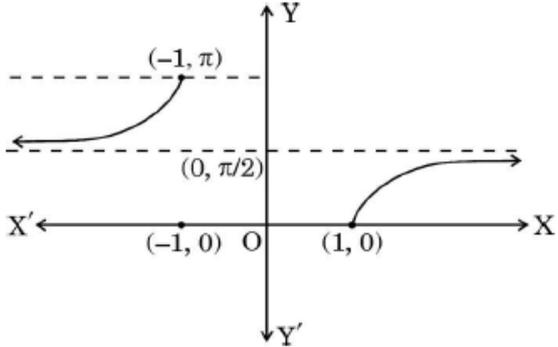


SOLUTIONS – 65/7/3

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
SECTION-A		
<i>This section comprises multiple choice questions (MCQs) of 1 mark each.</i>		
1.	<p>The given graph illustrates :</p>  <p>(A) $y = \sec^{-1} x$ (B) $y = \cot^{-1} x$ (C) $y = \tan^{-1} x$ (D) $y = \operatorname{cosec}^{-1} x$</p>	
Ans	(A) $\sec^{-1} x$	1
2.	<p>Let A be a square matrix of order 3. If $A = 5$, then $\operatorname{adj} A$ is :</p> <p>(A) 5 (B) 125 (C) 25 (D) -5</p>	
Ans	(C) 25	1
3	<p>If A and B are two square matrices each of order 3 with $A = 3$ and $B = 5$, then $2AB$ is :</p> <p>(A) 30 (B) 120 (C) 15 (D) 225</p>	
Ans	(B) 120	1
4.	<p>What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?</p> <p>(A) 9 (B) 512 (C) 615 (D) 64</p>	
Ans	(B) 512	1
5.	<p>Domain of $f(x) = \cos^{-1} x + \sin x$ is :</p> <p>(A) R (B) $(-1, 1)$ (C) $[-1, 1]$ (D) ϕ</p>	
Ans	(C) $[-1, 1]$	1

10.	Edge of a variable cube increases at the rate of 5 cm/s. The rate at which the surface area of the cube increases when the edge is 2 cm long is : (A) 24 cm ² /s (B) 120 cm ² /s (C) 12 cm ² /s (D) 5 cm ² /s	
Ans	(B) 120 cm ² /s	1
11.	$\int \frac{e^{9 \log x} - e^{8 \log x}}{e^{6 \log x} - e^{5 \log x}} dx$ is equal to : (A) $x + C$ (B) $\frac{x^2}{2} + C$ (C) $\frac{x^4}{4} + C$ (D) $\frac{x^3}{3} + C$	
Ans	(C) $\frac{x^4}{4} + C$	1
12.	If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = 2x - \sin x$, then f is : (A) a decreasing function (B) an increasing function (C) maximum at $x = \frac{\pi}{2}$ (D) maximum at $x = 0$	
Ans	(B) an increasing function	1
13.	A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is : (A) 6 (B) 1 (C) $\frac{1}{4}$ (D) 4	
Ans	(D) 4	1

14.	$\int \frac{e^{-x}}{16 + 9e^{-2x}} dx$ is equal to : (A) $\frac{16}{9} \tan^{-1}(e^{-x}) + C$ (B) $-\frac{1}{12} \tan^{-1}\left(\frac{3e^{-x}}{4}\right) + C$ (C) $\tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$ (D) $-\frac{1}{3} \tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$	
Ans	(B) $-\frac{1}{12} \tan^{-1}\left(\frac{3e^{-x}}{4}\right) + C$	1
15.	If $ \vec{a} + \vec{b} = \vec{a} - \vec{b} $ for any two vectors, then vectors \vec{a} and \vec{b} are : (A) orthogonal vectors (B) parallel to each other (C) unit vectors (D) collinear vectors	
Ans	(A) orthogonal vectors	1
16.	A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is : (A) $\frac{2}{13}$ (B) $\frac{3}{26}$ (C) $\frac{19}{26}$ (D) $\frac{3}{13}$	
Ans	(B) $\frac{3}{26}$	1
17.	If A and B are two events such that $P(B) = \frac{1}{5}$, $P(A B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{5}$, then P(A) is : (A) $\frac{10}{15}$ (B) $\frac{2}{15}$ (C) $\frac{1}{5}$ (D) $\frac{8}{15}$	

Ans	(D) $\frac{8}{15}$	1
18.	<p>For a function $f(x)$, which of the following holds true ?</p> <p>(A) $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$</p> <p>(B) $\int_{-a}^a f(x) dx = 0$, if f is an even function</p> <p>(C) $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$, if f is an odd function</p> <p>(D) $\int_0^{2a} f(x) dx = \int_0^a f(x) dx - \int_0^a f(2a + x) dx$</p>	
Ans	(A) $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$	1
	<p><i>Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.</i></p> <p>(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).</p> <p>(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).</p> <p>(C) Assertion (A) is true, but Reason (R) is false.</p> <p>(D) Assertion (A) is false, but Reason (R) is true.</p>	

19.	<p><i>Assertion (A)</i> : $f(x) = \begin{cases} x \sin \frac{1}{x} , & x \neq 0 \\ 0 & , x = 0 \end{cases}$ is continuous at $x = 0$.</p> <p><i>Reason (R)</i> : When $x \rightarrow 0$, $\sin \frac{1}{x}$ is a finite value between -1 and 1.</p>	
Ans	(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).	1
20.	<p><i>Assertion (A)</i> : Set of values of $\sec^{-1} \left(\frac{\sqrt{3}}{2} \right)$ is a null set.</p> <p><i>Reason (R)</i> : $\sec^{-1} x$ is defined for $x \in \mathbb{R} - (-1, 1)$.</p>	
Ans	(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).	1
SECTION-B <i>This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.</i>		
21.	<p>(a) Differentiate $\left(\frac{5^x}{x^5} \right)$ with respect to x.</p> <p style="text-align: center;">OR</p> <p>(b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.</p>	
Ans	(a) Let, $y = \frac{5^x}{x^5} = 5^x \cdot x^{-5} \Rightarrow \frac{dy}{dx} = (5^x)' \cdot x^{-5} + 5^x \cdot (x^{-5})'$	1

	$= \frac{5^x}{x^5} \log 5 - \frac{5^{x+1}}{x^6}$ <p style="text-align: center;">Or</p> <p>(b) Differentiating $-2x^2 - 5xy + y^3 = 76$, with respect to 'x'</p> $-4x - 5y - 5x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \frac{4x + 5y}{3y^2 - 5x}$	1 1 1										
22.	If $A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$, then find the value of K if $A^2 = 6A + KI_2$, where I_2 is an identity matrix.											
Ans	$A^2 = 6A + KI_2 \Rightarrow \begin{bmatrix} 1 & 0 \\ -6 & 25 \end{bmatrix} = \begin{bmatrix} 6+K & 0 \\ -6 & 30+K \end{bmatrix}$ $\Rightarrow 6 + K = 1 \Rightarrow K = -5, \text{ also satisfies } 30 + K = 25.$	$1\frac{1}{2}$ $\frac{1}{2}$										
23.	<p>(a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.</p> <p style="text-align: center;">OR</p> <p>(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?</p>											
Ans	<p>(a) Probability distribution table is:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td style="text-align: center;">X</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td style="text-align: center;">3</td> </tr> <tr> <td style="text-align: center;">P(X)</td> <td style="text-align: center;">$\frac{2}{10}$</td> <td style="text-align: center;">$\frac{3}{10}$</td> <td style="text-align: center;">$\frac{4}{10}$</td> <td style="text-align: center;">$\frac{1}{10}$</td> </tr> </tbody> </table>	X	0	1	2	3	P(X)	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{1}{2}$ 1
X	0	1	2	3								
P(X)	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{1}{10}$								

$$\text{Mean} = E(X) = \sum p_i x_i = 0 \cdot \frac{2}{10} + 1 \cdot \frac{3}{10} + 2 \cdot \frac{4}{10} + 3 \cdot \frac{1}{10} = \frac{14}{10} = \frac{7}{5} \text{ (or 1.4)}$$

OR

(b) A = A randomly chosen person is a woman

B = A randomly chosen person works outside village.

$$P(A) = \frac{4000}{8000} = \frac{1}{2}, P(B) = \frac{3000}{8000} = \frac{3}{8}, P(A \cap B) = \frac{1200}{8000} = \frac{3}{20}$$

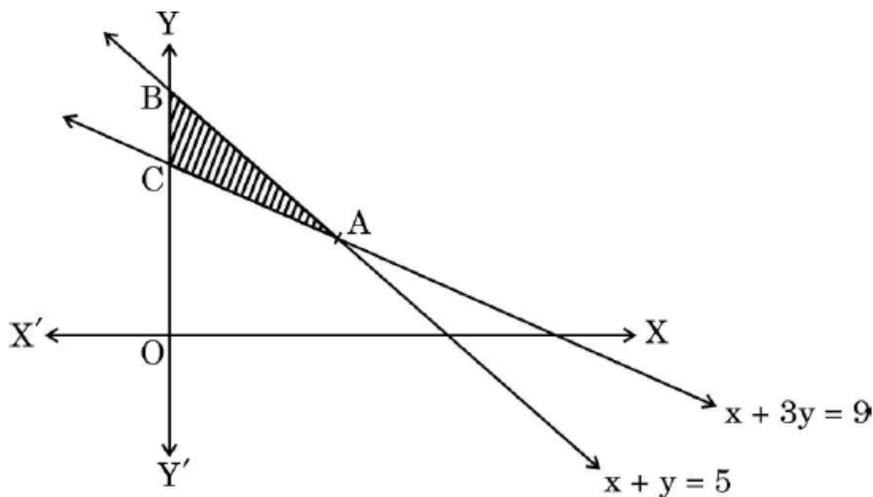
$$\text{Required probability} = P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{3}{8} - \frac{3}{20} = \frac{29}{40}$$

$\frac{1}{2}$

$1\frac{1}{2}$

$\frac{1}{2}$

24. For a Linear Programming Problem, find $\min Z = 5x + 3y$ (where Z is the objective function) for the feasible region shaded in the given figure.



(Note : The figure is not to scale)

Ans

Corner Points	Value of $Z = 5x + 3y$
A (3,2)	21
B (0,5)	15
C (0,3)	9

$$\text{Min } (Z) = 9$$

$1\frac{1}{2}$

$\frac{1}{2}$

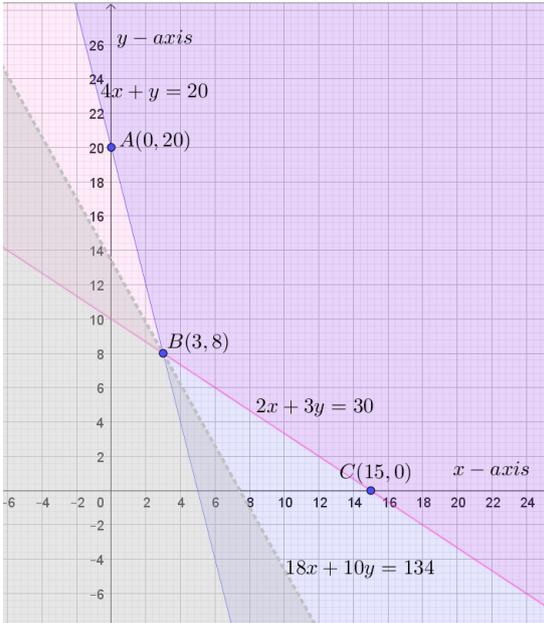
25. Let $f : A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$.
Discuss the bijectivity of the function.

Ans	<p>Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow x_1 = x_2, \therefore 'f'$ is one-one.</p> <p>For each $y \in B$, there exists $x = \frac{3y - 2}{y - 1} \in \mathbb{R} - \{3\}$, such that $f(x) = y, \therefore 'f'$ is onto</p> <p style="text-align: center;">$\Rightarrow 'f'$ is one-one & onto, or $'f'$ is a bijective function.</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
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SECTION-C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26.	<p>In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints</p> $4x + y \geq 20$ $2x + 3y \geq 30$ $x, y \geq 0$ <p>find the minimum value of Z.</p>	
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Ans		<p>Correct Fig.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Corner points</th> <th>Value of $Z = 18x + 10y$</th> </tr> </thead> <tbody> <tr> <td>A (0, 20)</td> <td>200</td> </tr> <tr> <td>B (3, 8)</td> <td>134</td> </tr> <tr> <td>C (15, 0)</td> <td>270</td> </tr> </tbody> </table> <p>Also, $Z < 134$, does not have any common point with the feasible region, $\therefore \text{Min}(Z) = 134$ at B (3, 8)</p>	Corner points	Value of $Z = 18x + 10y$	A (0, 20)	200	B (3, 8)	134	C (15, 0)	270	<p>$1\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>
Corner points	Value of $Z = 18x + 10y$										
A (0, 20)	200										
B (3, 8)	134										
C (15, 0)	270										

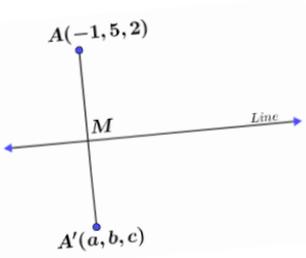
27.	<p>(a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.</p> <p style="text-align: center;">OR</p> <p>(b) Find the shortest distance between the lines :</p> $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$ $\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k}).$	
Ans	<p>(a) Let $\vec{d} = \vec{b} + \vec{c} = (2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k}$</p> $\hat{d} = \frac{(2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2 + \lambda)^2 + 40}}$ $\vec{a} \cdot \hat{d} = (\hat{i} - \hat{j} + 2\hat{k}) \cdot \frac{(2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2 + \lambda)^2 + 40}} = 1$ $\Rightarrow (2 + \lambda) + 6 + 4 = \sqrt{(2 + \lambda)^2 + 40} \Rightarrow \lambda = -5$ <p style="text-align: center;">OR</p> <p>(b) The two given lines are parallel with,</p> $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}, \vec{a}_2 = \hat{i} + 4\hat{k}$ <p>Then $\vec{a}_2 - \vec{a}_1 = -\hat{i} + \hat{j} + \hat{k}$ and the parallel vector is $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$</p> $\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ -1 & 1 & 1 \end{vmatrix} = -5\hat{i} - 4\hat{j} - \hat{k}$ $\text{Shortest Distance} = \frac{ \vec{b} \times (\vec{a}_2 - \vec{a}_1) }{ \vec{b} } = \frac{\sqrt{42}}{\sqrt{14}} = \sqrt{3}$	<p style="text-align: center;">1</p> <p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">1</p> <p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">OR</p> <p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">$1\frac{1}{2}$</p> <p style="text-align: center;">1</p>

28.	Differentiate $\log(x^x + \operatorname{cosec}^2 x)$ with respect to x .	
Ans	$\frac{d}{dx} \log(x^x + \operatorname{cosec}^2 x) = \frac{1}{x^x + \operatorname{cosec}^2 x} \frac{d}{dx} (e^{x \log x} + \operatorname{cosec}^2 x) \quad (\because x^x = e^{x \log x})$ $= \frac{1}{x^x + \operatorname{cosec}^2 x} [e^{x \log x} (1 + \log x) - 2 \operatorname{cosec}^2 x \cot x]$ $= \frac{1}{x^x + \operatorname{cosec}^2 x} [x^x (1 + \log x) - 2 \operatorname{cosec}^2 x \cot x]$	<p>1</p> <p>$1\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
29.	Show that of all the rectangles with a fixed perimeter, the square has the greatest area.	
Ans	<p>Let P be the perimeter of the rectangle, which is a constant. Also assume 'x' and 'y' be the length and breadth of the rectangle, then</p> $2(x+y) = P \text{ and } A(\text{Area}) = xy = \frac{x}{2}(P-2x) = \frac{1}{2}(Px - 2x^2)$ $A'(x) = \frac{1}{2}(P-4x), \therefore A'(x) = 0 \Rightarrow x = \frac{P}{4}, y = \frac{P}{4}$ $A''(x) = -2 < 0 \text{ at } x = \frac{P}{4}, \therefore \text{Area of the rectangle is max. if it is a square.}$	<p>$1\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>
30.	<p>(a) Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 4x^3 - 5, \forall x \in \mathbb{R}$ is one-one and onto.</p> <p style="text-align: center;">OR</p> <p>(b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y) : xy \text{ is a square of a natural number, } x, y \in N\}$. Determine if the relation R is an equivalence relation.</p>	
Ans	<p>(a) One-One: Let $x_1, x_2 \in \mathbb{R}$ such that</p> $f(x_1) = f(x_2) \Rightarrow 4x_1^3 - 5 = 4x_2^3 - 5 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2, \therefore \text{'f' is one-one}$ <p>Onto: $x \in \mathbb{R} (D_f) \Rightarrow x^3 \in \mathbb{R} \Rightarrow 4x^3 - 5 \in \mathbb{R} \Rightarrow f(x) \in \mathbb{R}, \therefore R_f = \text{Co-domain}(f)$</p> <p style="text-align: center;">\therefore 'f' is an onto function \Rightarrow 'f' is one-one & onto both</p> <p style="text-align: center;">OR</p> <p>(b) Reflexive: For any $x \in N, x \cdot x = x^2$, which is square of the natural number 'x'.</p> $\Rightarrow (x, x) \in R$ <p style="text-align: center;">\therefore 'R' is a Reflexive relation.</p>	<p>$1\frac{1}{2}$</p> <p>$1\frac{1}{2}$</p> <p>1</p>

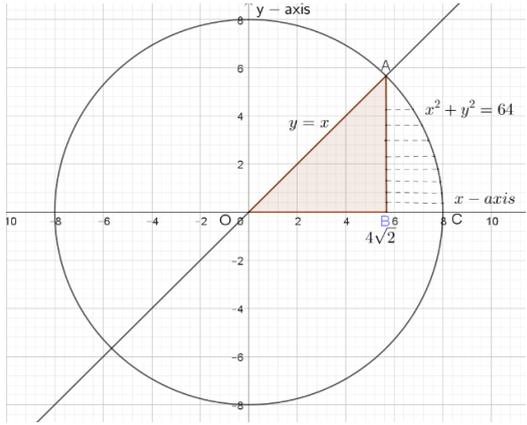
	<p>Symmetric: Let $(x, y) \in R \Rightarrow xy$ is a square of a natural number $\Rightarrow yx$ is a square of a natural number, $\therefore xy = yx$. $\Rightarrow (y, x) \in R$ \therefore 'R' is a Symmetric relation.</p> <p>Transitive: Let $(x, y), (y, z) \in R \Rightarrow xy = a^2, yz = b^2$ for some $a, b \in N$, $\therefore \frac{a^2}{y} = x, \frac{b^2}{y} = z \in N$ $\Rightarrow xz = \frac{a^2}{y} \cdot \frac{b^2}{y} = \left(\frac{ab}{y}\right)^2, \frac{ab}{y} \in N$ $\Rightarrow (x, z) \in R$ \therefore 'R' is a Transitive relation.</p> <p>Hence, R is an Equivalence relation</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
31.	<p>(a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.</p> <p style="text-align: center;">OR</p> <p>(b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days ?</p>	
Ans	<p>(a) The system of equations in matrices is:</p> $AX = B, \text{ where } A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ <p>The solution is given by $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{11} \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$</p> <p>Point common to paths of the ants is $(3, -1)$.</p> <p style="text-align: center;">OR</p>	<p>1</p> <p>$1\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>

	<p>(a) Let $A = \begin{bmatrix} 50 & 60 & 35 \\ 40 & 45 & 50 \end{bmatrix}$ Day I Day II, $B = \begin{bmatrix} 150 \\ 175 \\ 180 \end{bmatrix}$ be the day wise sale and the selling price per subject, matrices respectively.</p> <p>Total sales day wise = $\begin{bmatrix} 50 & 60 & 35 \\ 40 & 45 & 50 \end{bmatrix} \begin{bmatrix} 150 \\ 175 \\ 180 \end{bmatrix} = \begin{bmatrix} 24,300 \\ 22,875 \end{bmatrix}$ Day I Day II</p> <p>Total sales in two days = ₹ 24,300 + ₹ 22,875 = ₹ 47,175</p> <p>Profit = ₹ 47,175 – ₹ 35,000 = ₹ 12,175.</p>	<p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
<p>SECTION-D <i>This section comprises 4 Long Answer (LA) type questions of 5 marks each.</i></p>		
<p>32.</p>	<p>(a) Find :</p> $\int \frac{3x+1}{(x-2)^2(x+2)} dx$ <p style="text-align: center;">OR</p> <p>(b) Evaluate :</p> $\int_0^{\pi/2} \frac{x}{\cos x + \sin x} dx$	
<p>Ans</p>	<p>(a) Using Partial fractions,</p> $\int \frac{3x+1}{(x-2)^2(x+2)} dx = \frac{5}{16} \int \frac{1}{x-2} dx + \frac{7}{4} \int \frac{1}{(x-2)^2} dx - \frac{5}{16} \int \frac{1}{x+2} dx$ $= \frac{5}{16} \log x-2 - \frac{7}{4(x-2)} - \frac{5}{16} \log x+2 + C$ <p style="text-align: center;">or = $\frac{5}{16} \log \left \frac{x-2}{x+2} \right - \frac{7}{4(x-2)} + C$</p> <p style="text-align: center;">Or</p>	<p>$2\frac{1}{2}$</p> <p>$2\frac{1}{2}$</p>

	<p>(b) Let $I = \int_0^{\pi/2} \frac{x}{\cos x + \sin x} dx \Rightarrow I = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{\sin x + \cos x} dx \Rightarrow 2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{1}{\cos x + \sin x} dx$</p> <p>$\Rightarrow I = \frac{\pi}{4\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos(x - \pi/4)} dx = \frac{\pi}{4\sqrt{2}} \int_0^{\pi/2} \sec(x - \pi/4) dx$</p> <p>$\Rightarrow I = \frac{\pi}{4\sqrt{2}} \log \left[\sec(x - \pi/4) + \tan(x - \pi/4) \right] \Big _0^{\pi/2} = \frac{\pi}{4\sqrt{2}} \left[\log(\sqrt{2} + 1) - \log(\sqrt{2} - 1) \right]$</p> <p>Or, $I = \frac{\pi}{4\sqrt{2}} \log \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right)$</p>	<p>1½</p> <p>2</p> <p>1½</p>
33.	<p>(a) Find the point Q on the line $\frac{2x + 4}{6} = \frac{y + 1}{2} = \frac{-2z + 6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3).</p> <p style="text-align: center;">OR</p> <p>(b) Find the image of the point (-1, 5, 2) in the line $\frac{2x - 4}{2} = \frac{y}{2} = \frac{2 - z}{3}$. Find the length of the line segment joining the points (given point and the image point).</p>	
Ans	<p>(a) The general point on the line $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ is Q, from some $\lambda \in \mathbb{R}$</p> <p>$PQ = 3\sqrt{2} \Rightarrow (PQ)^2 = 18 \Rightarrow (3\lambda - 3)^2 + (2\lambda - 3)^2 + (2\lambda)^2 = 18$</p> <p>$17\lambda^2 - 30\lambda = 0 \Rightarrow \lambda = 0$ or $\lambda = \frac{30}{17}$</p> <p>Thus, the point is Q(-2, -1, 3) or Q($\frac{56}{17}, \frac{43}{17}, \frac{111}{17}$)</p> <p style="text-align: center;">Or</p> <p>(b) Let A'(a, b, c) be the image of the point A(-1, 5, 2) in the given line, also assume 'M' as the point of intersection of AA' with the given line, then 'M' is the mid-point of the line segment AA'</p>	<p>2</p> <p>1</p> <p>1</p> <p>1</p>

	 <p>The Line in the standard form is: $\frac{x-2}{1} = \frac{y}{2} = \frac{z-2}{-3}$, then</p> <p>M is the point $(\lambda + 2, 2\lambda, -3\lambda + 2)$, for some $\lambda \in \mathbb{R}$</p> <p>Direction Ratios of AM are $\lambda + 3, 2\lambda - 5, -3\lambda$</p> <p>AM \perp Line, $\therefore 1(\lambda + 3) + 2(2\lambda - 5) - 3(-3\lambda) = 0 \Rightarrow \lambda = \frac{1}{2}$</p> <p>$M\left(\frac{5}{2}, 1, \frac{1}{2}\right) = M\left(\frac{a-1}{2}, \frac{b+5}{2}, \frac{c+2}{2}\right) \Rightarrow a = 6, b = -3, c = -1$</p> <p>$\therefore$ The Image of A in the line is $A'(6, -3, -1)$</p> <p>And, $AA' = \sqrt{49 + 64 + 9} = \sqrt{122}$</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
<p>s</p> <p>34.</p>	<p>Solve the differential equation $(x - \sin y) dy + (\tan y) dx = 0$, given $y(0) = 0$.</p>	
<p>Ans</p>	<p>The differential equation can be written as:</p> $\frac{dx}{dy} + \cot y \cdot x = \cos y$, which is a linear order differential equation <p>Here, $P = \cot y$, $Q = \cos y$, I.F. (Integrating Factor) = $e^{\int \cot y \, dy} = e^{\log \sin y} = \sin y$</p> <p>The solution is, $x(\sin y) = \int \cos y \cdot \sin y \, dy$</p> $\Rightarrow x(\sin y) = \frac{(\sin y)^2}{2} + C$, For $x = 0, y = 0, C = 0$. <p>\therefore The Particular solution is: $x \sin y = \frac{\sin^2 y}{2}$ or $\sin y = 2x$ or $y = \sin^{-1} 2x$</p>	<p>1</p> <p>$1\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p>

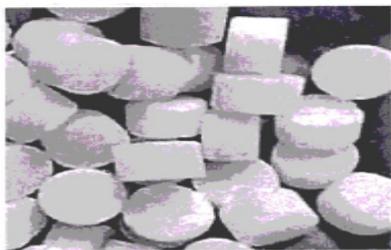
35.	A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.	
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Ans	 <p style="text-align: right;">Correct graph</p> <p style="text-align: right;">Equation of the circular tabletop: $x^2 + y^2 = 64$</p> <p style="text-align: right;">Equation of line (scratch): $x = y$</p> <p style="text-align: right;">The line and circle intersect at $x = 4\sqrt{2}$</p> <p>Area of the shaded region</p> $= \int_0^{4\sqrt{2}} x dx + \int_{4\sqrt{2}}^8 \sqrt{64 - x^2} dx$ $= \left[\frac{x^2}{2} \right]_0^{4\sqrt{2}} + \left[\frac{x}{2} \sqrt{64 - x^2} + 32 \sin^{-1} \frac{x}{8} \right]_{4\sqrt{2}}^8$ $= \frac{32}{2} + 32 \sin^{-1} 1 - 2\sqrt{2} \cdot 4\sqrt{2} - 32 \sin^{-1} \frac{1}{\sqrt{2}}$ $= 16 + 16\pi - 16 - 8\pi = 8\pi \text{ cm}^2$	<p style="text-align: right;">1</p> <p style="text-align: right;">$\frac{1}{2}$</p> <p style="text-align: right;">$\frac{1}{2}$</p> <p style="text-align: right;">$\frac{1}{2}$</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">$\frac{1}{2}$</p>
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Case Study – 1

36.

Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.



(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions :

- (i) Write the order and degree of the given differential equation. 1
- (ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm. 1
- (iii) (a) If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k . Hence, find t for $r = 0$ mm. 2
- OR**
- (iii) (b) If it is given that $r = 1$ mm when $t = 1$ hour, find the value of k . Hence, find t for $r = 0$ mm. 2

Ans	<p>(i) Order = 1, Degree = 1</p> <p>(ii) Separating the variable and integrating, $\int dr = \frac{2k}{3} \int dt \Rightarrow r = \frac{2}{3}kt + C$ Putting $t = 0, r = 5$, we get $C = 5$ $r = \frac{2}{3}kt + 5$</p> <p>(iii) (a) Putting $r = 3, t = 1$, $3 = \frac{2}{3}k(1) + 5 \Rightarrow k = -3$ $r = -2t + 5$, For $r = 0$, $t = \frac{5}{2}$ hrs or 2.5 hours Or</p> <p>(iii) (b) Putting $r = 1, t = 1$, $1 = \frac{2}{3}k + 5 \Rightarrow k = -6$ $\therefore r = -4t + 5$, For $r = 0$, $t = \frac{5}{4}$ hrs or 1.25 hours</p>	<p>$\frac{1}{2} + \frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
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Case Study – 2

37. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,
 A_2 : People with average health,
 and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1 , A_2 and A_3 are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions :

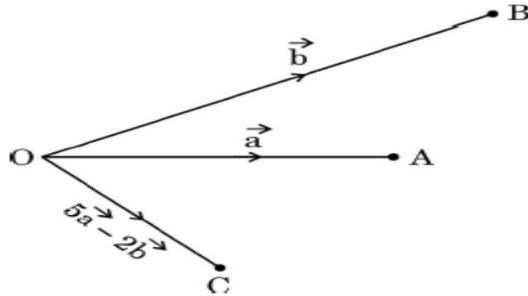
- (i) A person was tested randomly. What is the probability that he/she has contracted the disease ? 2
- (ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ? 2

Ans	<p>(i) Let A: Person contracted the disease</p> $P(A) = P(A_1) \cdot P(A A_1) + P(A_2) \cdot P(A A_2) + P(A_3) \cdot P(A A_3)$ $= \frac{7}{10} \left(\frac{25}{100} \right) + \frac{2}{10} \left(\frac{35}{100} \right) + \frac{1}{10} \left(\frac{50}{100} \right)$ $= \frac{295}{1000} = 0.295 \text{ or } \left(\frac{59}{200} \right)$	<p>$1\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
	<p>(ii) $P(A_2 \bar{A}) = \frac{P(A_2) \cdot P(\bar{A} A_2)}{P(A_1) \cdot P(\bar{A} A_1) + P(A_2) \cdot P(\bar{A} A_2)}$</p> $= \frac{\frac{2}{10} \times \frac{65}{100}}{\frac{7}{10} \times \frac{75}{100} + \frac{2}{10} \times \frac{65}{100} + \frac{1}{10} \times \frac{50}{100}}$ $= \frac{2 \times 13}{7 \times 15 + 2 \times 13 + 1 \times 10} = \frac{26}{141}$	<p>$1\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>

Case Study – 3

38.

Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$ and $\vec{OC} = 5\vec{a} - 2\vec{b}$ respectively.

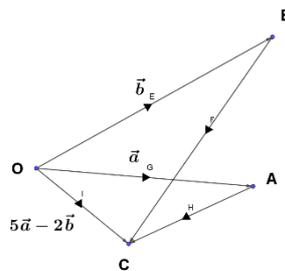


Based upon the above information, answer the following questions :

- (i) Complete the given figure to explain their entire movement plan along the respective vectors. 1
 - (ii) Find vectors \vec{AC} and \vec{BC} . 1
 - (iii) (a) If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between \vec{OA} and \vec{OB} . Also, find $|\vec{a} \times \vec{b}|$. 2
- OR**
- (iii) (b) If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$. 2

Ans

(i) The Complete figure of their entire movement plan is:



(ii) $\vec{AC} = \vec{OC} - \vec{OA} = 4\vec{a} - 2\vec{b}$, $\vec{BC} = \vec{OC} - \vec{OB} = 5\vec{a} - 3\vec{b}$ 1

(iii) (a) we are given: $|\vec{a}| = 1, |\vec{b}| = 2$, assuming 'θ' as the angle between \vec{OA} and \vec{OB} .

$$\theta = \cos^{-1} \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \right) = \cos^{-1} \frac{1}{1 \times 2} = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta = 1(2) \frac{\sqrt{3}}{2} = \sqrt{3}$$

	Or	
	(iii) (b) $\vec{a} + \vec{b} = 2\hat{i} + 3\hat{k}$, $\vec{a} - \vec{b} = 2\hat{i} - 2\hat{j} + 5\hat{k}$, let \vec{c} be \perp to both $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$	$\frac{1}{2}$
	Then, $\vec{c} = (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 3 \\ 2 & -2 & 5 \end{vmatrix} = 6\hat{i} - 4\hat{j} - 4\hat{k}$ and $ \vec{c} = \sqrt{68}$	1
	The required unit vector is, $\hat{c} = \frac{1}{2\sqrt{17}}(6\hat{i} - 4\hat{j} - 4\hat{k}) = \frac{1}{\sqrt{17}}(3\hat{i} - 2\hat{j} - 2\hat{k})$	$\frac{1}{2}$