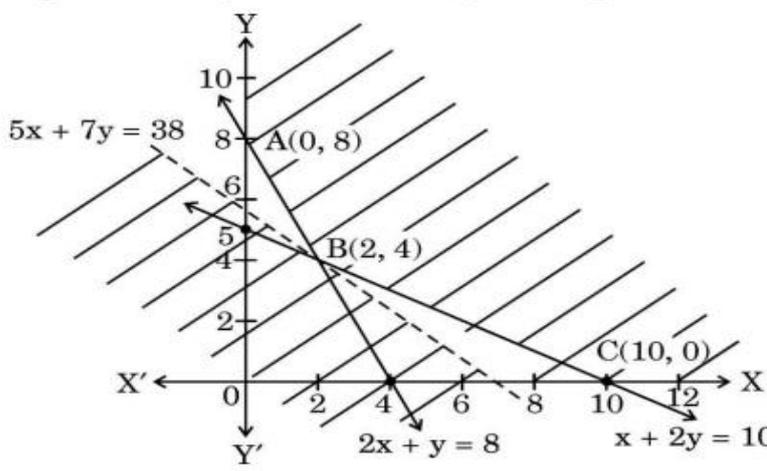




5.	<p>If <math>A = \begin{bmatrix} 5 &amp; 0 &amp; 0 \\ 0 &amp; 5 &amp; 0 \\ 0 &amp; 0 &amp; 5 \end{bmatrix}</math>, then <math>A^3</math> is :</p> <p>(A) <math>3 \begin{bmatrix} 5 &amp; 0 &amp; 0 \\ 0 &amp; 5 &amp; 0 \\ 0 &amp; 0 &amp; 5 \end{bmatrix}</math>                      (B) <math>\begin{bmatrix} 125 &amp; 0 &amp; 0 \\ 0 &amp; 125 &amp; 0 \\ 0 &amp; 0 &amp; 125 \end{bmatrix}</math></p> <p>(C) <math>\begin{bmatrix} 15 &amp; 0 &amp; 0 \\ 0 &amp; 15 &amp; 0 \\ 0 &amp; 0 &amp; 15 \end{bmatrix}</math>                      (D) <math>\begin{bmatrix} 5^3 &amp; 0 &amp; 0 \\ 0 &amp; 5 &amp; 0 \\ 0 &amp; 0 &amp; 5 \end{bmatrix}</math></p>	
Ans	(B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$	1
6.	<p>If <math>\begin{vmatrix} 2x &amp; 5 \\ 12 &amp; x \end{vmatrix} = \begin{vmatrix} 6 &amp; -5 \\ 4 &amp; 3 \end{vmatrix}</math>, then the value of x is :</p> <p>(A) 3    (B) 7</p> <p>(C) <math>\pm 7</math>    (D) <math>\pm 3</math></p>	
Ans	(C) $\pm 7$	1
7.	<p>If <math>P(A \cup B) = 0.9</math> and <math>P(A \cap B) = 0.4</math>, then <math>P(\bar{A}) + P(\bar{B})</math> is :</p> <p>(A) 0.3    (B) 1</p> <p>(C) 1.3    (D) 0.7</p>	
Ans	(D) 0.7	1
8.	<p>If a matrix A is both symmetric and skew-symmetric, then A is a :</p> <p>(A) diagonal matrix                              (B) zero matrix</p> <p>(C) non-singular matrix                        (D) scalar matrix</p>	
Ans	(B) zero matrix	1
9.	<p>The slope of the curve <math>y = -x^3 + 3x^2 + 8x - 20</math> is maximum at :</p> <p>(A) (1, -10)                                        (B) (1, 10)</p> <p>(C) (10, 1)                                        (D) (-10, 1)</p>	
Ans	(A) (1, -10)	1



14.	<p>The integrating factor of the differential equation</p> $\frac{dx}{dy} = \frac{x \log x}{\frac{2}{x} \log x - y}$ is : <p>(A) <math>\frac{1}{8x}</math> (B) <math>e</math></p> <p>(C) <math>e^{\log x}</math> (D) <math>\log x</math></p>	
Ans	(D) $\log x$	1
15.	<p>Let <math>\vec{a}</math> be a position vector whose tip is the point <math>(2, -3)</math>. If <math>\vec{AB} = \vec{a}</math>, where coordinates of A are <math>(-4, 5)</math>, then the coordinates of B are :</p> <p>(A) <math>(-2, -2)</math> (B) <math>(2, -2)</math> (C) <math>(-2, 2)</math> (D) <math>(2, 2)</math></p>	
Ans	(C) $(-2, 2)$	1
16.	<p>The respective values of <math> \vec{a} </math> and <math> \vec{b} </math>, if given</p> $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $ \vec{a}  = 3 \vec{b} $ , are : <p>(A) 48 and 16 (B) 3 and 1</p> <p>(C) 24 and 8 (D) 6 and 2</p>	
Ans	(C) 24 and 8	1
17.	<p>For a Linear Programming Problem (LPP), the given objective function <math>Z = 3x + 2y</math> is subject to constraints :</p> $x + 2y \leq 10$ $3x + y \leq 15$ $x, y \geq 0$ <p>The correct feasible region is :</p> <p>(A) ABC (B) AOEC</p> <p>(C) CED (D) Open unbounded region BCD</p>	

Ans	(B) AOEC	1
18.	<p>The sum of the order and degree of the differential equation</p> $\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^3 = \frac{d^2y}{dx^2} \text{ is :}$ <p>(A) 2                      (B) <math>\frac{5}{2}</math>                      (C) 3                      (D) 4</p>	
Ans	(C) 3	1
<p>Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.</p> <p>(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).</p> <p>(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is <b>not</b> the correct explanation of the Assertion (A).</p> <p>(C) Assertion (A) is true, but Reason (R) is false.</p> <p>(D) Assertion (A) is false, but Reason (R) is true.</p>		
19.	<p><b>Assertion (A) :</b> The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).</p>  <p>Min <math>Z = 50x + 70y</math>  subject to constraints  <math>2x + y \geq 8</math>, <math>x + 2y \geq 10</math>, <math>x, y \geq 0</math>  <math>Z = 50x + 70y</math> has a minimum value = 380 at B(2, 4).</p> <p><b>Reason (R) :</b> The region representing <math>50x + 70y &lt; 380</math> does not have any point common with the feasible region.</p>	

Ans	(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).	1
20.	<p><i>Assertion (A)</i> : Let <math>A = \{x \in \mathbb{R} : -1 \leq x \leq 1\}</math>. If <math>f : A \rightarrow A</math> be defined as <math>f(x) = x^2</math>, then <math>f</math> is not an onto function.</p> <p><i>Reason (R)</i> : If <math>y = -1 \in A</math>, then <math>x = \pm \sqrt{-1} \notin A</math>.</p>	
Ans	(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).	1
<b>SECTION-B</b> <i>This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.</i>		
21.	Find the domain of $\sec^{-1}(2x + 1)$ .	
Ans	<p>Domain of <math>\sec^{-1} x</math> is <math>(-\infty, -1] \cup [1, \infty)</math></p> <p><math>\Rightarrow 2x + 1 \leq -1</math> or <math>2x + 1 \geq 1 \Rightarrow x \leq -1</math> or <math>x \geq 0</math></p> <p><math>\Rightarrow</math> <b>Domain</b> = <math>(-\infty, -1] \cup [0, \infty)</math></p>	<p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
22.	The radius of a cylinder is decreasing at a rate of 2 cm/s and the altitude is increasing at the rate of 3 cm/s. Find the rate of change of volume of this cylinder when its radius is 4 cm and altitude is 6 cm.	
Ans	<p><math>\frac{dr}{dt} = -2 \text{ cm/s}, \frac{dh}{dt} = 3 \text{ cm/s}, \left(\frac{dV}{dt}\right)_{r=4, h=6} = ?</math></p> <p><math>V = \pi r^2 h \Rightarrow \frac{dV}{dt} = 2\pi r \cdot \frac{dr}{dt} \cdot h + \pi r^2 \frac{dh}{dt}</math></p> <p>When <math>r = 4 \text{ cm}</math> and <math>h = 6 \text{ cm}</math>,</p> <p><math>\frac{dV}{dt} = 2\pi(4)(-2)(6) + \pi(4)^2(3) = -48\pi \text{ cm}^3/\text{s}</math></p> <p>Volume is decreasing at the rate of <math>48\pi \text{ cm}^3/\text{s}</math></p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p>

23.	<p>(a) Find a vector of magnitude 5 which is perpendicular to both the vectors <math>3\hat{i} - 2\hat{j} + \hat{k}</math> and <math>4\hat{i} + 3\hat{j} - 2\hat{k}</math>.</p> <p style="text-align: center;"><b>OR</b></p> <p>(b) Let <math>\vec{a}, \vec{b}</math> and <math>\vec{c}</math> be three vectors such that <math>\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}</math> and <math>\vec{a} \times \vec{b} = \vec{a} \times \vec{c}</math>, <math>\vec{a} \neq 0</math>. Show that <math>\vec{b} = \vec{c}</math>.</p>	
Ans	<p>Let <math>\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}</math>, <math>\vec{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}</math></p> $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \hat{i} + 10\hat{j} + 17\hat{k}$ $ \vec{a} \times \vec{b}  = \sqrt{1^2 + 10^2 + 17^2} = \sqrt{390}$ <p>Unit vector <math>\hat{n} = \frac{\vec{a} \times \vec{b}}{ \vec{a} \times \vec{b} } = \frac{1}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})</math></p> <p><math>\therefore</math> Required vector = <math>\frac{5}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})</math></p> <p style="text-align: center;"><b>OR</b></p> <p>(b) <math>\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0</math>  <math>\Rightarrow</math> either <math>\vec{b} = \vec{c}</math> or <math>\vec{a} \perp (\vec{b} - \vec{c})</math>, since <math>\vec{a} \neq 0</math></p> <p>Also, <math>\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = 0</math>  <math>\Rightarrow</math> either <math>\vec{b} = \vec{c}</math> or <math>\vec{a} \parallel (\vec{b} - \vec{c})</math>, since <math>\vec{a} \neq 0</math></p> <p>Since vectors <math>\vec{a}</math> and <math>(\vec{b} - \vec{c})</math> cannot be <math>\parallel</math> and <math>\perp</math> simultaneously</p> <p>Hence <math>\vec{b} = \vec{c}</math></p>	<p style="text-align: right;">1/2</p> <p style="text-align: right;">1/2</p> <p style="text-align: right;">1/2</p> <p style="text-align: right;">1/2</p> <p style="text-align: right;"><b>1</b></p> <p style="text-align: right;">1/2</p> <p style="text-align: right;">1/2</p>
24.	<p>A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, -2) and B (6, 2, -3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.</p>	
Ans	<div style="text-align: center;">  </div> <p>Let P and Q trisect the wire AB.</p> <p>P divides AB in the ratio 1:2 then, coordinate of point P = <math>(\frac{14}{3}, \frac{4}{3}, -\frac{7}{3})</math></p> <p>Q divides AB in the ratio 2:1 then, coordinate of point Q = <math>(\frac{16}{3}, \frac{5}{3}, -\frac{8}{3})</math></p>	<p style="text-align: right;"><b>1</b></p> <p style="text-align: right;"><b>1</b></p>

25.	<p>(a) Differentiate <math>\frac{\sin x}{\sqrt{\cos x}}</math> with respect to x.</p> <p style="text-align: center;"><b>OR</b></p> <p>(b) If <math>y = 5 \cos x - 3 \sin x</math>, prove that <math>\frac{d^2y}{dx^2} + y = 0</math>.</p>	
Ans	<p>(a) Let <math>y = \frac{\sin x}{\sqrt{\cos x}}</math></p> $\frac{dy}{dx} = \frac{\sqrt{\cos x} \cdot \cos x - \sin x \cdot \left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{2\cos^2 x + \sin^2 x}{2(\cos x)^{3/2}} \text{ or } \frac{1 + \cos^2 x}{2(\cos x)^{3/2}}$ <p style="text-align: center;"><b>OR</b></p> <p>(b) <math>y = 5\cos x - 3\sin x</math>, then <math>\frac{dy}{dx} = -5 \cdot \sin x - 3 \cdot \cos x</math></p> $\Rightarrow \frac{d^2y}{dx^2} = -5 \cdot \cos x + 3 \cdot \sin x = -y$ $\Rightarrow \frac{d^2y}{dx^2} + y = 0$	<p style="text-align: right;"><b>1½</b></p> <p style="text-align: right;">½</p> <p style="text-align: right;"><b>1</b></p> <p style="text-align: right;">½</p> <p style="text-align: right;">½</p>
<p><b>SECTION-C</b></p> <p><i>This section comprises 6 Short Answer (SA) type questions of 3 marks each.</i></p>		
26.	<p>Show that <math>f(x) = \tan^{-1}(\sin x + \cos x)</math> is an increasing function in <math>\left[0, \frac{\pi}{4}\right]</math>.</p>	
Ans	$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} (\cos x - \sin x)$ <p>For <math>x \in \left[0, \frac{\pi}{4}\right]</math>, <math>\cos x \geq \sin x</math></p> $\Rightarrow f'(x) \geq 0, f \text{ is an increasing function in } \left[0, \frac{\pi}{4}\right]$	<p style="text-align: right;"><b>1½</b></p> <p style="text-align: right;"><b>1</b></p> <p style="text-align: right;">½</p>

27.	<p>(a) The probability that a student buys a colouring book is 0.7 and that she buys a box of colours is 0.2. The probability that she buys a colouring book, given that she buys a box of colours, is 0.3. Find the probability that the student :</p> <p>(i) Buys both the colouring book and the box of colours.</p> <p>(ii) Buys a box of colours given that she buys the colouring book.</p> <p style="text-align: center;"><b>OR</b></p> <p>(b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find :</p> <p>(i) The probability distribution of the number of oranges he draws.</p> <p>(ii) The expectation of the random variable (number of oranges).</p>	
Ans	<p>(a) Let A be the event of buying colouring book and B be the event of buying coloured box. } 1/2</p> <p><math>P(A) = 0.7, \quad P(B) = 0.2, \quad P(A/B) = 0.3</math> 1/2</p> <p>(i) <math>P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \Rightarrow 0.3 = \frac{P(A \cap B)}{0.2}</math></p> <p><math>\Rightarrow P(A \cap B) = 0.06</math> or <math>\frac{3}{50}</math> 1</p> <p>(ii) <math>P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}</math></p> <p><math>= \frac{0.06}{0.7} = \frac{3}{35}</math> or <math>0.086</math> 1</p> <p style="text-align: center;"><b>OR</b></p> <p>(b) Let X be random variable for number of oranges. 1/2</p> <p><math>X = 0, 1, 2, 3</math></p> <p>Let A be the event that orange is drawn.</p> <p><math>P(A) = \frac{4}{10} = \frac{2}{5}, \quad P(\bar{A}) = 1 - \frac{2}{5} = \frac{3}{5}</math> 1/2</p>	

	<p>(i)</p> <table border="1" data-bbox="336 259 1217 416"> <tr> <td><b>X</b></td> <td><b>0</b></td> <td><b>1</b></td> <td><b>2</b></td> <td><b>3</b></td> </tr> <tr> <td><b>P(X)</b></td> <td><math>\frac{27}{125}</math></td> <td><math>\frac{54}{125}</math></td> <td><math>\frac{36}{125}</math></td> <td><math>\frac{8}{125}</math></td> </tr> </table> <p>(ii) <math>E(X) = \sum p_i x_i = 0 \times \frac{27}{125} + 1 \times \frac{54}{125} + 2 \times \frac{36}{125} + 3 \times \frac{8}{125}</math>  <math>= \frac{150}{125}</math> or <math>\frac{6}{5}</math></p>	<b>X</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>P(X)</b>	$\frac{27}{125}$	$\frac{54}{125}$	$\frac{36}{125}$	$\frac{8}{125}$	<p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
<b>X</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>								
<b>P(X)</b>	$\frac{27}{125}$	$\frac{54}{125}$	$\frac{36}{125}$	$\frac{8}{125}$								
<p>28.</p>	<p>Find the particular solution of the differential equation  <math>\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0</math>; given that <math>y = 0</math>, when <math>x = 1</math>.</p>											
<p>Ans</p>	<p><math>\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec} \frac{y}{x}</math></p> <p>Put <math>y = vx</math></p> <p><math>\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}</math></p> <p><math>\Rightarrow v + x \frac{dv}{dx} = v - \operatorname{cosec} v</math></p> <p><math>\Rightarrow -\int \sin v \, dv = \int \frac{dx}{x}</math></p> <p><math>\Rightarrow \cos v = \log  x  + c</math></p> <p><math>\Rightarrow \cos \frac{y}{x} = \log  x  + c</math></p> <p><math>x = 1, y = 0 \Rightarrow c = 1</math></p> <p><math>\Rightarrow \cos \frac{y}{x} = \log  x  + 1</math></p>	<p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>										

<p><b>29.</b></p>	<p>(a) Find :</p> $\int \frac{2x}{(x^2 + 3)(x^2 - 5)} dx$ <p style="text-align: center;"><b>OR</b></p> <p>(b) Evaluate :</p> $\int_1^4 ( x - 2  +  x - 4 ) dx$	
<p><b>Ans</b></p>	<p>(a) Let <math>I = \int \frac{2x}{(x^2+3)(x^2-5)} dx</math></p> <p>Put <math>x^2 = t \Rightarrow 2x \cdot dx = dt</math></p> $\Rightarrow I = \int \frac{dt}{(t+3)(t-5)}$ $= \int \left( -\frac{1}{8(t+3)} + \frac{1}{8(t-5)} \right) dt$ $= \frac{1}{8} [\log  t - 5  - \log  t + 3 ] + c$ $= \frac{1}{8} \log \left  \frac{x^2-5}{x^2+3} \right  + c$ <p style="text-align: center;"><b>OR</b></p> <p>(b) <math>\int_1^4 ( x - 2  +  x - 4 ) dx</math></p> $= \int_1^2 (2 - x) dx + \int_2^4 (x - 2) dx - \int_1^4 (x - 4) dx$ $= \left[ \frac{(2-x)^2}{-2} \right]_1^2 + \left[ \frac{(x-2)^2}{2} \right]_2^4 - \left[ \frac{(x-4)^2}{2} \right]_1^4$ $= \frac{1}{2} + 2 + \frac{9}{2} = 7$	<p style="text-align: right;">1/2</p> <p style="text-align: right;"><b>1</b></p> <p style="text-align: right;"><b>1</b></p> <p style="text-align: right;">1/2</p> <p style="text-align: right;"><b>1 1/2</b></p> <p style="text-align: right;"><b>1</b></p> <p style="text-align: right;">1/2</p>

30.

In the Linear Programming Problem (LPP), find the point/points giving maximum value for  $Z = 5x + 10y$  subject to constraints

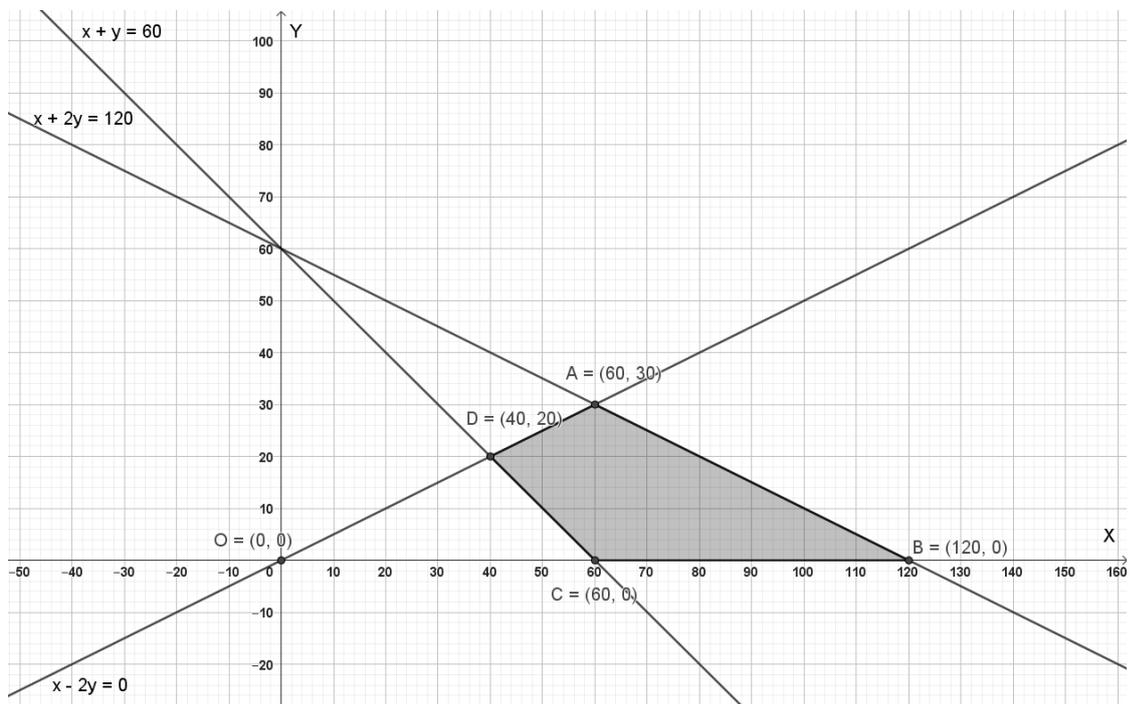
$$x + 2y \leq 120$$

$$x + y \geq 60$$

$$x - 2y \geq 0$$

$$x, y \geq 0$$

Ans



Corner Points	Value of Z
A (60, 30)	600
B (120, 0)	600
C (60, 0)	300
D (40, 20)	400

Since  $Z$  is maximum on points A and B

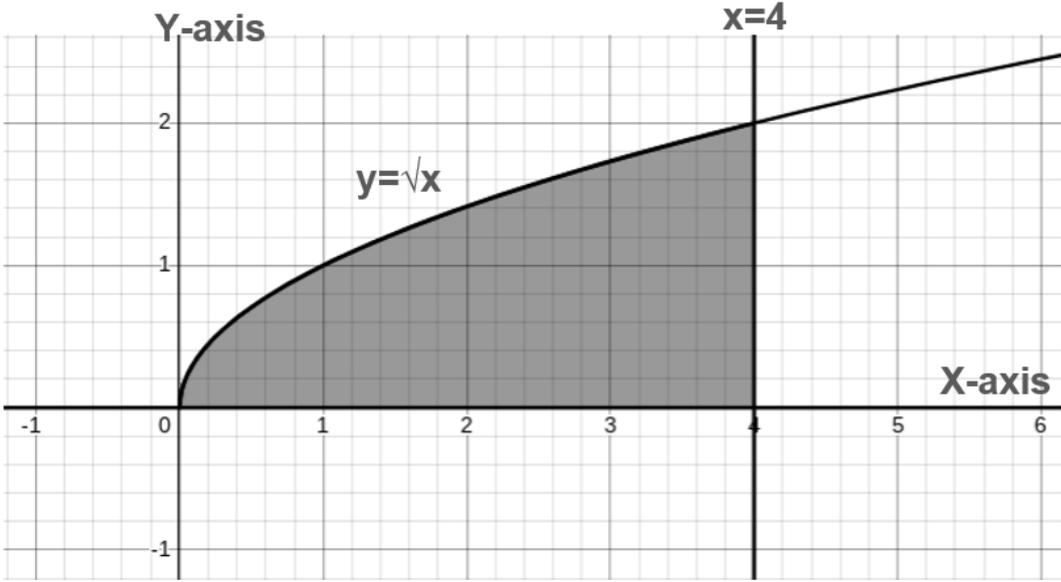
Hence all points lying on segment AB give maximum  $Z$ .

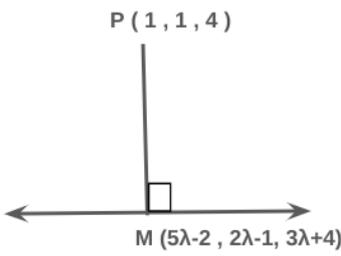
1½ for correct graph and correct feasible region

1

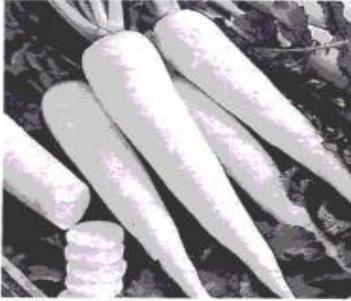
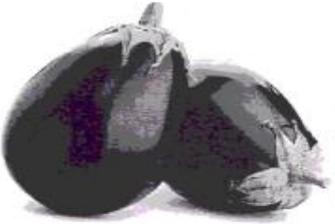
½

31.	<p>(a) If <math>\vec{a} + \vec{b} + \vec{c} = \vec{0}</math> such that <math> \vec{a}  = 3</math>, <math> \vec{b}  = 5</math>, <math> \vec{c}  = 7</math>, then find the angle between <math>\vec{a}</math> and <math>\vec{b}</math>.</p> <p style="text-align: center;"><b>OR</b></p> <p>(b) If <math>\vec{a}</math> and <math>\vec{b}</math> are unit vectors inclined with each other at an angle <math>\theta</math>, then prove that <math>\frac{1}{2}  \vec{a} - \vec{b}  = \sin \frac{\theta}{2}</math>.</p>	
Ans	<p>Given <math>\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow  \vec{a} + \vec{b}  =  -\vec{c} </math></p> <p><math>\Rightarrow  \vec{a} + \vec{b} ^2 =  \vec{c} ^2 \Rightarrow  \vec{a} ^2 +  \vec{b} ^2 + 2\vec{a} \cdot \vec{b} =  \vec{c} ^2</math></p> <p><math>\Rightarrow 9 + 25 + 2\vec{a} \cdot \vec{b} = 49</math></p> <p><math>\Rightarrow 2 \vec{a}  \vec{b} \cos \theta = 15</math></p> <p><math>\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}</math></p> <p style="text-align: center;"><b>OR</b></p> <p>(b) <math> \vec{a}  =  \vec{b}  = 1</math></p> <p><math> \vec{a} - \vec{b} ^2 =  \vec{a} ^2 +  \vec{b} ^2 - 2\vec{a} \cdot \vec{b}</math></p> <p><math>= 1 + 1 - 2 \vec{a}  \vec{b}  \cos \theta</math></p> <p><math>= 2 - 2 \cos \theta</math></p> <p><math>= 2 \left( 2 \sin^2 \frac{\theta}{2} \right) = 4 \sin^2 \frac{\theta}{2}</math></p> <p><math>\Rightarrow \sin \frac{\theta}{2} = \frac{1}{2}  \vec{a} - \vec{b} </math></p>	<p style="text-align: center;"><b>1</b></p>
<p><b>SECTION-D</b></p> <p><i>This section comprises 4 Long Answer (LA) type questions of 5 marks each.</i></p>		
32.	<p>Draw a rough sketch of the curve <math>y = \sqrt{x}</math>. Using integration, find the area of the region bounded by the curve <math>y = \sqrt{x}</math>, <math>x = 4</math> and x-axis, in the first quadrant.</p>	

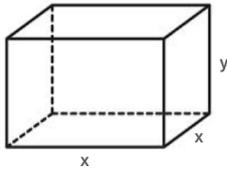
<p>Ans</p>	 <p>Required area = <math>\int_0^4 \sqrt{x} \, dx</math></p> $= \frac{2}{3} [x^{3/2}]_0^4$ $= \frac{2}{3} \times 8 = \frac{16}{3}$	<p>2 marks for correct figure and shading</p> <p>1 1 1</p>
<p>33.</p>	<p>An amount of ₹ 10,000 is put into three investments at the rate of 10%, 12% and 15% per annum. The combined annual income of all three investments is ₹ 1,310, however the combined annual income of the first and the second investments is ₹ 190 short of the income from the third. Use matrix method and find the investment amount in each at the beginning of the year.</p>	
<p>Ans</p>	<p>Let <math>x, y</math> and <math>z</math> (in ₹) be three investment amounts.</p> <p>Then <math>x + y + z = 10,000</math></p> $\frac{10}{100}x + \frac{12}{100}y + \frac{15}{100}z = 1310$ $- \frac{10}{100}x - \frac{12}{100}y + \frac{15}{100}z = 190$ <p>Let <math>A = \begin{bmatrix} 1 &amp; 1 &amp; 1 \\ 10 &amp; 12 &amp; 15 \\ -10 &amp; -12 &amp; 15 \end{bmatrix}</math>, <math>X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math>, <math>B = \begin{bmatrix} 10,000 \\ 1,310,000 \\ 19,000 \end{bmatrix}</math></p> <p><math> A  = 60 \neq 0</math></p> <p><math>\therefore A^{-1}</math> exists</p>	<p>} }</p> <p>1½</p> <p>½</p> <p>½</p>

	$AX = B \Rightarrow X = A^{-1}B$ $\text{adj}(A) = \begin{bmatrix} 360 & -27 & 3 \\ -300 & 25 & -5 \\ 0 & 2 & 2 \end{bmatrix}$ $A^{-1} = \frac{1}{60} \begin{bmatrix} 360 & -27 & 3 \\ -300 & 25 & -5 \\ 0 & 2 & 2 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{60} \begin{bmatrix} 360 & -27 & 3 \\ -300 & 25 & -5 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} 10,000 \\ 1,31,000 \\ 19,000 \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$ <p>Hence the investments are ₹ 2000, ₹ 3000 and ₹ 5000 respectively.</p>	<p>1½</p> <p>1</p>
<p>34.</p>	<p>(a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line <math>\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}</math>.</p> <p style="text-align: center;"><b>OR</b></p> <p>(b) Find the point on the line <math>\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}</math> at a distance of <math>2\sqrt{2}</math> units from the point (-1, -1, 2).</p>	
<p>Ans</p>	<p>(a) Let <math>\frac{x+2}{5} = \frac{y+1}{2} = \frac{z-4}{-3} = \lambda</math></p> <p>Coordinate of general point on the given line are M (5λ - 2, 2λ - 1, 3λ + 4)</p> <div style="text-align: center;">  </div> <p>Direction ratios of PM vector are <math>\langle 5\lambda - 3, 2\lambda - 2, 3\lambda \rangle</math></p> <p>Since, <math>PM \perp l</math></p> $\Rightarrow 5(5\lambda - 3) + 2(2\lambda - 2) + 3(3\lambda) = 0$ $\Rightarrow \lambda = \frac{1}{2}$ <p>Hence, coordinates of M are <math>(\frac{1}{2}, 0, \frac{11}{2})</math></p> <p style="text-align: center;"><b>OR</b></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>

	<p>(b) Equation of given line be <math>\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda</math> (say)</p> <p>Coordinate of any general point on the line are <math>P(3\lambda + 1, 2\lambda - 1, 3\lambda + 4)</math>.</p> <p>Let distance of point P from <math>(-1, -1, 2)</math> is <math>2\sqrt{2}</math>.</p> $\Rightarrow \sqrt{(3\lambda + 2)^2 + (2\lambda)^2 + (3\lambda + 2)^2} = 2\sqrt{2}$ $\Rightarrow 22\lambda^2 + 24\lambda = 0$ $\Rightarrow \lambda = 0 \text{ or } \lambda = -\frac{12}{11}$ <p>Hence, coordinates of point P are <math>(1, -1, 4)</math> or <math>(-\frac{25}{11}, -\frac{35}{11}, \frac{8}{11})</math></p>	<p>1</p> <p>1½</p> <p>1</p> <p>1½</p>
35.	<p>(a) For a positive constant 'a', differentiate <math>a^{t+\frac{1}{t}}</math> with respect to <math>(t+\frac{1}{t})^a</math>, where t is a non-zero real number.</p> <p style="text-align: center;"><b>OR</b></p> <p>(b) Find <math>\frac{dy}{dx}</math> if <math>y^x + x^y + x^x = a^b</math>, where a and b are constants.</p>	
Ans	<p>(a) Let <math>u = a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt} = a^{t+\frac{1}{t}} \cdot \log a \cdot (1 - \frac{1}{t^2})</math></p> $v = (t + \frac{1}{t})^a \Rightarrow \frac{dv}{dt} = a(t + \frac{1}{t})^{a-1} \cdot (1 - \frac{1}{t^2})$ $\frac{du}{dv} = \frac{du/dt}{dv/dt} = \frac{a^{t+\frac{1}{t}} \cdot \log a}{a(t+\frac{1}{t})^{a-1}}$ <p style="text-align: center;"><b>OR</b></p> <p>(b) Let <math>u = y^x</math>, <math>v = x^y</math> and <math>w = x^x</math></p> $\Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0 \dots\dots\dots(i)$ $u = y^x \Rightarrow \log u = x \cdot \log y \Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = \frac{x}{y} \cdot \frac{dy}{dx} + \log y$ $\Rightarrow \frac{du}{dx} = y^x \left( \frac{x}{y} \cdot \frac{dy}{dx} + \log y \right) = xy^{x-1} \frac{dy}{dx} + y^x \log y$ $v = x^y \Rightarrow \log v = y \cdot \log x \Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \frac{y}{x} + \log x \cdot \frac{dy}{dx}$	<p>2</p> <p>2</p> <p>1</p> <p>1</p> <p>1</p>

	$\Rightarrow \frac{dv}{dx} = x^y \left( \frac{y}{x} + \log x \cdot \frac{dy}{dx} \right) = yx^{y-1} + x^y \log x \frac{dy}{dx}$ $w = x^x \Rightarrow \log w = x \cdot \log x \Rightarrow \frac{1}{w} \cdot \frac{dw}{dx} = 1 + \log x$ $\Rightarrow \frac{dw}{dx} = x^x \cdot (1 + \log x)$ <p><math>\therefore</math> From (i), we get</p> $xy^{x-1} \cdot \frac{dy}{dx} + y^x \cdot \log y + yx^{y-1} + x^y \cdot \log x \cdot \frac{dy}{dx} + x^x \cdot (1 + \log x) = 0$ $\Rightarrow \frac{dy}{dx} = - \frac{x^x \cdot (1 + \log x) + y^x \cdot \log y + yx^{y-1}}{x \cdot y^{x-1} + x^y \cdot \log x}$	<p>1</p> <p>1</p> <p>1</p>
<p><b>SECTION-E</b></p> <p><i>This section comprises 3 case study-based questions of 4 marks each</i></p>		
36.	<p style="text-align: center;"><b>Case Study – 1</b></p> <p>A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>Radish</p> </div> <div style="text-align: center;">  <p>Cabbage</p> </div> <div style="text-align: center;">  <p>Brinjal</p> </div> </div> <p>Based upon the above information, answer the following questions :</p> <p>(i) Calculate the probability of a randomly chosen seed to germinate. <span style="float: right;">2</span></p> <p>(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ? <span style="float: right;">2</span></p>	
Ans	<p><b>Let A: Event that chosen seed germinates.</b></p> <p><b>B: Event that Brinjal seed is chosen.</b></p> <p><b>C: Event that Cabbage seed is chosen.</b></p> <p><b>R: Event that Radish seed is chosen.</b></p>	

	$P(B) = \frac{10}{30}; P(C) = \frac{12}{30}; P(R) = \frac{8}{30};$ $P\left(\frac{A}{B}\right) = \frac{25}{100}; P\left(\frac{A}{C}\right) = \frac{35}{100}; P\left(\frac{A}{R}\right) = \frac{40}{100}$ <p>(i) <math display="block">P(A) = P(B) \cdot P\left(\frac{A}{B}\right) + P(C) \cdot P\left(\frac{A}{C}\right) + P(R) \cdot P\left(\frac{A}{R}\right)</math></p> $= \frac{10}{30} \times \frac{25}{100} + \frac{12}{30} \times \frac{35}{100} + \frac{8}{30} \times \frac{40}{100}$ $= \frac{990}{3000} \text{ or } \frac{33}{100}$ <p>(ii) (a) <math display="block">P\left(\frac{C}{A}\right) = \frac{P(C) \cdot P\left(\frac{A}{C}\right)}{P(B) \cdot P\left(\frac{A}{B}\right) + P(C) \cdot P\left(\frac{A}{C}\right) + P(R) \cdot P\left(\frac{A}{R}\right)}</math></p> $= \frac{\frac{12}{30} \times \frac{35}{100}}{\frac{990}{3000}}$ $= \frac{42}{99} \text{ or } \frac{14}{33}$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
37.	<p style="text-align: center;"><b>Case Study – 2</b></p> <p>A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.</p> <p>On the basis of the above information, answer the following questions :</p> <p>(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant.</p> <p>(ii) Find <math>\frac{dS}{dx}</math>.</p> <p>(iii) (a) Find a relation between x and y such that the surface area (S) is minimum.</p> <p style="text-align: center;"><b>OR</b></p> <p>(iii) (b) If surface area (S) is constant, the volume (V) = <math>\frac{1}{4}(Sx - 2x^3)</math>, x being the edge of base. Show that volume (V) is maximum for <math>x = \sqrt{\frac{S}{6}}</math>.</p>	<p>1</p> <p>1</p> <p>2</p> <p>2</p>
Ans		



(i)  $V = x^2y \Rightarrow y = \frac{V}{x^2} \dots \dots \dots$  (i)

Hence,  $S = 2x^2 + 4xy = 2x^2 + \frac{4V}{x}$

(ii)  $\frac{dS}{dx} = 4 \left( x - \frac{V}{x^2} \right)$

(iii) (a)  $\frac{dS}{dx} = 0 \Rightarrow V = x^3 \Rightarrow x^2y = x^3 \Rightarrow y = x$

$\frac{d^2S}{dx^2} = 4 \left( 1 + \frac{2V}{x^3} \right) = 12 > 0 \Rightarrow S$  is minimum if  $y = x$ .

OR

(iii) (b)  $V = \frac{1}{4}(Sx - 2x^3) \Rightarrow \frac{dV}{dx} = \frac{1}{4}(S - 6x^2)$

Put  $\frac{dV}{dx} = 0 \Rightarrow x = \sqrt{\frac{S}{6}}$

$\left( \frac{d^2V}{dx^2} \right)_{x=\sqrt{\frac{S}{6}}} = -3\sqrt{\frac{S}{6}} < 0 \Rightarrow$  Volume is maximum for  $x = \sqrt{\frac{S}{6}}$ .

1

1

1

1

1

1/2

1/2

**Case Study – 3**

Let A be the set of 30 students of class XII in a school. Let  $f : A \rightarrow N$ , N is a set of natural numbers such that function  $f(x) =$  Roll Number of student x.

On the basis of the given information, answer the following :

(i) Is f a bijective function ?

1

(ii) Give reasons to support your answer to (i).

1

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where

$R = \{(x, y) : x, y \text{ are Roll Numbers of students such that } y = 3x\}$ .

List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer.

2

OR

(iii) (b) Let R be a relation defined by

$R = \{(x, y) : x, y \text{ are Roll Numbers of students such that } y = x^3\}$ .

List the elements of R. Is R a function ? Justify your answer.

2

38.

Ans

(i) No, f is not bijective function

1

	<p>(ii) <b>Range = {1, 2, 3, 4, ....., 30} and codomain= N</b>  <b>Since, Range <math>\neq</math> codomain <math>\Rightarrow</math> f is not onto and hence f is not bijective.</b></p> <p>(iii) (a)  <b><math>R = \{(1, 3), (2, 6), (3, 9), (4, 12), (5, 15), (6, 18), (7, 21), (8, 24), (9, 27), (10, 30)\}</math></b>  <b>Since <math>(1, 1) \notin R \Rightarrow R</math> is not reflexive.</b>  <b><math>(1, 3) \in R</math> but <math>(3, 1) \notin R \Rightarrow R</math> is not symmetric</b>  <b><math>(1, 3) \in R, (3, 9) \in R</math> but <math>(1, 9) \notin R \Rightarrow R</math> is not transitive.</b></p> <p style="text-align: center;"><b>OR</b></p> <p>(iii) (b) <b><math>R = \{(1, 1), (2, 8), (3, 27)\}</math></b>  <b><math>\because</math> elements 4, 5, 6 ... 30 do not have an image. Hence the above relation is not a function.</b></p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><b>1</b></p> <p><b>1</b></p> <p><b>1</b></p> <p><b>1</b></p>
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