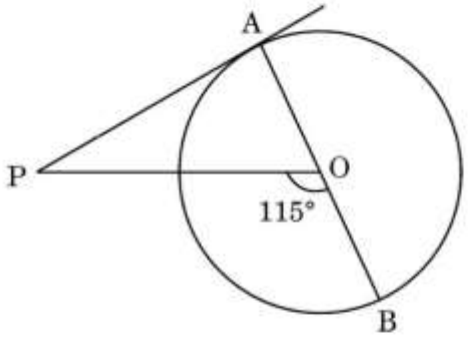
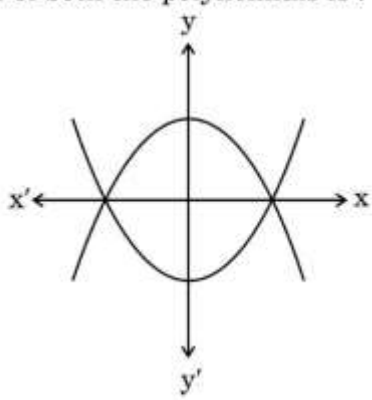
SOLUTIONS
MATHEMATICS (Subject Code–041) (PAPER CODE: 30/1/2)

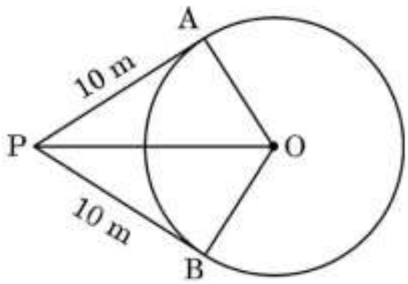
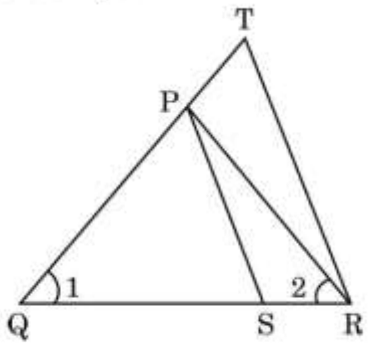
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A This section has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.	
1.	In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle$ POB = 115°, then $\angle$ APO is equal to :  (A) 25° (B) 65° (C) 90° (D) 35°	
Sol.	(A) 25°	**1**
2.	A piece of wire 20 cm long is bent into the form of an arc of a circle of radius $\frac{60}{\pi}$ cm. The angle subtended by the arc at the centre of the circle is : (A) 30° (B) 60° (C) 90° (D) 50°	
Sol.	(B) 60°	**1**
3.	Three numbers in AP have the sum 30. What is its middle term ? (A) 4 (B) 10 (C) 16 (D) 8	
Sol.	(B) 10	**1**

4.	An arc of a circle is of length 5π cm and the sector it bounds has an area of 20π cm^2. Its radius is : (A) 10 cm (B) 1 cm (C) 5 cm (D) 8 cm	
Sol.	(D) 8 cm	**1**
5.	If x = 1 and y = 2 is a solution of the pair of linear equations $2x - 3y + a = 0$ and $2x + 3y - b = 0$, then : (A) $a = 2b$ (B) $2a = b$ (C) $a + 2b = 0$ (D) $2a + b = 0$	
Sol.	(B) $2a = b$	**1**
6.	Two polynomials are shown in the graph below. The number of distinct zeroes of both the polynomials is :  (A) 3 (B) 5 (C) 2 (D) 4	
Sol.	(C) 2	**1**
7.	If $\alpha + \beta = 90°$ and $\alpha = 2\beta$, then $\cos^2 \alpha + \sin^2 \beta$ is equal to : (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2	
Sol.	(B) $\frac{1}{2}$	**1**

8.	A card is selected at random from a deck of 52 playing cards. The probability of it being a red face card is : (A) $\frac{3}{13}$ (B) $\frac{2}{13}$ (C) $\frac{1}{2}$ (D) $\frac{3}{26}$	
Sol.	(D) $\frac{3}{26}$	**1**
9.	If α and β are the zeroes of polynomial $3x^2 + 6x + k$ such that $\alpha + \beta + \alpha\beta = -\frac{2}{3}$, then the value of k is : (A) -8 (B) 8 (C) -4 (D) 4	
Sol.	(D) 4	**1**
10.	The value of $\tan^2\theta - \left(\frac{1}{\cos\theta} \times \sec\theta\right)$ is : (A) 1 (B) 0 (C) -1 (D) 2	
Sol.	(C) -1	**1**
11.	Which of the following is a rational number between $\sqrt{3}$ and $\sqrt{5}$? (A) 1·4142387954012 .... (B) $2{\cdot}32\overline{6}$ (C) π (D) 1·857142	
Sol.	(D) 1.857142	**1**
12.	If HCF(98, 28) = m and LCM(98, 28) = n, then the value of $n - 7m$ is : (A) 0 (B) 28 (C) 98 (D) 198	
Sol.	(C) 98	**1**

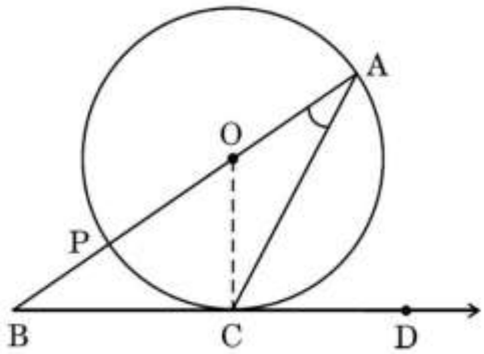
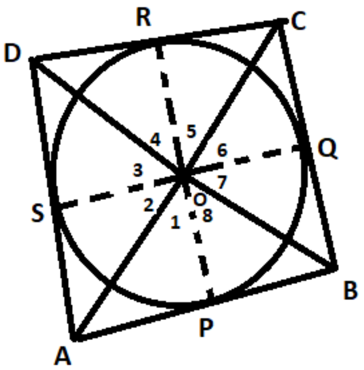
13.	If the length of a chord of a circle is equal to its radius, then the angle subtended by chord at the centre is : (A) 60° (B) 30° (C) 120° (D) 90°	
Sol.	(A) 60°	**1**
14.	The greatest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is : (A) 13 (B) 65 (C) 875 (D) 1750	
Sol.	(A) 13	**1**
15.	A ladder 14 m long leans against a wall. If the foot of the ladder is 7 m from the wall, then the angle of elevation of the top of the wall is : (A) 15° (B) 30° (C) 45° (D) 60°	
Sol.	(D) 60°	**1**
16.	In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE. Then, the two triangles are : (A) congruent but not similar (B) congruent as well as similar (C) neither congruent nor similar (D) similar but not congruent	
Sol.	(D) similar but not congruent	**1**
17.	The mid-point of the line segment joining the points P(– 4, 5) and Q(4, 6) lies on : (A) x-axis (B) y-axis (C) origin (D) neither x-axis nor y-axis	
Sol.	(B) y – axis	**1**

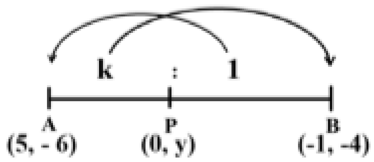
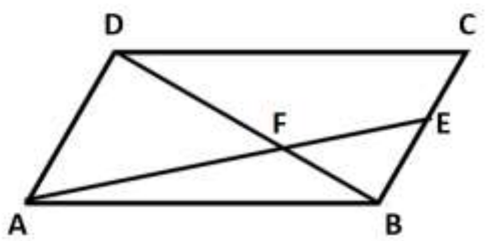
18.	Mode and Mean of a data are 15x and 18x, respectively. Then the median of the data is : (A) x (B) 11x (C) 17x (D) 34x	
Sol.	(C) 17x	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* If we join two hemispheres of same radius along their bases, then we get a sphere. *Reason (R):* Total Surface Area of a sphere of radius r is $3\pi r^2$.	
Sol.	(C) Assertion (A) is true, but Reason (R) is false.	**1**
20.	*Assertion (A) :* The probability of selecting a number at random from the numbers 1 to 20 is 1. *Reason (R):* For any event E, if P(E) = 1, then E is called a sure event.	
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
	SECTION B This section has **5** Very Short Answer (VSA) type questions carrying **2** marks each.	
21.	If the zeroes of the polynomial $x^2 + ax + b$ are in the ratio 3 : 4, then prove that $12a^2 = 49b$.	
Sol.	Let the zeroes are 3α and 4α $3\alpha + 4\alpha = -a$ $\Rightarrow 7\alpha = -a$ Also, $12\alpha^2 = b$ $LHS = 12a^2 = 12(-7\alpha)^2 = 49 \times 12(\alpha)^2 = 49b$ = RHS	 ½ ½ **1**

22.	A person is standing at P outside a circular ground at a distance of 26 m from the centre of the ground. He found that his distances from the points A and B on the ground are 10 m (PA and PB are tangents to the circle). Find the radius of the circular ground. 	
Sol.	$\angle$ OAP = 90° In right Δ OAP, $(26)^2 = OA^2 + (10)^2$ $\Rightarrow OA = \sqrt{576} = 24$ $\therefore$ radius = 24 m	½ 1 ½
23 (a)	If Δ ABC ~ Δ PQR in which AB = 6 cm, BC = 4 cm, AC = 8 cm and PR = 6 cm, then find the length of (PQ + QR).	
Sol.	$\frac{6}{PQ} = \frac{4}{QR} = \frac{8}{6}$ $\Rightarrow PQ = \frac{9}{2}$ cm or 4.5 cm and QR = 3 cm $\therefore$ PQ + QR = 7.5cm	½ ½ ½ ½
	OR	
23 (b)	In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$, show that $\Delta PQS \sim \Delta TQR$. 	
Sol.	In Δ PQR, $\angle 1 = \angle 2$ $\therefore$ PR = PQ $\therefore \frac{QR}{QS} = \frac{QT}{PR} \Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$ Also, $\angle 1 = \angle 1$ $\therefore \Delta PQS \sim \Delta TQR$	½ ½ ½ ½

24 (a)	If $x \cos 60° + y \cos 0° + \sin 30° - \cot 45° = 5$, then find the value of $x + 2y$.	
Sol.	$x\left(\frac{1}{2}\right) + y\,(1) + \frac{1}{2} - 1 = 5$ $\Rightarrow \quad x + 2y = 11$	**1½** ½
	OR	
24 (b)	Evaluate : $\dfrac{\tan^2 60°}{\sin^2 60° + \cos^2 30°}$	
Sol.	$\dfrac{(\sqrt{3})^2}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$ $= \ 2$	**1½** ½
25.	The coordinates of the centre of a circle are $(2a, a - 7)$. Find the value(s) of 'a' if the circle passes through the point $(11, -9)$ and has diameter $10\sqrt{2}$ units.	
Sol.	radius = $5\sqrt{2}$ units $(2a - 11)^2 + (a - 7 + 9)^2 = 50$ $\Rightarrow a^2 - 8a + 15 = 0$ $\Rightarrow (a - 5)(a - 3) = 0$ $\Rightarrow a = 5, 3$	½ ½ ½ ½
	SECTION C This section has **6** Short Answer (SA) type questions carrying **3** marks each.	
26.	If the radii of the bases of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3, find the ratio of their volumes.	
Sol.	Let radius of cylinder = r_1 & height of cylinder = h_1 Let radius of cone = r_2 & height of cone = h_2 $\frac{r_1}{r_2} = \frac{3}{4}, \quad \frac{h_1}{h_2} = \frac{2}{3}$ $\frac{volume\ of\ cylinder}{volume\ of\ cone} = \frac{\pi {r_1}^2 h_1}{\frac{1}{3}\pi {r_2}^2 h_2} = 3 \times \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right)$ $= 3 \times \left(\frac{3}{4}\right)^2 \times \left(\frac{2}{3}\right)$ $= \frac{9}{8}$ Hence, required ratio is 9:8	**1** **1** **1**

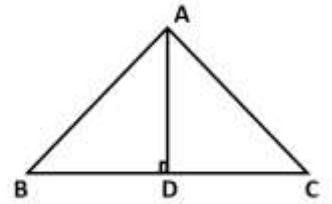
27.	Three sets of Physics, Chemistry and Mathematics books have to be stacked in such a way that all the books are stored subject-wise and the height of each stack is the same. The number of Physics books is 144, the number of Chemistry books is 180 and the number of Mathematics books is 192. Assuming that the books are of same thickness, determine the number of stacks of Physics, Chemistry and Mathematics books.	
Sol.	$144 = 2^4 \times 3^2$ $180 = 2^2 \times 3^2 \times 5$ $192 = 2^6 \times 3$ HCF $= 2^2 \times 3 = 12$ Number of stacks of Physics Books = $\frac{144}{12} = 12$ Number of stacks of Chemistry Books = $\frac{180}{12} = 15$ Number of stacks of Mathematics Books = $\frac{192}{12} = 16$	**1½** ½ **1**
28	Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.	
Sol.	Total outcomes = 36 Number of Outcomes with difference of the numbers on the two dice is 2 = 8 (1,3) (3,1) (4,2) (2,4) (5,3) (3,5) (4,6) (6,4) P (difference of the numbers on the two dice is 2) = $\frac{8}{36}$ or $\frac{2}{9}$	**1** **1** **1**
29 (a)	Prove that : $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$	
Sol.	LHS = $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ $= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$ $= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta\,(\sin\theta-\cos\theta)}$ $= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$ $= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$ $= \frac{(1+\sin\theta\cos\theta)}{\sin\theta\cos\theta}$ $= 1+\sec\theta\,\text{cosec}\,\theta$ = RHS	½ **1** ½ ½ ½
	OR	
29 (b)	Prove that : $\frac{\sin A+\cos A}{\sin A-\cos A} + \frac{\sin A-\cos A}{\sin A+\cos A} = \frac{2}{2\sin^2 A-1}$	
Sol.	LHS = $\frac{\sin A+\cos A}{\sin A-\cos A} + \frac{\sin A-\cos A}{\sin A+\cos A}$	

	$= \dfrac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$ $= \dfrac{\sin^2 A + \cos^2 A + 2\sin A\cos A + \sin^2 A + \cos^2 A - 2\sin A\cos A}{\sin^2 A - \cos^2 A}$ $= \dfrac{1+1}{\sin^2 A - (1 - \sin^2 A)}$ $= \dfrac{2}{2\sin^2 A - 1}$ = RHS	**1** **1** ½ ½
30 (a)	In the given figure, O is the centre of the circle and BCD is tangent to it at C. Prove that ∠ BAC + ∠ ACD = 90°.  	
Sol.	In Δ OAC, OA = OC ⇒ ∠ OCA = ∠ OAC Now, ∠ OCD = 90° ⇒∠ OCA + ∠ ACD = 90° ⇒∠ OAC + ∠ ACD = 90° or ∠ BAC + ∠ ACD = 90°	**1** **1** ½ ½
	OR	
30 (b)	Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.	
Sol.	 Correct Figure Δ OAP ≅ Δ OAS ∴ ∠ 1 = ∠ 2 Similarly, ∠ 3 = ∠ 4 , ∠ 5 = ∠ 6 , ∠ 7 = ∠ 8 Also, ∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 + ∠ 5 + ∠ 6 + ∠ 7 + ∠ 8 = 360°	½ **1** ½

	$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360°$ $\Rightarrow \angle AOB + \angle COD = 180°$ Similarly, $\angle BOC + \angle AOD = 180°$	½ ½
31.	Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (– 1, – 4). Also find the point of intersection.	
Sol.	 Let the ratio be k:1 and point on y- axis be P(0, y) $0 = \frac{-k+5}{k+1}$ k = 5 Hence, ratio is 5:1 $y = \frac{-4(5)-6}{5+1} = \frac{-26}{6} = \frac{-13}{3}$ Coordinates of point of intersection are $P\left(0, -\frac{13}{3}\right)$	½ 1 ½ ½ ½
	SECTION D This section has **4** Long Answer (LA) type questions carrying **5** marks each.	
32 (a)	The diagonal BD of a parallelogram ABCD intersects the line segment AE at the point F, where E is any point on the side BC. Prove that DF × EF = FB × FA.	
Sol.	 Correct figure In Δ ADF and Δ EBF, $\angle DFA = \angle EFB$ $\angle ADF = \angle FBE$ $\therefore \Delta ADF \sim \Delta EBF$ $\therefore \frac{DF}{FB} = \frac{FA}{EF}$ $\Rightarrow DF \times EF = FB \times FA$	1 2 1 1
	OR	

32 (b) In ΔABC, if $AD \perp BC$ and $AD^2 = BD \times DC$, then prove that $\angle BAC = 90°$.

Sol.



Correct figure — **1**

$AD^2 = BD \times DC$

$\frac{AD}{DC} = \frac{BD}{AD}$ — **1**

Also, $\angle ADB = \angle ADC$

$\therefore \Delta DBA \sim \Delta DAC$ — **1**

$\angle DBA = \angle DAC$

$\angle BAD = \angle DCA$ } — **1**

Adding both

$\angle DBA + \angle DCA = \angle DAC + \angle BAD$

$\therefore \quad \angle BAC = 90°$ — **1**

33. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the mean and mode of the data :

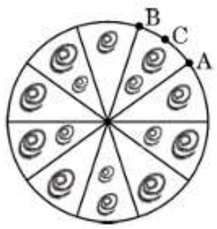
Monthly Consumption (in units)	*Number of Consumers*
65 – 85	4
85 – 105	5
105 – 125	13
125 – 145	20
145 – 165	14
165 – 185	8
185 – 205	4

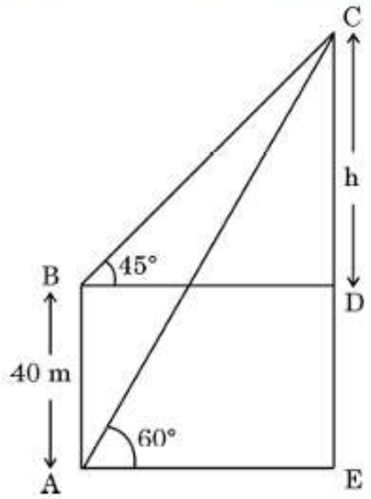
Sol.

Monthly Consumption (in units)	f_i	x_i	$u_i = \frac{x_i - 135}{h}$	$f_i u_i$
65-85	4	75	-3	-12
85-105	5	95	-2	-10
105-125	13	115	-1	-13
125-145	20	135=a	0	0
145-165	14	155	1	14
165-185	8	175	2	16
185-205	4	195	3	12
Total	68			7

Correct Table — **1½**

	Mean = $135 + \frac{7}{68} \times 20$ $= 137.06$ Modal Class is 125-145 Mode = $125 + \left(\frac{20-13}{40-13-14}\right) \times 20$ $= 135.77$ Hence, Mean = 137.06 units and Mode = 135.77 units	**1** ½ ½ **1** ½
34.	Vijay invested certain amounts of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. He received ₹ 1,860 as the total annual interest. However, had he interchanged the amounts of investments in the two schemes, he would have received ₹ 20 more as annual interest. How much money did he invest in each scheme ?	
Sol.	Let Vijay invested ₹ x at 8% rate of interest & ₹ y at 9% rate of interest ATQ, $\frac{8x}{100} + \frac{9y}{100} = 1860$ or $8x + 9y = 186000$ …………..(i) $\frac{9x}{100} + \frac{8y}{100} = 1880$ or $9x + 8y = 188000$ …………..(ii) On solving (i) and (ii), we get $x = 12000$ $y = 10000$ Hence, money invested in scheme A is ₹ 12000 and scheme B is ₹ 10000.	**1½** **1½** **1** **1**
35 (a)	A two-digit number is such that the product of its digits is 12. When 36 is added to this number, the digits interchange their places. Find the number.	
Sol.	Let unit digit be y and ten's digit = x hence, the two digit number = $10x + y$, ATQ $xy = 12$ ….. (i) $10x + y + 36 = 10y + x$ $x - y + 4 = 0$ ……. (ii) From (i) and (ii) $x^2 + 4x - 12 = 0$ $(x + 6)(x - 2) = 0$ Hence, $x = 2$ and $y = 6$ ∴ Number = 26	½ ½ **1** **1** ½ ½ ½ ½
	OR	

35 (b)	A student scored a total of 32 marks in class tests in Mathematics and Science. Had he scored 2 marks less in Science and 4 marks more in Mathematics, the product of his marks would have been 253. Find his marks in the two subjects.	
Sol.	Let marks scored in Mathematics be x and marks scored in Science be y ATQ, $x + y = 32$ …… (i) $and\ (x+4)(y-2) = 253$ ….. (ii) from (i) and (ii) $x^2 - 26x + 133 = 0$ $(x-19)(x-7) = 0$ $x = 19, 7$ $x = 19 \Rightarrow y = 13$ $x = 7 \Rightarrow y = 25$ Hence, marks in Mathematics and Science are 19, 13 or 7, 25	 **1** **1** **1** **1** ½ ½
	SECTION E This section has **3** case study based carrying **4** marks each.	
36.	**Case Study – 1** A brooch is a decorative piece often worn on clothing like jackets, blouses or dresses to add elegance. Made from precious metals and decorated with gemstones, brooches come in many shapes and designs.  One such brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure.  Based on the above given information, answer the following questions : (i) Find the central angle of each sector. (ii) Find the length of the arc ACB. (iii) (a) Find the area of each sector of the brooch. **OR** (iii) (b) Find the total length of the silver wire used.	
Sol.	(i) central angle = $\frac{360^0}{10}$ = 36°	**1**

	(ii) length of arc ACB = $\frac{1}{10} \times 2 \times \frac{22}{7} \times \frac{35}{2}$ = 11mm (iii)(a) Area of each sector of the brooch = $\frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$ $= \frac{385}{4}$ mm^2 or 96.25 mm^2 **OR** (iii) (b) length of silver wire used = $2 \times \frac{22}{7} \times \frac{35}{2} + 5 \times 35$ = 285 mm	**1** **1** **1** **1** **1**
37.	**Case Study – 2** Amrita stood near the base of a lighthouse, gazing up at its towering height. She measured the angle of elevation to the top and found it to be 60°. Then, she climbed a nearby observation deck, 40 metres higher than her original position and noticed the angle of elevation to the top of lighthouse to be 45°.   Based on the above given information, answer the following questions : (i) **If CD is h metres, find the distance BD in terms of 'h'.** (ii) **Find distance BC in terms of 'h'.** (iii) (a) Find the height CE of the lighthouse [Use $\sqrt{3}$ = 1·73] **OR** (iii) (b) Find distance AE, if AC = 100 m.	
Sol.	(i) $\frac{h}{BD}$ = tan 45° = 1 $\Rightarrow$BD = h m (ii) $\frac{h}{BC}$ = sin 45° = $\frac{1}{\sqrt{2}}$ $\Rightarrow$BC = $\sqrt{2}$h m	½ ½ ½ ½

	(iii)(a) $\tan 60° = \frac{EC}{AE}$ $\Rightarrow \sqrt{3} = \frac{h+40}{h}$ $\Rightarrow$ h = 20 ($\sqrt{3}$ + 1) = 20 × 2.73 = 54.6 m ∴ CE = 54.6 + 40 = 94.6 m **OR** (iii)(b) $\cos 60° = \frac{AE}{AC}$ $\Rightarrow \frac{1}{2} = \frac{AE}{100}$ ∴ AE = 50 m	**1** ½ ½ **1** **1**
38.	**Case Study – 3** A school is organizing a charity run to raise funds for a local hospital. The run is planned as a series of rounds around a track, with each round being 300 metres. To make the event more challenging and engaging, the organizers decide to increase the distance of each subsequent round by 50 metres. For example, the second round will be 350 metres, the third round will be 400 metres and so on. The total number of rounds planned is 10.  Based on the information given above, answer the following questions : (i) Write the fourth, fifth and sixth term of the Arithmetic Progression so formed. (ii) Determine the distance of the 8^{th} round. (iii) (a) Find the total distance run after completing all 10 rounds. **OR** (iii) (b) If a runner completes only the first 6 rounds, what is the total distance run by the runner ?	
Sol.	A.P formed is 300, 350, 400…… (i) $a_4 = 450$ $a_5 = 500$ $a_6 = 550$ (ii) $a_8 = 300 + 7 \times 50$ = 650 m	 **1** ½ ½

	(iii) (a) $S_{10} = \frac{10}{2} \times (2 \times 300 + 9 \times 50)$ $= 5250$ m **OR** (iii) (b) $S_6 = \frac{6}{2} \times (2 \times 300 + 5 \times 50)$ $= 2250$ m	**1** **1** **1** **1**