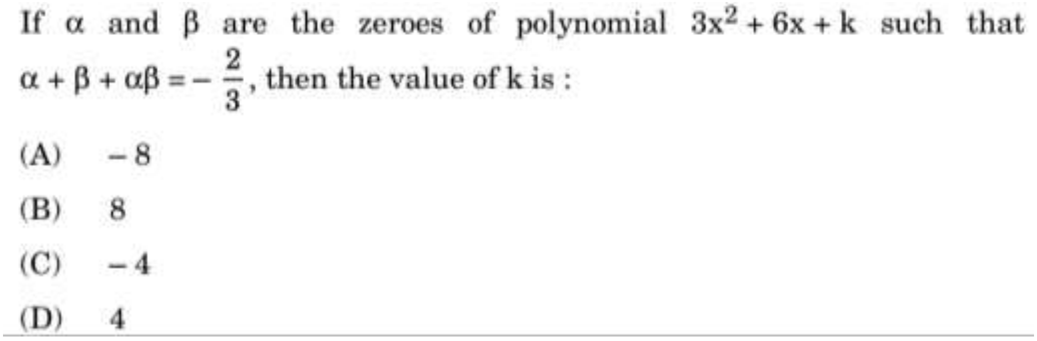
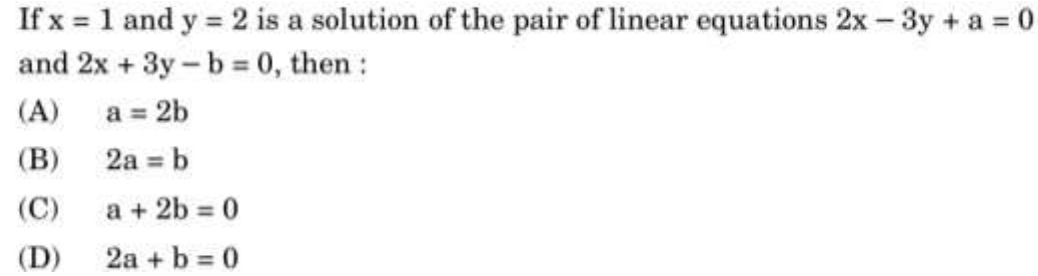
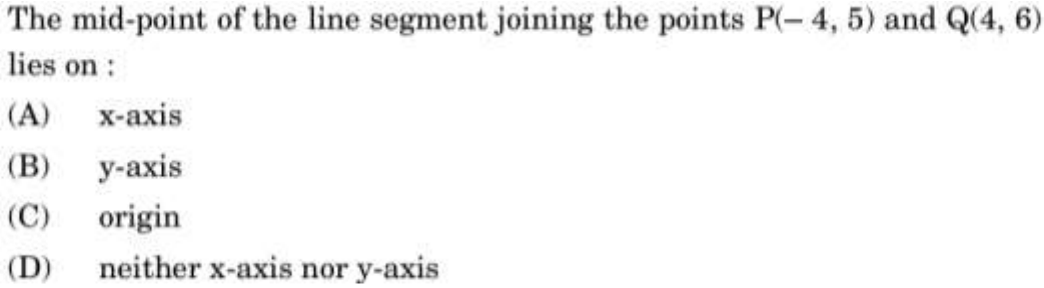
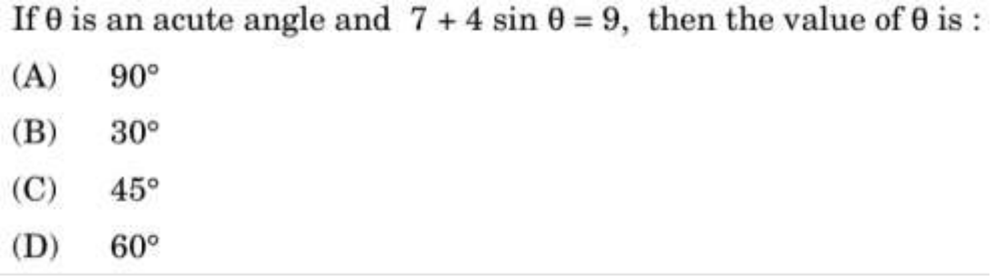
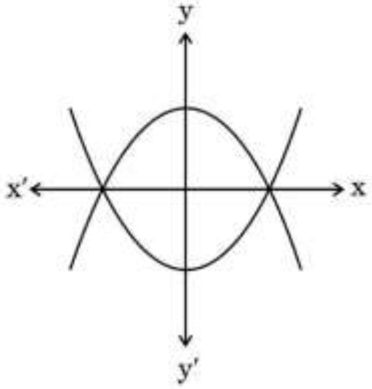
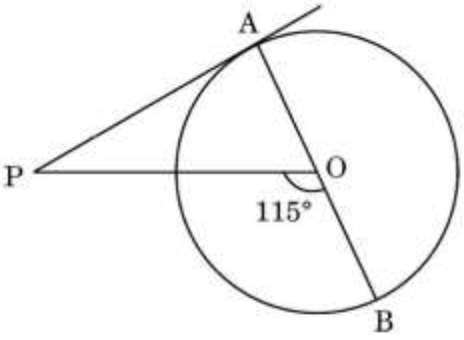
SOLUTIONS
MATHEMATICS (Subject Code– 041) (PAPER CODE: 30/1/1)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each.	
1.	 If α and β are the zeroes of polynomial $3x^2 + 6x + k$ such that $\alpha + \beta + \alpha\beta = -\frac{2}{3}$, then the value of k is : (A) -8 (B) 8 (C) -4 (D) 4	
Sol.	(D) 4	**1**
2.	 If x = 1 and y = 2 is a solution of the pair of linear equations $2x - 3y + a = 0$ and $2x + 3y - b = 0$, then : (A) $a = 2b$ (B) $2a = b$ (C) $a + 2b = 0$ (D) $2a + b = 0$	
Sol.	(B) 2a = b	**1**
3.	 The mid-point of the line segment joining the points P(– 4, 5) and Q(4, 6) lies on : (A) x-axis (B) y-axis (C) origin (D) neither x-axis nor y-axis	
Sol.	(B) y – axis	**1**
4.	 If θ is an acute angle and $7 + 4 \sin \theta = 9$, then the value of θ is : (A) 90° (B) 30° (C) 45° (D) 60°	
Sol.	(B) 30^0	**1**

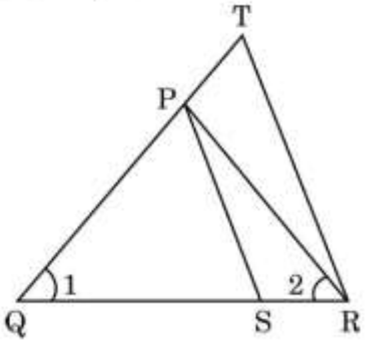
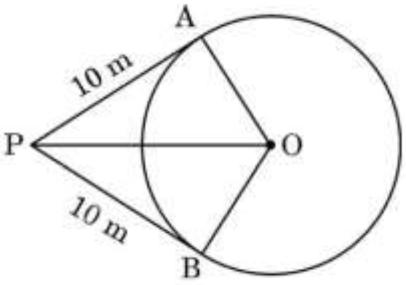
5.	The value of $\tan^2\theta - \left(\frac{1}{\cos\theta} \times \sec\theta\right)$ is : (A) 1 (B) 0 (C) -1 (D) 2	
Sol.	(C) -1	**1**
6.	If HCF(98, 28) = m and LCM(98, 28) = n, then the value of $n - 7m$ is : (A) 0 (B) 28 (C) 98 (D) 198	
Sol.	(C) 98	**1**
7.	The tangents drawn at the extremities of the diameter of a circle are always : (A) parallel (B) perpendicular (C) equal (D) intersecting	
Sol.	(A) parallel	**1**
8.	In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE. Then, the two triangles are : (A) congruent but not similar (B) congruent as well as similar (C) neither congruent nor similar (D) similar but not congruent	
Sol.	(D) similar but not congruent	**1**
9.	If $(-1)^n + (-1)^8 = 0$, then n is : (A) any positive integer (B) any negative integer (C) any odd number (D) any even number	
Sol.	(C) any odd number	**1**

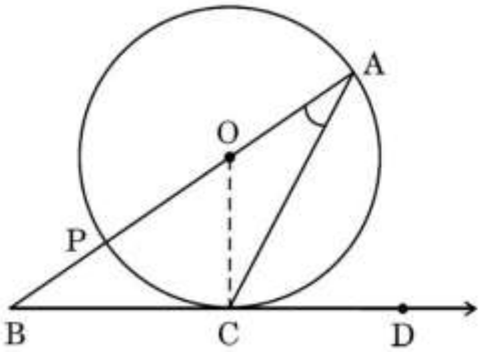
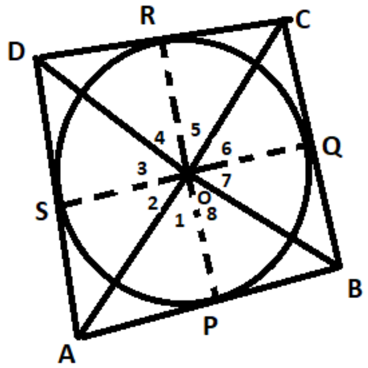
10.	Two polynomials are shown in the graph below. The number of distinct zeroes of both the polynomials is :  (A) 3 (B) 5 (C) 2 (D) 4	
Sol.	(C) 2	**1**
11.	If the sum of first m terms of an AP is $2m^2 + 3m$, then its second term is : (A) 10 (B) 9 (C) 12 (D) 4	
Sol.	(B) 9	**1**
12.	Mode and Mean of a data are 15x and 18x, respectively. Then the median of the data is : (A) x (B) 11x (C) 17x (D) 34x	
Sol.	(C) 17x	**1**
13.	A card is selected at random from a deck of 52 playing cards. The probability of it being a red face card is : (A) $\frac{3}{13}$ (B) $\frac{2}{13}$ (C) $\frac{1}{2}$ (D) $\frac{3}{26}$	
Sol.	(D) $\frac{3}{26}$	**1**

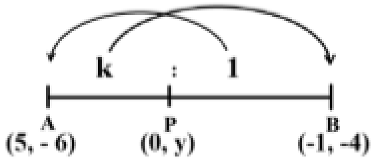
14.	Which of the following is a rational number between $\sqrt{3}$ and $\sqrt{5}$? (A) 1·4142387954012 …. (B) $2{\cdot}32\overline{6}$ (C) π (D) 1·857142	
Sol.	(D) 1.857142	**1**
15.	If a sector of a circle has an area of 40π sq. units and a central angle of 72°, the radius of the circle is : (A) 200 units (B) 100 units (C) 20 units (D) $10\sqrt{2}$ units	
Sol.	(D) $10\sqrt{2}$ units	**1**
16.	In the given figure, PA is a tangent from an external point P to a circle with centre O. If ∠ POB = 115°, then ∠ APO is equal to :  (A) 25° (B) 65° (C) 90° (D) 35°	
Sol.	(A) 25°	**1**
17.	A kite is flying at a height of 150 m from the ground. It is attached to a string inclined at an angle of 30° to the horizontal. The length of the string is : (A) $100\sqrt{3}$ m (B) 300 m (C) $150\sqrt{2}$ m (D) $150\sqrt{3}$ m	
Sol.	(B) 300 m	**1**

18.	A piece of wire 20 cm long is bent into the form of an arc of a circle of radius $\frac{60}{\pi}$ cm. The angle subtended by the arc at the centre of the circle is : (A) 30° (B) 60° (C) 90° (D) 50°	
Sol.	(B) 60°	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* The probability of selecting a number at random from the numbers 1 to 20 is 1. *Reason (R):* For any event E, if P(E) = 1, then E is called a sure event.	
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
20	*Assertion (A) :* If we join two hemispheres of same radius along their bases, then we get a sphere. *Reason (R):* Total Surface Area of a sphere of radius r is $3\pi r^2$.	
Sol.	(C) Assertion (A) is true, but Reason (R) is false.	**1**
	SECTION B This section has **5** Very Short Answer (VSA) type questions carrying **2** marks each.	
21.(a)	If $x \cos 60° + y \cos 0° + \sin 30° - \cot 45° = 5$, then find the value of $x + 2y$.	
Sol.	$x\left(\frac{1}{2}\right) + y\,(1) + \frac{1}{2} - 1 = 5$ $\Rightarrow \quad x + 2y = 11$	**1½** ½
	OR	

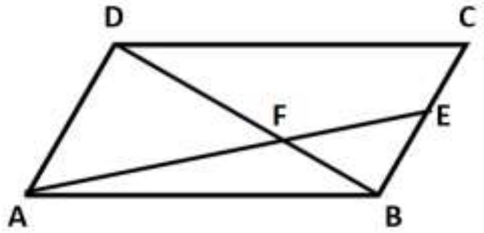
21. (b)	Evaluate : $\frac{\tan^2 60^\circ}{\sin^2 60^\circ + \cos^2 30^\circ}$	
Sol.	$\frac{(\sqrt{3})^2}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$ $= 2$	**1½** ½
22.	Find the zeroes of the polynomial $p(x) = x^2 + \frac{4}{3}x - \frac{4}{3}$.	
Sol.	$\frac{1}{3}(3x^2 + 4x - 4)$ $= \frac{1}{3}(3x^2 + 6x - 2x - 4)$ $= \frac{1}{3}(3x - 2)(x + 2)$ Zeroes are $\frac{2}{3}, -2$	½ ½ ½ ½
23.	The coordinates of the centre of a circle are $(2a, a - 7)$. Find the value(s) of 'a' if the circle passes through the point $(11, -9)$ and has diameter $10\sqrt{2}$ units.	
Sol.	radius $= 5\sqrt{2}$ units $(2a - 11)^2 + (a - 7 + 9)^2 = 50$ $\Rightarrow a^2 - 8a + 15 = 0$ $\Rightarrow (a - 5)(a - 3) = 0$ $\Rightarrow a = 5, 3$	½ ½ ½ ½
24.(a)	If Δ ABC ~ Δ PQR in which AB = 6 cm, BC = 4 cm, AC = 8 cm and PR = 6 cm, then find the length of (PQ + QR).	
Sol.	$\frac{6}{PQ} = \frac{4}{QR} = \frac{8}{6}$ $\Rightarrow PQ = \frac{9}{2}$ cm or 4.5 cm and QR = 3 cm ∴ PQ + QR = 7.5cm	½ ½ ½ ½
	OR	

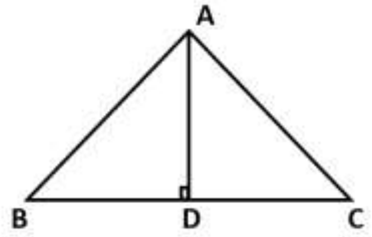
24.(b)	In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$, show that $\Delta PQS \sim \Delta TQR$.  	
Sol.	In Δ PQR, $\angle 1 = \angle 2$ $\therefore$ PR = PQ $\therefore \frac{QR}{QS} = \frac{QT}{PR} \Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$ Also, $\angle 1 = \angle 1$ $\therefore \Delta PQS \sim \Delta TQR$	½ ½ ½ ½
25.	A person is standing at P outside a circular ground at a distance of 26 m from the centre of the ground. He found that his distances from the points A and B on the ground are 10 m (PA and PB are tangents to the circle). Find the radius of the circular ground.  	
Sol.	$\angle OAP = 90°$ In right Δ OAP, $(26)^2 = OA^2 + (10)^2$ $\Rightarrow OA = \sqrt{576} = 24$ $\therefore$ radius = 24 m	½ **1** ½

	SECTION C This section has **6** Short Answer (SA) type questions carrying **3** marks each.	
26. (a)	In the given figure, O is the centre of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90^\circ$. 	
Sol.	In Δ OAC,	
	OA = OC	
	$\Rightarrow \angle OCA = \angle OAC$	**1**
	Now, $\angle OCD = 90^\circ$	**1**
	$\Rightarrow \angle OCA + \angle ACD = 90^\circ$	½
	$\Rightarrow \angle OAC + \angle ACD = 90^\circ$	½
	or $\angle BAC + \angle ACD = 90^\circ$	
	OR	
26.(b)	Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.	
Sol.		
	Correct Figure	½
	$\Delta OAP \cong \Delta OAS$	**1**
	$\therefore \angle 1 = \angle 2$	
	Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$	½
	Also, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$	
	$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^\circ$	½
	$\Rightarrow \angle AOB + \angle COD = 180^\circ$	½
	Similarly, $\angle BOC + \angle AOD = 180^\circ$	

27. (a)	Prove that : $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$	
Sol.	LHS = $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$ $=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$ $=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta\,(\sin\theta-\cos\theta)}$ $=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$ $=\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$ $=\frac{(1+\sin\theta\cos\theta)}{\sin\theta\cos\theta}$ = 1+ sec θ cosec θ = RHS	 ½ **1** ½ ½ ½
	OR	
27.(b)	Prove that : $\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}=\frac{2}{2\sin^2A-1}$	
Sol.	LHS = $\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}$ $=\frac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{(\sin A-\cos A)(\sin A+\cos A)}$ $=\frac{\sin^2A+\cos^2A+2\sin A\cos A+\sin^2A+\cos^2A-2\sin A\cos A}{\sin^2A-\cos^2A}$ $=\frac{1+1}{\sin^2A-(1-\sin^2A)}$ $=\frac{2}{2\sin^2A-1}$ = RHS	 **1** **1** ½ ½
28.	Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (– 1, – 4). Also find the point of intersection.	
Sol.	 Let the ratio be k:1 and point on y- axis be P(0, y) $0=\frac{-k+5}{k+1}$ k = 5 Hence, ratio is 5:1 $y=\frac{-4(5)-6}{5+1}=\frac{-26}{6}=\frac{-13}{3}$ Coordinates of point of intersection are $P\left(0,-\frac{13}{3}\right)$	 ½ **1** ½ ½ ½

29.	Prove that $\frac{1}{\sqrt{5}}$ is an irrational number.	
Sol.	Let $\frac{1}{\sqrt{5}}$ be a rational number.	
	$\therefore \frac{1}{\sqrt{5}} = \frac{p}{q}$, where $q \neq 0$ and let p & q be the co-primes.	½
	$5p^2 = q^2 \Rightarrow q^2$ is divisible by 5.	
	$\Rightarrow$ q is divisible by 5. ----- (i)	1
	let q = 5a, where 'a' is some integer.	
	$25a^2 = 5p^2 \Rightarrow p^2 = 5a^2 \Rightarrow p^2$ is divisible by 5.	
	$\Rightarrow$ p is divisible by 5 ----- (ii)	1
	(i) and (ii) leads to contradiction as p and q are coprimes.	½
	$\therefore \frac{1}{\sqrt{5}}$ is an irrational number	
30.	A room is in the form of a cylinder surmounted by a hemispherical dome. The base radius of the hemisphere is half of the height of the cylindrical part. If the room contains $\frac{1408}{21}$ m^3 of air, find the height of the cylindrical part. (Use $\pi = \frac{22}{7}$).	
Sol.	Let r is the radius of hemisphere and cylinder and h is the height of cylinder	
	h = 2r	½
	Volume of air in room $= \frac{2}{3}\pi r^3 + \pi r^2 h$	
	$\frac{1408}{21} = \frac{2}{3}\pi r^3 + \pi r^2(2r)$	½
	$\frac{1408}{21} = \frac{8}{3} \times \frac{22}{7} \times r^3$	
	$r^3 = 8$	**1**
	$\therefore$ r = 2 m	½
	and h = 4 m	½
31.	Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.	
Sol.	Total outcomes = 36	**1**
	Number of Outcomes with difference of the numbers on the two dice is 2 = 8	**1**
	(1,3) (3,1) (4,2) (2,4) (5,3) (3,5) (4,6) (6,4)	
	P (difference of the numbers on the two dice is 2) = $\frac{8}{36}$ or $\frac{2}{9}$	**1**

	SECTION D This section has **4** Long Answer (LA) type questions carrying **5** marks each.	
32.	Vijay invested certain amounts of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. He received ₹ 1,860 as the total annual interest. However, had he interchanged the amounts of investments in the two schemes, he would have received ₹ 20 more as annual interest. How much money did he invest in each scheme ?	
Sol.	Let Vijay invested ₹ x at 8% rate of interest & ₹ y at 9% rate of interest	
	ATQ,	
	$\frac{8x}{100} + \frac{9y}{100} = 1860$	**1½**
	or $8x + 9y = 186000$ …………..(i)	
	$\frac{9x}{100} + \frac{8y}{100} = 1880$	**1½**
	or $9x + 8y = 188000$ …………..(ii)	
	On solving (i) and (ii), we get	
	$x = 12000$	**1**
	$y = 10000$	**1**
	Hence, money invested in scheme A is ₹ 12000 and scheme B is ₹ 10000.	
33.(a)	The diagonal BD of a parallelogram ABCD intersects the line segment AE at the point F, where E is any point on the side BC. Prove that DF × EF = FB × FA.	
Sol.		
	Correct figure	**1**
	In Δ ADF and Δ EBF,	
	$\angle$ DFA = $\angle$ EFB	
	$\angle$ ADF = $\angle$ FBE	
	$\therefore$ Δ ADF ~ Δ EBF	**2**
	$\therefore \frac{DF}{FB} = \frac{FA}{EF}$	**1**
	$\Rightarrow$ DF ×EF = FB ×FA	**1**
	OR	

33.(b)	In Δ ABC, if AD ⊥ BC and $AD^2 = BD \times DC$, then prove that ∠ BAC = 90°.	
Sol.	 Correct figure $AD^2 = BD \times DC$ $\frac{AD}{DC} = \frac{BD}{AD}$ Also, ∠ ADB = ∠ ADC ∴ Δ DBA ~ Δ DAC ∠ DBA = ∠ DAC ∠ BAD = ∠ DCA Adding both ∠ DBA + ∠ DCA = ∠ DAC + ∠ BAD ∴ ∠ BAC = 90°	**1** **1** **1** **1** **1**
34.(a)	The perimeter of a right triangle is 60 cm and its hypotenuse is 25 cm. Find the lengths of other two sides of the triangle.	
Sol.	Let the other two sides be x cm and y cm ATQ $x + y + 25 = 60$ $y = 35 - x$ Now, $x^2 + y^2 = (25)^2$ $x^2 + (35 - x)^2 = 625$ $x^2 - 35x + 300 = 0$ $(x - 20)(x - 15) = 0$ $\Rightarrow x = 20, 15$ $x = 20 \Rightarrow y = 15$ $x = 15 \Rightarrow y = 20$ Hence sides are 15 cm and 20 cm.	**1** ½ **1** **1** **1** ½
	OR	
34.(b)	A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the speed of the train.	
Sol.	Let the speed of train be x km/h Reduced speed of train = (x – 8) km/h ATQ $\frac{480}{x - 8} - \frac{480}{x} = 3$	½ 1½

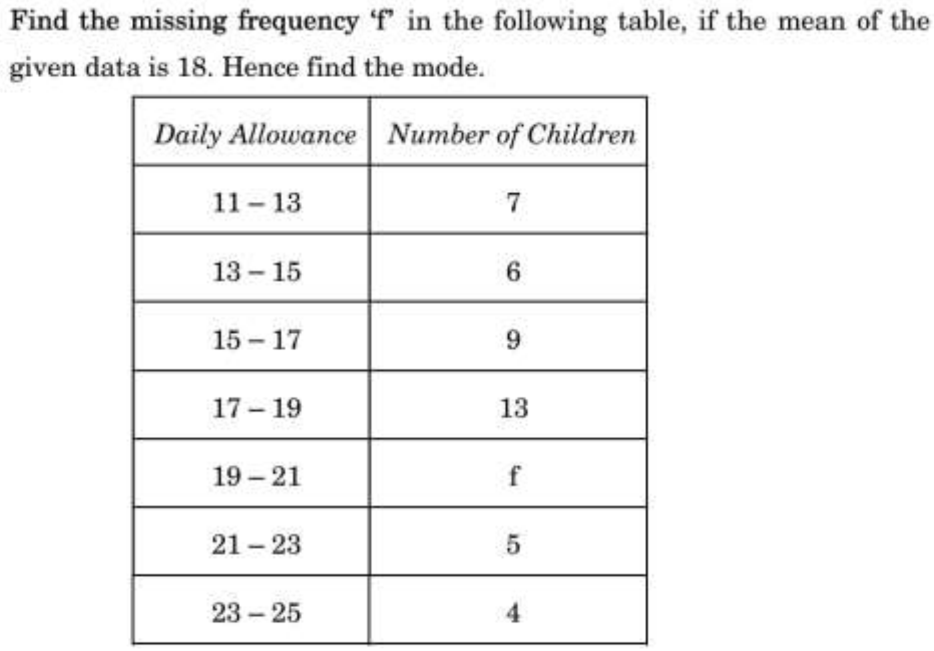
$x^2 - 8x - 1280 = 0$ **1½**

$(x - 40)(x + 32) = 0$ **1**

$\Rightarrow x = 40$ **½**

$\therefore$ Speed of train = 40 km/h

35.



Find the missing frequency 'f' in the following table, if the mean of the given data is 18. Hence find the mode.

Daily Allowance	*Number of Children*
11 – 13	7
13 – 15	6
15 – 17	9
17 – 19	13
19 – 21	f
21 – 23	5
23 – 25	4

Sol.

Daily Allowance	x_i	f_i	$f_i x_i$
11 – 13	12	7	84
13 – 15	14	6	84
15 – 17	16	9	144
17 – 19	18	13	234
19 – 21	20	f	20f
21 – 23	22	5	110
23 – 25	24	4	96
Total		44 + f	752 + 20f

Correct table **1½**

Mean = $\frac{752+20f}{44+f}$ **1**

$\Rightarrow 18 = \frac{752+20f}{44+f}$

$\therefore$ f = 20 **½**

modal class is 19 – 21 **½**

mode = $19 + \frac{20-13}{40-13-5} \times 3$ **1**

= 19.95 approx. **½**

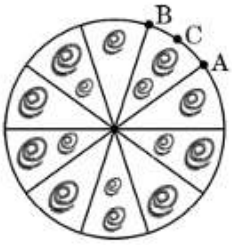
	SECTION E This section has **3** case study based carrying **4** marks each.	
36.	**Case Study – 1** A school is organizing a charity run to raise funds for a local hospital. The run is planned as a series of rounds around a track, with each round being 300 metres. To make the event more challenging and engaging, the organizers decide to increase the distance of each subsequent round by 50 metres. For example, the second round will be 350 metres, the third round will be 400 metres and so on. The total number of rounds planned is 10.  Based on the information given above, answer the following questions : (i) Write the fourth, fifth and sixth term of the Arithmetic Progression so formed. (ii) Determine the distance of the 8th round. (iii) (a) Find the total distance run after completing all 10 rounds. **OR** (iii) (b) If a runner completes only the first 6 rounds, what is the total distance run by the runner ?	
Sol.	A.P formed is 300, 350, 400……	
	(i) $a_4 = 450$	**1**
	$a_5 = 500$	
	$a_6 = 550$	
	(ii) $a_8 = 300 + 7 \times 50$	½
	$= 650$ m	½
	(iii) (a) $S_{10} = \frac{10}{2} \times (2 \times 300 + 9 \times 50)$	**1**
	$= 5250$ m	**1**
	OR	
	(iii) (b) $S_6 = \frac{6}{2} \times (2 \times 300 + 5 \times 50)$	**1**
	$= 2250$ m	**1**

37.

Case Study – 2

A brooch is a decorative piece often worn on clothing like jackets, blouses or dresses to add elegance. Made from precious metals and decorated with gemstones, brooches come in many shapes and designs.

One such brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure.



Based on the above given information, answer the following questions :

(i) Find the central angle of each sector.

(ii) Find the length of the arc ACB.

(iii) (a) Find the area of each sector of the brooch.

OR

(iii) (b) Find the total length of the silver wire used.

Sol.

(i) central angle = $\frac{360^0}{10}$ = 36° **1**

(ii) length of arc ACB = $\frac{1}{10} \times 2 \times \frac{22}{7} \times \frac{35}{2}$ = 11mm **1**

(iii)(a) Area of each sector of the brooch = $\frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$ **1**

= $\frac{385}{4}$ mm^2 or 96.25 mm^2 **1**

OR

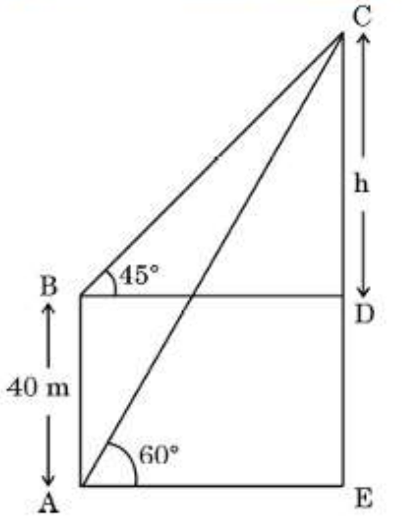
(iii) (b) length of silver wire used = $2 \times \frac{22}{7} \times \frac{35}{2} + 5 \times 35$ **1**

= 285 mm **1**

38.

Case Study – 3

Amrita stood near the base of a lighthouse, gazing up at its towering height. She measured the angle of elevation to the top and found it to be 60°. Then, she climbed a nearby observation deck, 40 metres higher than her original position and noticed the angle of elevation to the top of lighthouse to be 45°.



Based on the above given information, answer the following questions :

(i) **If CD is h metres, find the distance BD in terms of 'h'.**

(ii) **Find distance BC in terms of 'h'.**

(iii) (a) Find the height CE of the lighthouse [Use $\sqrt{3} = 1{\cdot}73$]

OR

(iii) (b) Find distance AE, if AC = 100 m.

Sol.

(i) $\frac{h}{BD} = \tan 45° = 1$ ½

$\Rightarrow BD = h$ m ½

(ii) $\frac{h}{BC} = \sin 45° = \frac{1}{\sqrt{2}}$ ½

$\Rightarrow BC = \sqrt{2}h$ m ½

(iii)(a) $\tan 60° = \frac{EC}{AE}$

$\Rightarrow \sqrt{3} = \frac{h+40}{h}$ **1**

$\Rightarrow h = 20\,(\sqrt{3} + 1) = 20 \times 2.73 = 54.6$ m ½

$\therefore$ CE = 54.6 + 40 = 94.6 m ½

OR

	(iii)(b) $\cos 60° = \frac{AE}{AC}$ $\Rightarrow \frac{1}{2} = \frac{AE}{100}$ $\therefore$ AE = 50 m	**1** **1**