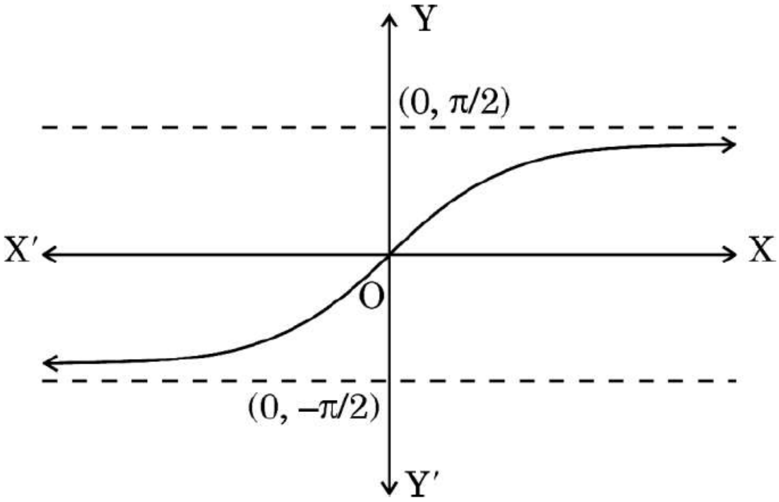
SOLUTIONS – 65/7/1

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A ***This section comprises multiple choice questions (MCQs) of 1 mark each.***	
1.	The given graph illustrates :  (A) $y = \tan^{-1} x$ (B) $y = \text{cosec}^{-1} x$ (C) $y = \cot^{-1} x$ (D) $y = \sec^{-1} x$	
Ans	**(A)** $y = \tan^{-1} x$	**1**
2.	Domain of $f(x) = \cos^{-1} x + \sin x$ is : (A) R (B) (–1, 1) (C) [–1, 1] (D) ϕ	
Ans	**(C)** $[-1, 1]$	**1**
3.	What is the total number of possible matrices of order 3 × 3 with each entry as $\sqrt{2}$ or $\sqrt{3}$? (A) 9 (B) 512 (C) 615 (D) 64	
Ans	**(B) 512**	**1**

4.	The matrix $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ is a/an : (A) scalar matrix (B) identity matrix (C) null matrix (D) symmetric matrix	
Ans	**(D) symmetric matrix**	**1**
5.	If A and B are two square matrices each of order 3 with $\|A\| = 3$ and $\|B\| = 5$, then $\|2AB\|$ is : (A) 30 (B) 120 (C) 15 (D) 225	
Ans	**(B) 120**	**1**
6.	Let A be a square matrix of order 3. If $\|A\| = 5$, then $\|\text{adj } A\|$ is : (A) 5 (B) 125 (C) 25 (D) – 5	
Ans	**(C) 25**	**1**
7.	If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$, then the value of $(x-y)$ is : (A) 2 or 10 (B) – 2 or 10 (C) 2 or – 10 (D) – 2 or – 10	
Ans	**(B) –2 or 10**	**1**
8.	If $f(x) = \begin{cases} 1, & \text{if } x \le 3 \\ ax+b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \le x \end{cases}$ is continuous in R, then the values of a and b are : (A) $a = 3,\ b = -8$ (B) $a = 3,\ b = 8$ (C) $a = -3,\ b = -8$ (D) $a = -3,\ b = 8$	
Ans	**(A) $a = 3, b = -8$**	**1**

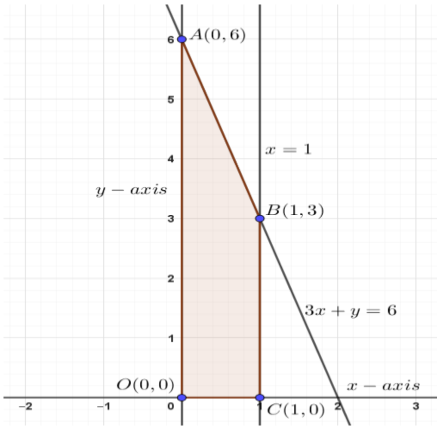
9.	If $f(x) = -2x^8$, then the correct statement is : (A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$ (C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$	
Ans	**(B)** $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$	**1**
10.	A spherical ball has a variable diameter $\frac{5}{2}(3x + 1)$. The rate of change of its volume w.r.t. x, when x = 1, is : (A) 225π (B) 300π (C) 375π (D) 125π	
Ans	**(C) 375π**	**1**
11.	If $f : R \to R$ is defined as $f(x) = 2x - \sin x$, then f is : (A) a decreasing function (B) an increasing function (C) maximum at $x = \frac{\pi}{2}$ (D) maximum at x = 0	
Ans	**(B) an increasing function**	**1**
12.	$\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}}\,dx$ is equal to : (A) $x + C$ (B) $\frac{x^2}{2} + C$ (C) $\frac{x^4}{4} + C$ (D) $\frac{x^3}{3} + C$	
Ans	**(C)** $\frac{x^4}{4} + C$	**1**

13.	For a function f(x), which of the following holds true ? (A) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ (B) $\int_{-a}^{a} f(x)\,dx = 0$, if f is an even function (C) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an odd function (D) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(2a+x)\,dx$	
Ans	**(A)** $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$	**1**
14.	$\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$ is equal to : (A) $\frac{1}{2}\cos^{-1}(e^x) + C$ (B) $\frac{1}{2}\sin^{-1}(e^x) + C$ (C) $\frac{e^x}{2} + C$ (D) $\sin^{-1}\left(\frac{e^x}{2}\right) + C$	
Ans	**(D)** $\sin^{-1}\left(\frac{e^x}{2}\right) + C$	**1**

15.	A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is : (A) 6 (B) 1 (C) $\frac{1}{4}$ (D) 4	
Ans	**(D) 4**	**1**
16.	If $\lvert \vec{a} + \vec{b} \rvert = \lvert \vec{a} - \vec{b} \rvert$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are : (A) orthogonal vectors (B) parallel to each other (C) unit vectors (D) collinear vectors	
Ans	**(A) orthogonal vectors**	**1**
17.	If $P(A) = \frac{1}{7}$, $P(B) = \frac{5}{7}$ and $P(A \cap B) = \frac{4}{7}$, then $P(\bar{A} \mid B)$ is : (A) $\frac{6}{7}$ (B) $\frac{3}{4}$ (C) $\frac{4}{5}$ (D) $\frac{1}{5}$	
Ans	**(D)** $\mathbf{\frac{1}{5}}$	**1**
18.	A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is : (A) $\frac{2}{13}$ (B) $\frac{3}{26}$ (C) $\frac{19}{26}$ (D) $\frac{3}{13}$	
Ans	**(B)** $\mathbf{\frac{3}{26}}$	**1**

	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* $f(x) = \begin{cases} x \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is continuous at x = 0. *Reason (R) :* When $x \to 0$, $\sin \frac{1}{x}$ is a finite value between – 1 and 1.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**
20.	*Assertion (A) :* Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set. *Reason (R) :* $\sec^{-1} x$ is defined for $x \in R - (-1, 1)$.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**

SECTION-B *This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.*		
21.	Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = R - \{3\}$ and $B = R - \{1\}$. Discuss the bijectivity of the function.	
Ans	**Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow x_1 = x_2$, $\therefore$ 'f' is one-one.** **For each $y \in B$, there exists $x = \frac{3y-2}{y-1} \in R - \{3\}$, such that $f(x) = y$, $\therefore$ 'f' is onto** **$\Rightarrow$ 'f' is one-one & onto, or 'f' is a bijective function.**	**1** **½** **½**
22.	If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$.	
Ans	$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$. **L.H.S.=** $A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ **=R.H.S.**	**1** **1**
23.	(a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x. **OR** (b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.	
Ans	**(a) Let,** $y = \frac{5^x}{x^5} = 5^x \cdot x^{-5} \Rightarrow \frac{dy}{dx} = (5^x)' \cdot x^{-5} + 5^x \cdot (x^{-5})'$ $= \frac{5^x}{x^5}\log 5 - \frac{5^{x+1}}{x^6}$ **OR** **(b) Differentiating** $-2x^2 - 5xy + y^3 = 76$, **with respect to 'x'** $-4x - 5y - 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \frac{4x+5y}{3y^2-5x}$	**1** **1** 1½ ½

24.	In a Linear Programming Problem, the objective function Z = 5x + 4y needs to be maximised under constraints $3x + y \le 6$, $x \le 1$, $x, y \ge 0$. Express the LPP on the graph and shade the feasible region and mark the corner points.	
Ans	 **Correct Plotting of the two lines** **Correct shading of the feasible region and marking the corner points**	1½ ½
25.	(a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean. **OR** (b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?	
Ans	**(a) Probability distribution table is:**	

X	0	1	2	3
P(X)	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{1}{10}$

½ 1

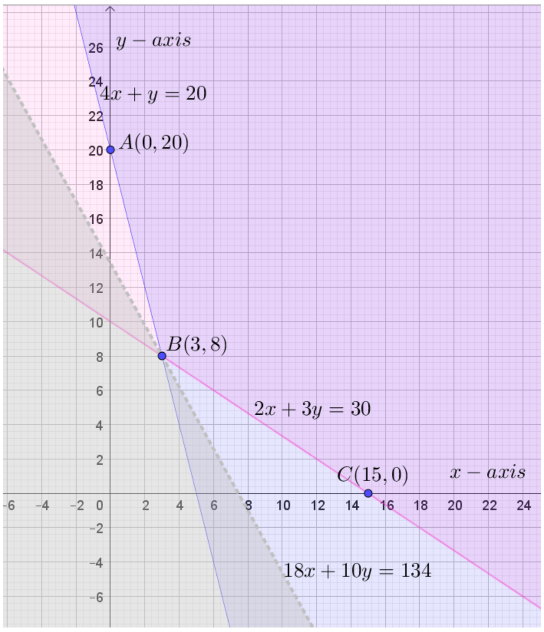
Mean = E(X) = $\sum p_i x_i = 0 \cdot \frac{2}{10} + 1 \cdot \frac{3}{10} + 2 \cdot \frac{4}{10} + 3 \cdot \frac{1}{10} = \frac{14}{10} = \frac{7}{5}$ **(or 1.4)** ½

	OR **(b) A = A randomly chosen person is a woman** **B = A randomly chosen person works outside village.** $P(A)=\frac{4000}{8000}=\frac{1}{2},\ P(B)=\frac{3000}{8000}=\frac{3}{8},\ P(A\cap B)=\frac{1200}{8000}=\frac{3}{20}$ **Required probability =** $P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{1}{2}+\frac{3}{8}-\frac{3}{20}=\frac{29}{40}$	$1\frac{1}{2}$ $\frac{1}{2}$
	SECTION-C ***This section comprises 6 Short Answer (SA) type questions of 3 marks each.***	
26.	(a) Show that the function $f : R \to R$ defined by $f(x) = 4x^3 - 5$, $\forall\ x \in R$ is one-one and onto. **OR** (b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y) : xy$ is a square of a natural number, $x, y \in N\}$. Determine if the relation R is an equivalence relation.	
Ans	**(a) One-One: Let** $x_1, x_2 \in R$ **such that** $f(x_1)=f(x_2)\Rightarrow 4x_1^3-5=4x_2^3-5\Rightarrow x_1^3=x_2^3\Rightarrow x_1=x_2$, $\therefore$ **'f' is one-one** **Onto:** $x\in R\ (D_f)\Rightarrow x^3\in R\Rightarrow 4x^3-5\in R\Rightarrow f(x)\in R,\ \therefore\ R_f=\text{Co-domain}(f)$ $\therefore$ **'f' is an onto function** $\Rightarrow$ **'f' is one-one & onto both** **OR** **(b) Reflexive: For any** $x\in N$, $x\cdot x=x^2$, **which is square of the natural number 'x'.** $\Rightarrow (x,x)\in R$ $\therefore$ **'R' is a Reflexive relation.** **Symmetric: Let** $(x,y)\in R\Rightarrow xy$ **is a square of a natural number** $\Rightarrow yx$ **is a square of a natural number,** $\because xy=yx$. $\Rightarrow (y,x)\in R$ $\therefore$ **'R' is a Symmetric relation.** **Transitive: Let** $(x,y),(y,z)\in R\Rightarrow xy=a^2, yz=b^2$ **for some** $a,b\in N$, $\therefore \frac{a^2}{y}=x, \frac{b^2}{y}=z\in N$ $\Rightarrow xz=\frac{a^2}{y}\cdot\frac{b^2}{y}=\left(\frac{ab}{y}\right)^2, \frac{ab}{y}\in N$ $\Rightarrow (x,z)\in R$ $\therefore$ **'R' is a Transitive relation.** **Hence, R is an Equivalence relation**	$1\frac{1}{2}$ $1\frac{1}{2}$ 1 1 $\frac{1}{2}$ $\frac{1}{2}$

27.	(a) Let 2x + 5y – 1 = 0 and 3x + 2y – 7 = 0 represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants. **OR** (b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days ?	
Ans	**(a) The system of equations in matrices is:** $\mathbf{AX = B}$, **where** $\mathbf{A}=\begin{bmatrix}2 & 5\\3 & 2\end{bmatrix}, \mathbf{X}=\begin{bmatrix}x\\y\end{bmatrix}, \mathbf{B}=\begin{bmatrix}1\\7\end{bmatrix}$	**1**
	The solution is given by $\mathbf{X = A^{-1}B} \Rightarrow \begin{bmatrix}x\\y\end{bmatrix}=\frac{-1}{11}\begin{bmatrix}2 & -5\\-3 & 2\end{bmatrix}\begin{bmatrix}1\\7\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$	**1½**
	Point common to paths of the ants is $(3,-1)$.	**½**
	OR	
	(a) Let $\mathbf{A}=\begin{bmatrix}50 & 60 & 35\\40 & 45 & 50\end{bmatrix}\begin{matrix}\text{Day I}\\\text{Day II}\end{matrix}$, $\mathbf{B}=\begin{bmatrix}150\\175\\180\end{bmatrix}$ **be the day wise sale and the selling price per subject, matrices respectively.**	**1**
	Total sales day wise $=\begin{bmatrix}50 & 60 & 35\\40 & 45 & 50\end{bmatrix}\begin{bmatrix}150\\175\\180\end{bmatrix}=\begin{bmatrix}24,300\\22,875\end{bmatrix}\begin{matrix}\text{Day I}\\\text{Day II}\end{matrix}$	**1**
	Total sales in two days = ₹ 24,300 + ₹ 22,875 = ₹ 47,175	**½**
	Profit = ₹ 47,175 – ₹ 35,000 = ₹ 12,175.	**½**

28.	Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x.	
Ans	$\frac{dy}{dx} = \frac{1}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}} \cdot \frac{1}{\sin\left(\frac{x^3}{3}-1\right)} \cdot \cos\left(\frac{x^3}{3}-1\right) \cdot \frac{3x^2}{3}$ $= \frac{x^2 \cot\left(\frac{x^3}{3}-1\right)}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}}$	$1+1+\frac{1}{2}$ $\frac{1}{2}$
29.	Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.	
Ans	**Let numbers be 'x' and 'y' such that** $xy = 289 \Rightarrow y = \frac{289}{x}$**, 'S' be their sum, then** $S = x + y = x + \frac{289}{x}$ $\frac{dS}{dx} = 1 - \frac{289}{x^2}, \frac{dS}{dx} = 0 \Rightarrow x = 17$**, a positive integer** $\left.\frac{d^2S}{dx^2}\right]_{x=17} = 289\left.\left(\frac{2}{x^3}\right)\right]_{x=17} > 0$, $\therefore$ **S is minimum when** $x = 17, y = 17$	1 1½ ½
30.	In the Linear Programming Problem for objective function Z = 18x + 10y subject to constraints $4x + y \geq 20$ $2x + 3y \geq 30$ $x, y \geq 0$ find the minimum value of Z.	

Ans



Correct Fig. — 1½

Corner points	Value of Z = 18x + 10y
A (0, 20)	200
B (3, 8)	134
C (15, 0)	270

1

Also, $Z < 134$, does not have any common point with the feasible region,
$\therefore \text{Min}(Z) = 134$ at B (3, 8) — ½

31. (a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.

OR

(b) Find the shortest distance between the lines :

$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k})$.

Ans

(a) Let $\vec{d} = \vec{b} + \vec{c} = (2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}$ — 1

$$\hat{d} = \frac{(2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}}$$ — ½

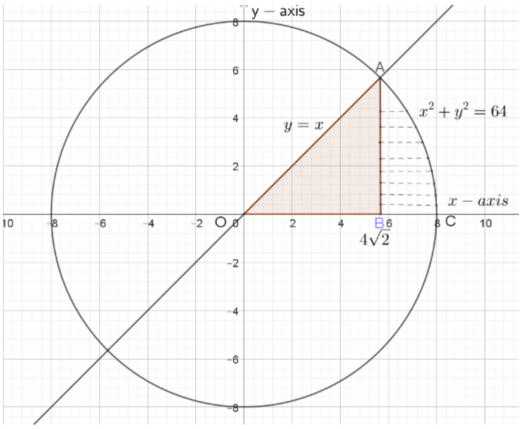
$$\vec{a}\cdot\hat{d} = (\hat{i} - \hat{j} + 2\hat{k})\cdot\frac{(2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}} = 1$$ — 1

$$\Rightarrow (2+\lambda) + 6 + 4 = \sqrt{(2+\lambda)^2 + 40} \Rightarrow \lambda = -5$$ — ½

OR

	(b) The two given lines are parallel with, $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} + 4\hat{k}$ **Then** $\vec{a}_2 - \vec{a}_1 = -\hat{i} + \hat{j} + \hat{k}$ **and the parallel vector is** $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$	½
	$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ -1 & 1 & 1 \end{vmatrix} = -5\hat{i} - 4\hat{j} - \hat{k}$	1½
	Shortest Distance $= \dfrac{\left\lvert \vec{b} \times (\vec{a}_2 - \vec{a}_1) \right\rvert}{\lvert \vec{b} \rvert} = \dfrac{\sqrt{42}}{\sqrt{14}} = \sqrt{3}$	1
	SECTION-D ***This section comprises 4 Long Answer (LA) type questions of 5 marks each.***	
32.	(a) Find : $\displaystyle\int \frac{x^2+1}{(x^2+2)(2x^2+1)}\,dx$ **OR** (b) Evaluate : $\displaystyle\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\,dx$	
Ans	**(a)** $\displaystyle\int \frac{x^2+1}{(x^2+2)(2x^2+1)}dx = \frac{1}{3}\int \frac{1}{x^2+2}dx + \frac{1}{3}\int \frac{1}{2x^2+1}dx$ **(Using Partial Fractions)**	2½
	$= \dfrac{1}{3}\cdot\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{x}{\sqrt{2}}\right) + \dfrac{1}{3}\cdot\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}x\right) + C$	2½
	or $= \dfrac{1}{3\sqrt{2}}\left(\tan^{-1}\dfrac{x}{\sqrt{2}} + \tan^{-1}\sqrt{2}x\right) + C$ **OR**	
	(b) Let $I = \displaystyle\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}dx$ -- (i) $\Rightarrow I = \displaystyle\int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \tan x}dx$ -- (ii)	1½

	Adding (i) & (ii), we get	
	$2I = \pi\int_0^{\pi}\frac{\tan x}{\sec x + \tan x}dx \Rightarrow 2I = \pi\int_0^{\pi}\frac{\tan x(\sec x - \tan x)}{\sec^2 x - \tan^2 x}dx$	1
	$= \pi\int_0^{\pi}(\sec x\tan x - \sec^2 x + 1)dx$	1
	$= \pi(\sec x - \tan x + x)\Big]_0^{\pi}$	1
	$= \pi(-1 + \pi - 1) = \pi(\pi - 2)$	
	$\therefore I = \frac{\pi}{2}(\pi - 2)$ or $\pi\left(\frac{\pi}{2} - 1\right)$	½
33.	A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.	

Ans	 **Correct graph** **Equation of the circular tabletop:** $x^2+y^2=64$ **Equation of line (scratch):** $x = y$ **The line and circle intersect at** $x = 4\sqrt{2}$ **Area of the shaded region** $= \int_0^{4\sqrt{2}} x\,dx + \int_{4\sqrt{2}}^{8} \sqrt{64-x^2}\,dx$ $= \left.\frac{x^2}{2}\right]_0^{4\sqrt{2}} + \left[\frac{x}{2}\sqrt{64-x^2} + 32\sin^{-1}\frac{x}{8}\right]_{4\sqrt{2}}^{8}$ $= \frac{32}{2} + 32\sin^{-1}1 - 2\sqrt{2}\cdot 4\sqrt{2} - 32\sin^{-1}\frac{1}{\sqrt{2}}$ $= 16 + 16\pi - 16 - 8\pi = 8\pi \text{ cm}^2$	1 ½ ½ ½ 1 1 ½
34.	Solve the differential equation $\frac{dy}{dx} = \cos x - 2y$.	
Ans	**The given differential equation can be written as:** $\frac{dy}{dx} + 2y = \cos x$, **Taking** $P = 2, Q = \cos x$ **Integrating factor is given by,** $I = e^{\int 2dx} = e^{2x}$ $\therefore$ **The solution is,** $y \cdot e^{2x} = \int e^{2x}\cos x\,dx$ **Let,** $I_1 = \int \cos x \cdot e^{2x}dx = \cos x\frac{e^{2x}}{2} - \int(-\sin x)\frac{e^{2x}}{2}dx$ $= \frac{e^{2x}\cos x}{2} + \frac{1}{2}\left[\sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2}dx\right]$ $\Rightarrow I_1 = \frac{e^{2x}\cos x}{2} + \frac{e^{2x}\sin x}{4} - \frac{1}{4}I_1 \Rightarrow I_1 = \frac{e^{2x}}{5}(2\cos x + \sin x)$ $\therefore$ **The solution of the differential equation is** $y \cdot e^{2x} = \frac{e^{2x}}{5}(2\cos x + \sin x) + C \;\Rightarrow\; y = \frac{1}{5}(2\cos x + \sin x) + Ce^{-2x}$	½ 1 1½ 1½ ½

35.

(a) Find the point Q on the line $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3).

OR

(b) Find the image of the point (–1, 5, 2) in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).

Ans

(a) The general point on the line $(3\lambda-2, 2\lambda-1, 2\lambda+3)$ is Q, from some $\lambda \in R$ — 2

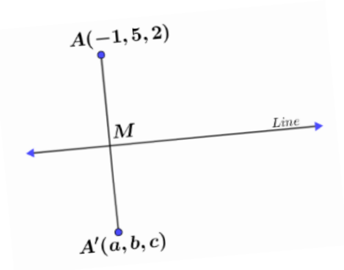
$$PQ = 3\sqrt{2} \Rightarrow (PQ)^2 = 18 \Rightarrow (3\lambda-3)^2 + (2\lambda-3)^2 + (2\lambda)^2 = 18$$ — 1

$$17\lambda^2 - 30\lambda = 0 \Rightarrow \lambda = 0 \text{ or } \lambda = \frac{30}{17}$$ — 1

Thus, the point is $Q(-2,-1,3)$ **or** $Q\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ — 1

OR

(b) Let $A'(a,b,c)$ be the image of the point $A(-1,5,2)$ in the given line, also assume 'M' as the point of intersection of AA' with the given line, then 'M' is the mid-point of the line segment AA'



The Line in the standard form is: $\frac{x-2}{1} = \frac{y}{2} = \frac{z-2}{-3}$**, then** — 1

M is the point $(\lambda+2, 2\lambda, -3\lambda+2)$, **for some** $\lambda \in R$ — ½

Direction Ratios of AM are $\lambda+3, 2\lambda-5, -3\lambda$ — ½

$AM \perp \text{Line}, \therefore 1(\lambda+3) + 2(2\lambda-5) - 3(-3\lambda) = 0 \Rightarrow \lambda = \frac{1}{2}$ — 1

$M\left(\frac{5}{2}, 1, \frac{1}{2}\right) = M\left(\frac{a-1}{2}, \frac{b+5}{2}, \frac{c+2}{2}\right) \Rightarrow a = 6, b = -3, c = -1$ — 1

$\therefore$ **The Image of A in the line is** $A'(6,-3,-1)$ — ½

And, $AA' = \sqrt{49+64+9} = \sqrt{122}$ — ½

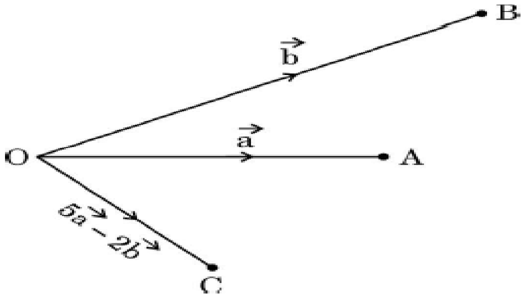
SECTION-E

This section comprises 3 case study-based questions of 4 marks each

36.

Case Study – 1

Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based upon the above information, answer the following questions :

(i) Complete the given figure to explain their entire movement plan along the respective vectors. *1*

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$. *1*

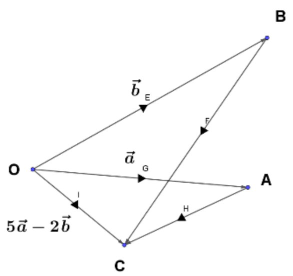
(iii) (a) If $\vec{a} . \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a} \times \vec{b}|$. *2*

OR

(iii) (b) If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$. *2*

Ans

(i) The Complete figure of their entire movement plan is: **1**



(ii) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 4\vec{a} - 2\vec{b}$, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 5\vec{a} - 3\vec{b}$ **½+½**

(iii) (a) we are given: $|\vec{a}| = 1, |\vec{b}| = 2$, **assuming '** θ **' as the angle between** $\overrightarrow{OA}$ **and** $\overrightarrow{OB}$**.**

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right) = \cos^{-1}\frac{1}{1 \times 2} = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$ **1**

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta = 1(2)\frac{\sqrt{3}}{2} = \sqrt{3}$$ **1**

OR

(iii) (b) $\vec{a}+\vec{b}=2\hat{i}+3\hat{k}$, $\vec{a}-\vec{b}=2\hat{i}-2\hat{j}+5\hat{k}$, **let** $\vec{c}$ **be** $\perp$ **to both** $(\vec{a}+\vec{b})$ **and** $(\vec{a}-\vec{b})$ — $\frac{1}{2}+\frac{1}{2}$

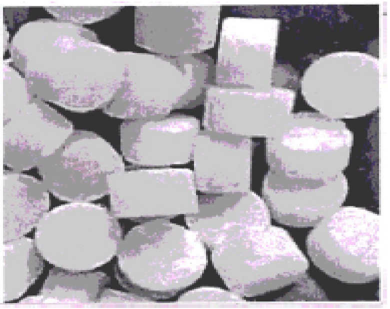
Then, $\vec{c}=(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 3\\ 2 & -2 & 5\end{vmatrix}=6\hat{i}-4\hat{j}-4k$ **and** $|\vec{c}|=\sqrt{68}$ — $\frac{1}{2}$

The required unit vector is, $\hat{c}=\frac{1}{2\sqrt{17}}(6\hat{i}-4\hat{j}-4\hat{k})=\frac{1}{\sqrt{17}}(3\hat{i}-2\hat{j}-2\hat{k})$ — $\frac{1}{2}$

37.

Case Study – 2

Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.



(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt}=kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions :

(i) Write the order and degree of the given differential equation. *1*

(ii) Substituting $V=\pi r^3$ and $S=2\pi r^2$, we get the differential equation $\frac{dr}{dt}=\frac{2}{3}k$. Solve it, given that r(0) = 5 mm. *1*

(iii) (a) If it is given that r = 3 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm. *2*

OR

(iii) (b) If it is given that r = 1 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm. *2*

Ans	**(i) Order = 1, Degree = 1** **(ii) Separating the variable and integrating,** $\int dr = \frac{2k}{3}\int dt \Rightarrow r = \frac{2}{3}kt + C$ **Putting** $t = 0, r = 5$, **we get** $C = 5$ $r = \frac{2}{3}kt + 5$ **(iii) (a) Putting** $r = 3, t = 1$, $3 = \frac{2}{3}k(1) + 5 \Rightarrow k = -3$ $r = -2t + 5$, **For** $r = 0$, $t = \frac{5}{2}$ **hours or 2.5 hours** **OR** **(iii) (b) Putting** $r = 1, t = 1$, $1 = \frac{2}{3}k + 5 \Rightarrow k = -6$ $\therefore\ r = -4t + 5$, **For** $r = 0$, $t = \frac{5}{4}$ **hours or 1.25 hours**	½ + ½ ½ ½ 1 1 1 1
38.	**Case Study – 3** Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record. Let A_1 : People with good health, A_2 : People with average health, and A_3 : People with poor health. During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively. Based upon the above information, answer the following questions : (i) A person was tested randomly. What is the probability that he/she has contracted the disease ? *2* (ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ? *2*	
Ans	**(i) Let A: Person contracted the disease** $P(A) = P(A_1)\cdot P(A\mid A_1) + P(A_2)\cdot P(A\mid A_2) + P(A_3)\cdot P(A\mid A_3)$ $= \frac{7}{10}\left(\frac{25}{100}\right) + \frac{2}{10}\left(\frac{35}{100}\right) + \frac{1}{10}\left(\frac{50}{100}\right)$ $= \frac{295}{1000} = 0.295$ **or** $\left(\frac{59}{200}\right)$	 1½ ½

	(ii) $P(A_2 \mid \bar{A}) = \dfrac{P(A_2)\cdot P(\bar{A}/A_2)}{P(A_1)\cdot P(\bar{A}/A_1) + P(A_2)\cdot P(\bar{A}/A_2)}$ $= \dfrac{\frac{2}{10}\times\frac{65}{100}}{\frac{7}{10}\times\frac{75}{100}+\frac{2}{10}\times\frac{65}{100}+\frac{1}{10}\times\frac{50}{100}}$ $= \dfrac{2\times13}{7\times15+2\times13+1\times10} = \dfrac{26}{141}$	$1\frac{1}{2}$ $\frac{1}{2}$