are 9·6 and 10·5, respectively ? (A) 7·8 (B) 12·3 (C) 8·4 (D) 7	
Sol.	(A) 7.8	**1**
2.	The value of $(\tan A \operatorname{cosec} A)^2 - (\sin A \sec A)^2$ is : (A) 0 (B) 1 (C) –1 (D) 2	
Sol.	(B) 1	**1**
3.	A kite is flying at a height of 150 m from the ground. It is attached to a string inclined at an angle of 30° to the horizontal. The length of the string is : (A) $100\sqrt{3}$ m (B) 300 m (C) $150\sqrt{2}$ m (D) $150\sqrt{3}$ m	
Sol.	(B) 300 m	**1**
4.	In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE. Then, the two triangles are : (A) congruent but not similar (B) congruent as well as similar (C) neither congruent nor similar (D) similar but not congruent	
Sol.	(D) similar but not congruent	**1**

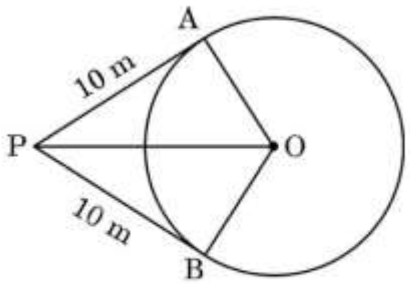
5.	If θ is an acute angle and $7 + 4\sin\theta = 9$, then the value of θ is : (A) 90° (B) 30° (C) 45° (D) 60°	
Sol.	(B) 30^0	**1**
6.	Two polynomials are shown in the graph below. The number of distinct zeroes of both the polynomials is :  (A) 3 (B) 5 (C) 2 (D) 4	
Sol.	(C) 2	**1**
7.	In the given figure, PA is a tangent from an external point P to a circle with centre O. If ∠ POB = 115°, then ∠ APO is equal to :  (A) 25° (B) 65° (C) 90° (D) 35°	
Sol.	(A) 25°	**1**

8.	A piece of wire 20 cm long is bent into the form of an arc of a circle of radius $\frac{60}{\pi}$ cm. The angle subtended by the arc at the centre of the circle is : (A) 30° (B) 60° (C) 90° (D) 50°	
Sol.	(B) 60°	**1**
9.	If HCF(98, 28) = m and LCM(98, 28) = n, then the value of $n - 7m$ is : (A) 0 (B) 28 (C) 98 (D) 198	
Sol.	(C) 98	**1**
10.	Which of the following is a rational number between $\sqrt{3}$ and $\sqrt{5}$? (A) 1·4142387954012 .... (B) $2{\cdot}32\overline{6}$ (C) π (D) 1·857142	
Sol.	(D) 1.857142	**1**
11.	The sum of the zeroes of the polynomial $p(x) = 5x - 7x^2 + 3$ is : (A) $\frac{-7}{5}$ (B) $\frac{7}{5}$ (C) $\frac{5}{7}$ (D) $\frac{-5}{7}$	
Sol.	(C) $\frac{5}{7}$	**1**
12.	If x = 1 and y = 2 is a solution of the pair of linear equations $2x - 3y + a = 0$ and $2x + 3y - b = 0$, then : (A) $a = 2b$ (B) $2a = b$ (C) $a + 2b = 0$ (D) $2a + b = 0$	
Sol.	(B) 2a = b	**1**

13.	If a sector of a circle has an area of 40π sq. units and a central angle of $72°$, the radius of the circle is : (A) 200 units (B) 100 units (C) 20 units (D) $10\sqrt{2}$ units	
Sol.	(D) $10\sqrt{2}$ units	**1**
14.	The tangents drawn at the extremities of the diameter of a circle are always : (A) parallel (B) perpendicular (C) equal (D) intersecting	
Sol.	(A) parallel	**1**
15.	If $(-1)^n + (-1)^8 = 0$, then n is : (A) any positive integer (B) any negative integer (C) any odd number (D) any even number	
Sol.	(C) any odd number	**1**
16.	The end points of a diameter of circle are (2, 4) and (– 3, – 1). The length of its radius is : (A) $\frac{5\sqrt{2}}{2}$ units (B) $5\sqrt{2}$ units (C) $3\sqrt{2}$ units (D) $\pm\frac{5\sqrt{2}}{2}$ units	
Sol.	(A) $\frac{5\sqrt{2}}{2}$ units	**1**
17.	The 11^{th} and 13^{th} term of an AP are 39 and 45, respectively. What is the common difference of the AP ? (A) 42 (B) 21 (C) 6 (D) 3	
Sol.	(D) 3	**1**

18.	A card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a spade or a king ? (A) $\frac{1}{13}$ (B) $\frac{2}{13}$ (C) $\frac{4}{13}$ (D) $\frac{9}{13}$	
Sol.	(C) $\frac{4}{13}$	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* The probability of selecting a number at random from the numbers 1 to 20 is 1. *Reason (R):* For any event E, if P(E) = 1, then E is called a sure event.	
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
20.	*Assertion (A) :* If we join two hemispheres of same radius along their bases, then we get a sphere. *Reason (R):* Total Surface Area of a sphere of radius r is $3\pi r^2$.	
Sol.	(C) Assertion (A) is true, but Reason (R) is false.	**1**
	SECTION B This section has **5** Very Short Answer (VSA) type questions carrying **2** marks each.	
21 (a)	If Δ ABC ~ Δ PQR in which AB = 6 cm, BC = 4 cm, AC = 8 cm and PR = 6 cm, then find the length of (PQ + QR).	
Sol.	$\frac{6}{PQ} = \frac{4}{QR} = \frac{8}{6}$ $\Rightarrow PQ = \frac{9}{2}$ cm or 4.5 cm	½ ½

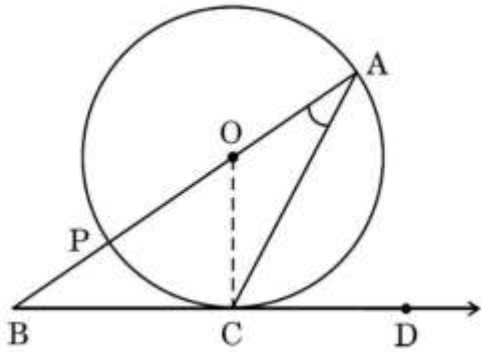
	and QR = 3 cm $\therefore$ PQ + QR = 7.5cm	½ ½
	OR	
21 (b)	In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$, show that Δ PQS ~ Δ TQR. 	
Sol.	In Δ PQR, $\angle 1 = \angle 2$ $\therefore$ PR = PQ $\therefore \frac{QR}{QS} = \frac{QT}{PR} \Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$ Also, $\angle 1 = \angle 1$ $\therefore$ Δ PQS ~ Δ TQR	½ ½ ½ ½
22 (a)	If $x \cos 60° + y \cos 0° + \sin 30° - \cot 45° = 5$, then find the value of $x + 2y$.	
Sol.	$x\left(\frac{1}{2}\right) + y\,(1) + \frac{1}{2} - 1 = 5$ $\Longrightarrow \quad x + 2y = 11$	**1½** ½
	OR	
22 (b)	Evaluate : $\frac{\tan^2 60°}{\sin^2 60° + \cos^2 30°}$	
Sol.	$\frac{(\sqrt{3})^2}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$ $= \quad 2$	**1½** ½

23.	A person is standing at P outside a circular ground at a distance of 26 m from the centre of the ground. He found that his distances from the points A and B on the ground are 10 m (PA and PB are tangents to the circle). Find the radius of the circular ground. 	
Sol.	∠ OAP = 90° In right Δ OAP, $(26)^2 = OA^2 + (10)^2$ $\Rightarrow OA = \sqrt{576} = 24$ ∴ radius = 24 m	½ 1 ½
24.	Find the zeroes of the polynomial $p(x) = x^2 + \frac{4}{3}x - \frac{4}{3}$.	
Sol.	$\frac{1}{3}(3x^2 + 4x - 4)$ $= \frac{1}{3}(3x^2 + 6x - 2x - 4)$ $= \frac{1}{3}(3x - 2)(x + 2)$ Zeroes are $\frac{2}{3}, -2$	½ ½ ½ ½
25.	Find the length of the median through the vertex B of Δ ABC with vertices A(9, – 2), B(– 3, 7) and C(–1, 10).	
Sol.	Mid point of AC = (4,4) Length of median from B to AC = $\sqrt{(4+3)^2 + (4-7)^2}$ $= \sqrt{58}$ Hence the length of median is $\sqrt{58}$ units	**1** ½ ½
	SECTION C This section has **6** Short Answer (SA) type questions carrying **3** marks each.	
26.	Prove that $\sqrt{5}$ is an irrational number.	
Sol.	Let $\sqrt{5}$ be a rational number. $\therefore \sqrt{5} = \frac{p}{q}$, where q ≠ 0 and let p & q are co-primes. $5q^2 = p^2 \Rightarrow p^2$ is divisible by 5 ⇒ p is divisible by 5----- (i) ⇒let p = 5a, where ‘a’ is some integer $25a^2 = 5q^2 \Rightarrow q^2 = 5a^2 \Rightarrow q^2$ is divisible by 5. ⇒ q is divisible by 5. ----- (ii)	 ½ 1 1

	(i) and (ii) leads to contradiction as p and q are coprimes. $\therefore \sqrt{5}$ is an irrational number	½
27.	Two dice are rolled together. Find the probability of getting : (i) a multiple of 2 on one and a multiple of 3 on the other die. (ii) the product of two numbers on the top of the two dice is a perfect square number.	
Sol.	Total outcomes = 36 (i) (2, 3), (2, 6), (3, 2), (3, 4), (3, 6), (4, 3), (4, 6), (6, 2), (6, 3), (6, 4), (6, 6) Number of outcomes having multiple of 2 on one die and a multiple of 3 on other die = 11 Hence, P(E) = $\frac{11}{36}$ (ii) (1, 1), (2, 2), (3, 3), (1, 4), (4, 1), (4, 4), (5, 5), (6, 6) Number of outcomes having product of two numbers on the top of the dice is a perfect square number = 8 P(E) = $\frac{8}{36}$ or $\frac{2}{9}$	1 ½ ½ ½ ½
28 (a)	Prove that : $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$	
Sol.	LHS = $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$ $=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$ $=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta\,(\sin\theta-\cos\theta)}$ $=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$ $=\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$ $=\frac{(1+\sin\theta\cos\theta)}{\sin\theta\cos\theta}$ $=1+\sec\theta\,\mathrm{cosec}\,\theta$ = RHS	 ½ 1 ½ ½ ½
	OR	
28.(b)	Prove that : $\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}=\frac{2}{2\sin^2 A-1}$	
Sol.	LHS = $\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}$ $=\frac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{(\sin A-\cos A)(\sin A+\cos A)}$ $=\frac{\sin^2A+\cos^2A+2\sin A\cos A+\sin^2A+\cos^2A-2\sin A\cos A}{\sin^2A-\cos^2A}$ $=\frac{1+1}{\sin^2A-(1-\sin^2A)}$ $=\frac{2}{2\sin^2A-1}$ = RHS	 1 1 ½ ½

<table>
<tr><td>29.</td><td>A room is in the form of a cylinder surmounted by a hemispherical dome. The base radius of the hemisphere is half of the height of the cylindrical part. If the room contains $\frac{1408}{21}$ m^3 of air, find the height of the cylindrical part. (Use $\pi = \frac{22}{7}$).</td><td></td></tr>
<tr><td>Sol.</td><td>Let r is the radius of hemisphere and cylinder and h is the height of cylinder
$h = 2r$
Volume of air in room $= \frac{2}{3}\pi r^3 + \pi r^2 h$
$\frac{1408}{21} = \frac{2}{3}\pi r^3 + \pi r^2(2r)$
$\frac{1408}{21} = \frac{8}{3} \times \frac{22}{7} \times r^3$
$r^3 = 8$
$\therefore \quad r = 2$ m
and $h = 4$ m</td><td>
½

½

1
½
½</td></tr>
<tr><td>30 (a)</td><td>In the given figure, O is the centre of the circle and BCD is tangent to it at C. Prove that ∠ BAC + ∠ ACD = 90°.

</td><td></td></tr>
<tr><td>Sol.</td><td>In Δ OAC,
OA = OC
⇒ ∠ OCA = ∠ OAC
Now, ∠ OCD = 90°
⇒∠ OCA + ∠ ACD = 90°
⇒∠ OAC + ∠ ACD = 90°
or ∠ BAC + ∠ ACD = 90°</td><td>

1
1
½
½</td></tr>
<tr><td></td><td>OR</td><td></td></tr>
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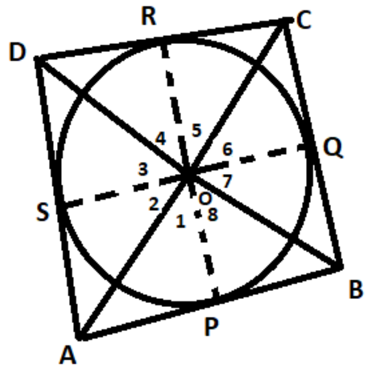
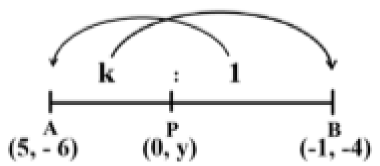
30 (b)	Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.	
Sol.	 Correct Figure Δ OAP $\cong$ Δ OAS $\therefore \angle 1 = \angle 2$ Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$ Also, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ $\Rightarrow 2 (\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360°$ $\Rightarrow \angle AOB + \angle COD = 180°$ Similarly, $\angle BOC + \angle AOD = 180°$	 ½ 1 ½ ½ ½
31.	Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (– 1, – 4). Also find the point of intersection.	
Sol.	 Let the ratio be k:1 and point on y- axis be P(0, y) $0 = \frac{-k+5}{k+1}$ k = 5 Hence, ratio is 5:1 $y = \frac{-4(5)-6}{5+1} = \frac{-26}{6} = \frac{-13}{3}$ Coordinates of point of intersection are $P\left(0, -\frac{13}{3}\right)$	 ½ 1 ½ ½ ½

	SECTION D This section has **4** Long Answer (LA) type questions carrying **5** marks each.	
32 (a)	The perimeter of a right triangle is 60 cm and its hypotenuse is 25 cm. Find the lengths of other two sides of the triangle.	
Sol.	Let the other two sides be x cm and y cm ATQ $x + y + 25 = 60$ $y = 35 - x$ Now, $x^2 + y^2 = (25)^2$ $x^2 + (35 - x)^2 = 625$ $x^2 - 35x + 300 = 0$ $(x - 20)(x - 15) = 0$ $\Rightarrow x = 20, 15$ $\left.\begin{array}{l} x = 20 \Rightarrow y = 15 \\ x = 15 \Rightarrow y = 20 \end{array}\right\}$ Hence sides are 15 cm and 20 cm.	 **1** **½** **1** **1** **1** **½**
	OR	
32 (b)	A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the speed of the train.	
Sol.	Let the speed of train be x km/h Reduced speed of train = (x – 8) km/h ATQ $\frac{480}{x-8} - \frac{480}{x} = 3$ $x^2 - 8x - 1280 = 0$ $(x - 40)(x + 32) = 0$ $\Rightarrow x = 40$ $\therefore$ Speed of train = 40 km/h	 **½** **1½** **1½** **1** **½**
33.	A bag contains some red and blue balls. Ten percent of the red balls, when added to twenty percent of the blue balls, give a total of 24. If three times the number of red balls exceeds the number of blue balls by 20, find the number of red and blue balls.	
Sol.	Let number of red balls be x & number of blue balls be y A.T.Q. $\frac{10x}{100} + \frac{20y}{100} = 24$ or $x + 2y = 240$ …..(i) Also, $3x - y = 20$ ……(ii) Solving (i) and (ii) , we get x = 40, y = 100 $\therefore$ Number of red balls = 40 and Number of blue balls = 100	 **1½** **1½** **1+1**

34. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table :

Length (in mm)	*Number of Leaves*
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Find the median length of the leaves.

Sol.

Length (mm)	f_i	*cf*
117.5 – 126.5	3	3
126.5 – 135.5	5	8
135.5 – 144.5	9	17
144.5 – 153.5	12	29
153.5 – 162.5	5	34
162.5 – 171.5	4	38
171.5 – 180.5	2	40

Correct Table **2**

Median class = 144.5 – 153.5 **1**

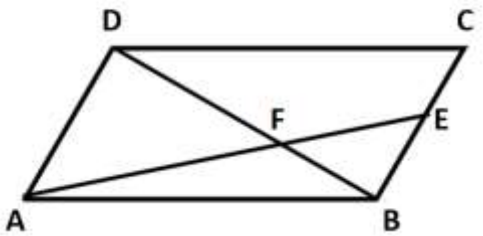
Median = $144.5 + \frac{20-17}{12} \times 9$ **1½**

= 146.75 **½**

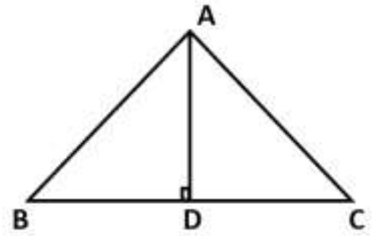
Hence, median length is 146.75 mm

35 (a) The diagonal BD of a parallelogram ABCD intersects the line segment AE at the point F, where E is any point on the side BC. Prove that DF × EF = FB × FA.

Sol.



Correct figure **1**

	In Δ ADF and Δ EBF, ∠ DFA = ∠ EFB ∠ ADF = ∠ FBE ∴ Δ ADF ~ Δ EBF ∴ $\frac{DF}{FB} = \frac{FA}{EF}$ ⇒ DF ×EF = FB ×FA	**2** **1** **1**
	OR	
35 (b)	In Δ ABC, if AD ⊥ BC and $AD^2 = BD \times DC$, then prove that ∠ BAC = 90°.	
Sol.	 Correct figure $AD^2 = BD \times DC$ $\frac{AD}{DC} = \frac{BD}{AD}$ Also, ∠ ADB = ∠ ADC ∴ Δ DBA ~ Δ DAC ∠ DBA = ∠ DAC ∠ BAD = ∠ DCA Adding both ∠ DBA + ∠ DCA = ∠ DAC + ∠ BAD ∴ ∠ BAC = 90°	**1** **1** **1** **1** **1**

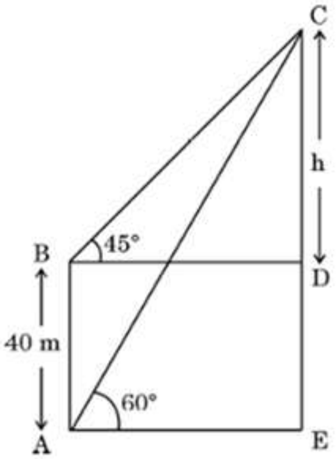
SECTION E

This section has **3** case study based carrying **4** marks each.

36.

Case Study – 1

Amrita stood near the base of a lighthouse, gazing up at its towering height. She measured the angle of elevation to the top and found it to be 60°. Then, she climbed a nearby observation deck, 40 metres higher than her original position and noticed the angle of elevation to the top of lighthouse to be 45°.



Based on the above given information, answer the following questions :

(i) **If CD is h metres, find the distance BD in terms of 'h'.**

(ii) **Find distance BC in terms of 'h'.**

(iii) (a) Find the height CE of the lighthouse [Use $\sqrt{3}$ = 1·73]

OR

(iii) (b) Find distance AE, if AC = 100 m.

Sol.

(i) $\frac{h}{BD} = \tan 45° = 1$ ½

BD = h m ½

(ii) $\frac{h}{BC} = \sin 45° = \frac{1}{\sqrt{2}}$ ½

BC = $\sqrt{2}$h m ½

(iii)(a) $\tan 60° = \frac{EC}{AE}$

$\sqrt{3} = \frac{h+40}{h}$

h $= 20(\sqrt{3}+1) = 20 \times 2.73 = 54.6\,m$ **1**

CE = 54.6 + 40 = 94.6 m ½

OR ½

(iii)(b) $\cos 60° = \frac{AE}{AC}$

$\frac{1}{2} = \frac{AE}{100}$ **1**

AE = 50 m **1**

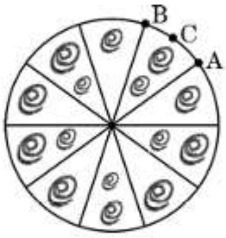
37.	**Case Study – 2** A school is organizing a charity run to raise funds for a local hospital. The run is planned as a series of rounds around a track, with each round being 300 metres. To make the event more challenging and engaging, the organizers decide to increase the distance of each subsequent round by 50 metres. For example, the second round will be 350 metres, the third round will be 400 metres and so on. The total number of rounds planned is 10.  Based on the information given above, answer the following questions : (i) Write the fourth, fifth and sixth term of the Arithmetic Progression so formed. (ii) Determine the distance of the 8^{th} round. (iii) (a) Find the total distance run after completing all 10 rounds. **OR** (iii) (b) If a runner completes only the first 6 rounds, what is the total distance run by the runner ?	
Sol.	A.P formed is 300, 350, 400…… (i) $a_4 = 450$ $a_5 = 500$ $a_6 = 550$ (ii) $a_8 = 300 + 7 \times 50$ $= 650$ m (iii) (a) $S_{10} = \frac{10}{2} \times (2 \times 300 + 9 \times 50)$ $= 5250$ m **OR** (iii) (b) $S_6 = \frac{6}{2} \times (2 \times 300 + 5 \times 50)$ $= 2250$ m	 **1** ½ ½ **1** **1** **1** **1**

38.

Case Study – 3

A brooch is a decorative piece often worn on clothing like jackets, blouses or dresses to add elegance. Made from precious metals and decorated with gemstones, brooches come in many shapes and designs.

One such brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure.



Based on the above given information, answer the following questions :

(i) Find the central angle of each sector.

(ii) Find the length of the arc ACB.

(iii) (a) Find the area of each sector of the brooch.

OR

(iii) (b) Find the total length of the silver wire used.

Sol.

(i) central angle = $\frac{360^0}{10}$ = 36° **1**

(ii) length of arc ACB = $\frac{1}{10} \times 2 \times \frac{22}{7} \times \frac{35}{2}$ = 11mm **1**

(iii)(a) Area of each sector of the brooch = $\frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$ **1**

= $\frac{385}{4}$ mm^2 or 96.25 mm^2 **1**

OR

(iii) (b) length of silver wire used = $2 \times \frac{22}{7} \times \frac{35}{2} + 5 \times 35$ **1**

= 285 mm **1**