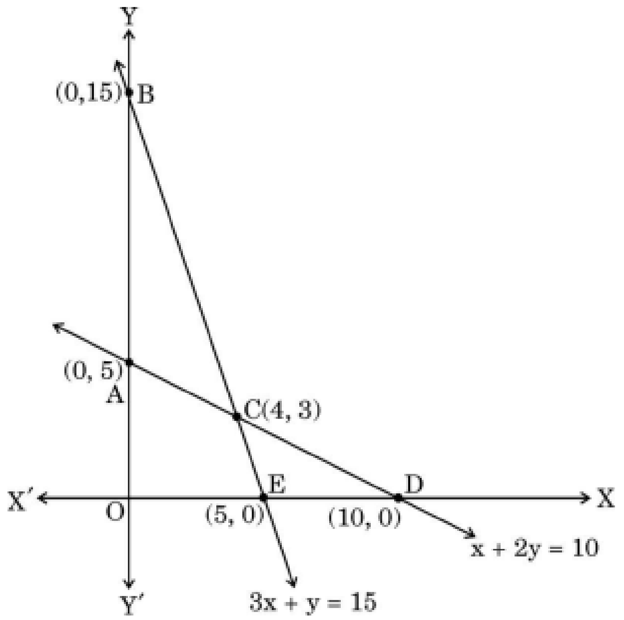
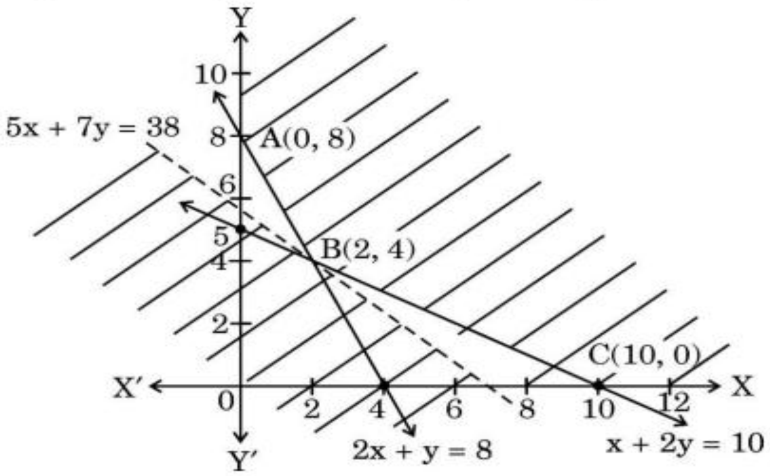
Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A ***This section comprises multiple choice questions (MCQs) of 1 mark each.***	
1.	If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is : (A) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$ (C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ (D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$	
Ans	(B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$	**1**
2.	If $P(A \cup B) = 0{\cdot}9$ and $P(A \cap B) = 0{\cdot}4$, then $P(\overline{A}) + P(\overline{B})$ is : (A) 0·3 (B) 1 (C) 1·3 (D) 0·7	
Ans	**(D) 0.7**	**1**
3.	If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ -1 & 2 \\ 0 & 5 \end{bmatrix}$, then the correct statement is : (A) Only AB is defined. (B) Only BA is defined. (C) AB and BA, both are defined. (D) AB and BA, both are not defined.	
Ans	**(C) AB and BA, both are defined.**	**1**
4.	If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is : (A) 3 (B) 7 (C) ± 7 (D) ± 3	
Ans	**(C) ± 7**	**1**

5.	If $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at x = 0, then the value of a is : (A) 1 (B) –1 (C) ± 1 (D) 0	
Ans	**(C) ± 1**	**1**
6.	If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $\|A\|$ is : (A) 0 (B) –10 (C) 10 (D) 1	
Ans	**(B) -10**	**1**
7.	The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is : (A) $-\frac{\pi}{3}$ (B) $-\frac{2\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$	
Ans	**(D) $\frac{2\pi}{3}$**	**1**
8.	If $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$ is a singular matrix, then the value of x is : (A) 0 (B) 1 (C) – 2 (D) – 4	
Ans	**(C) -2**	**1**
9.	If $f(x) = \{[x], x \in R\}$ is the greatest integer function, then the correct statement is : (A) f is continuous but not differentiable at x = 2. (B) f is neither continuous nor differentiable at x = 2. (C) f is continuous as well as differentiable at x = 2. (D) f is not continuous but differentiable at x = 2.	
Ans	**(B) f is neither continuous nor differentiable at x=2.**	**1**

10.	The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at : (A) (1, –10) (B) (1, 10) (C) (10, 1) (D) (–10, 1)	
Ans	**(A) (1, -10)**	**1**
11.	$\int \sqrt{1+\sin x}\, dx$ is equal to : (A) $2\left(-\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$ (B) $2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + C$ (C) $-2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$ (D) $2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$	
Ans	**(B) $2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + C$**	1
12.	$\int_0^{\pi/2} \cos x . e^{\sin x}\, dx$ is equal to : (A) 0 (B) $1 - e$ (C) $e - 1$ (D) e	
Ans	**(C) e - 1**	**1**
13.	The area of the region enclosed between the curve $y = x\|x\|$, x-axis, $x = -2$ and $x = 2$ is : (A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) 0 (D) 8	
Ans	**(B) $\frac{16}{3}$**	**1**
14.	The integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ is : (A) $e^{-1/\sqrt{x}}$ (B) $e^{2/\sqrt{x}}$ (C) $e^{2\sqrt{x}}$ (D) $e^{-2\sqrt{x}}$	
Ans	**(C) $e^{2\sqrt{x}}$**	**1**

15.	The sum of the order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ is : (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4	
Ans	**(C) 3**	**1**
16.	For a Linear Programming Problem (LPP), the given objective function $Z = 3x + 2y$ is subject to constraints : $x + 2y \le 10$ $3x + y \le 15$ $x, y \ge 0$  The correct feasible region is : (A) ABC (B) AOEC (C) CED (D) Open unbounded region BCD	
Ans	**(B) AOEC**	**1**
17.	Let $\vec{a}$ be a position vector whose tip is the point $(2, -3)$. If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are $(-4, 5)$, then the coordinates of B are : (A) $(-2, -2)$ (B) $(2, -2)$ (C) $(-2, 2)$ (D) $(2, 2)$	
Ans	**(C) (−2, 2)**	**1**
18.	The respective values of $\lvert\vec{a}\rvert$ and $\lvert\vec{b}\rvert$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $\lvert\vec{a}\rvert = 3\lvert\vec{b}\rvert$, are : (A) 48 and 16 (B) 3 and 1 (C) 24 and 8 (D) 6 and 2	
Ans	**(C) 24 and 8**	**1**

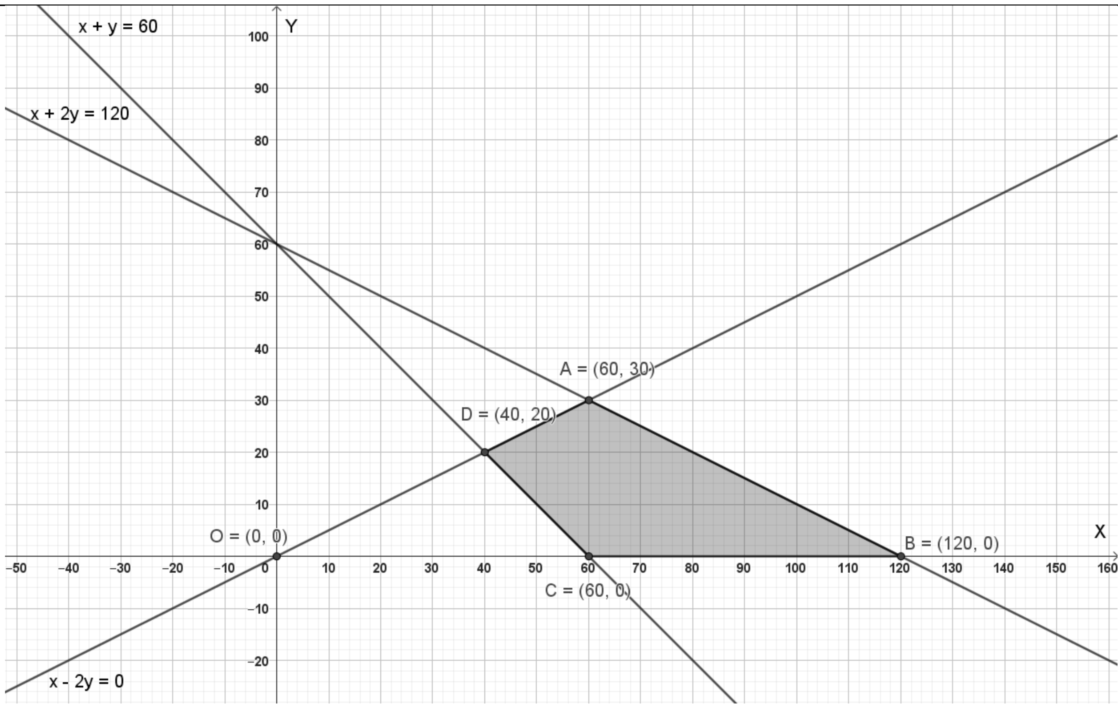
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).  Min Z = 50x + 70y subject to constraints $2x + y \geq 8,\ x + 2y \geq 10,\ x, y \geq 0$ Z = 50x + 70y has a minimum value = 380 at B(2, 4). *Reason (R) :* The region representing 50x + 70y < 380 does not have any point common with the feasible region.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**
20.	*Assertion (A) :* Let $A = \{x \in R : -1 \leq x \leq 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function. *Reason (R) :* If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**

SECTION-B ***This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.***		
21.	Find the domain of the function $f(x) = \cos^{-1}(x^2 - 4)$.	
Ans	**Domain of** $\cos^{-1}x$ **is** $[-1, 1]$ $\Rightarrow -1 \leq x^2 - 4 \leq 1 \Rightarrow 3 \leq x^2 \leq 5$ $\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$	1 ½ ½
22.	Surface area of a balloon (spherical), when air is blown into it, increases at a rate of 5 mm^2/s. When the radius of the balloon is 8 mm, find the rate at which the volume of the balloon is increasing.	
Ans	$\frac{dS}{dt} = 5\ mm^2/s$, $\left(\frac{dV}{dt}\right)_{r=8} = ?$ $S=4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r.\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{5}{8\pi r}$ $V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2.\frac{dr}{dt} \Rightarrow \frac{dV}{dt} = \frac{5}{2}r$ $\Rightarrow \left(\frac{dV}{dt}\right)_{r=8} = 20\ mm^3/s$	 ½ 1 ½
23.	(a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x. **OR** (b) If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.	
Ans	**(a) Let** $y = \frac{\sin x}{\sqrt{\cos x}}$ $\frac{dy}{dx} = \frac{\sqrt{\cos x}.\cos x - \sin x.\left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{2\cos^2 x+\sin^2 x}{2(\cos x)^{3/2}}$ or $\frac{1+\cos^2 x}{2(\cos x)^{3/2}}$ **OR** **(b)** $y = 5\cos x - 3\sin x$, **then** $\frac{dy}{dx} = -5.\sin x - 3.\cos x$ $\Rightarrow \frac{d^2y}{dx^2} = -5.\cos x + 3.\sin x = -y$ $\Rightarrow \frac{d^2y}{dx^2} + y = 0$	 1½ ½ 1 ½ ½

24.	(a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$. **OR** (b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, $\vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.	
Ans	**Let** $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}, \quad \vec{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$ $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \hat{i} + 10\hat{j} + 17\hat{k}$ $\lvert\vec{a}\times\vec{b}\rvert = \sqrt{1^2 + 10^2 + 17^2} = \sqrt{390}$ **Unit vector** $\hat{n} = \frac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert} = \frac{1}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})$ $\therefore$ **Required vector** $= \frac{5}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})$	 ½ ½ ½ ½
	OR	
	(b) $\vec{a}.\vec{b} = \vec{a}.\vec{c} \Rightarrow \vec{a}.(\vec{b} - \vec{c}) = 0$ $\Rightarrow$ **either** $\vec{b} = \vec{c}$ **or** $\vec{a} \perp (\vec{b} - \vec{c})$, **since** $\vec{a} \neq 0$ **Also,** $\vec{a}\times\vec{b} = \vec{a}\times\vec{c} \Rightarrow \vec{a}\times(\vec{b} - \vec{c}) = 0$ $\Rightarrow$ **either** $\vec{b} = \vec{c}$ **or** $\vec{a} \parallel (\vec{b} - \vec{c})$, **since** $\vec{a} \neq 0$ **Since vectors** $\vec{a}$ **and** $(\vec{b} - \vec{c})$ **cannot be ∥ and ⊥ simultaneously** **Hence** $\vec{b} = \vec{c}$	 1 ½ ½
25.	A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, – 2) and B (6, 2, – 3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.	
Ans	 **Let P and Q trisect the wire AB.** **P divides AB in the ratio 1:2 then, coordinate of point P** $= \left(\frac{14}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ **Q divides AB in the ratio 2:1 then, coordinate of point Q** $= \left(\frac{16}{3}, \frac{5}{3}, -\frac{8}{3}\right)$	 1 1

SECTION-C		
This section comprises 6 Short Answer (SA) type questions of 3 marks each.		
26.	Find the value of 'a' for which $f(x) = \sqrt{3}\sin x - \cos x - 2ax + 6$ is decreasing in R.	
Ans	Since $f(x)$ is a decreasing function $\Rightarrow f'(x) \le 0$ $\Rightarrow \sqrt{3}.\cos x + \sin x - 2a \le 0$ $\Rightarrow 2\left(\frac{\sqrt{3}}{2}.\cos x + \frac{1}{2}\sin x\right) - 2a \le 0$ $\Rightarrow \cos\left(x - \frac{\pi}{6}\right) \le a$ Since, $-1 \le \cos\left(x - \frac{\pi}{6}\right) \le 1 \Rightarrow a \ge 1$ i.e. $a \in [1, \infty)$ or $(1, \infty)$	1 1 1
27.	(a) Find : $\int \frac{2x}{(x^2+3)(x^2-5)}\,dx$ **OR** (b) Evaluate : $\int_1^4 (\lvert x-2\rvert + \lvert x-4\rvert)\,dx$	
Ans	**(a)** Let $I = \int \frac{2x}{(x^2+3)(x^2-5)}dx$ Put $x^2 = t \Rightarrow 2x.dx = dt$ $\Longrightarrow I = \int \frac{dt}{(t+3)(t-5)}$ $= \int\left(-\frac{1}{8\,(t+3)} + \frac{1}{8(t-5)}\right)dt$ $= \frac{1}{8}[\log\lvert t-5\rvert - \log\lvert t+3\rvert] + c$ $= \frac{1}{8}\log\left\lvert\frac{x^2-5}{x^2+3}\right\rvert + c$ **OR** **(b)** $\int_1^4 (\lvert x-2\rvert + \lvert x-4\rvert)\,dx$ $= \int_1^2 (2-x)\,dx + \int_2^4 (x-2)dx - \int_1^4 (x-4)\,dx$ $= \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^4 - \left[\frac{(x-4)^2}{2}\right]_1^4$ $= \frac{1}{2} + 2 + \frac{9}{2} = 7$	½ 1 1 ½ 1½ 1 ½

28.	Find the particular solution of the differential equation $$\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$$ given that $y = \frac{\pi}{4}$, when $x = 1$.	
Ans	$\left[x.\sin^2\frac{y}{x} - y\right].dx + x.dy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$ Put $y = vx \implies \frac{dy}{dx} = x\frac{dv}{dx} + v$ $\therefore\ x\frac{dv}{dx} + v = v - \sin^2 v$ $\implies -\int \operatorname{cosec}^2 v\,dv = \int \frac{dx}{x}$ $\implies \cot v = \log\lvert x\rvert + c$ $\implies \cot\frac{y}{x} = \log\lvert x\rvert + c$ $x = 1, y = \frac{\pi}{4} \implies c = 1$ $\implies \cot\frac{y}{x} = \log\lvert x\rvert + 1$	1 1 ½ ½
29.	In the Linear Programming Problem (LPP), find the point/points giving maximum value for $Z = 5x + 10y$ subject to constraints $x + 2y \le 120$ $x + y \ge 60$ $x - 2y \ge 0$ $x, y \ge 0$	
Ans		



1½ for correct graph and correct feasible region

Corner Points	**Value of Z**
A (60, 30)	**600**
B (120, 0)	**600**
C (60, 0)	**300**
D (40, 20)	**400**

1

Since Z is maximum on points A and B

Hence all points lying on segment AB give maximum Z. ½

30.

(a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$.

Ans

Given $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow |\vec{a} + \vec{b}| = |-\vec{c}|$ **1**

$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{c}|^2 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}.\vec{b} = |\vec{c}|^2$

$\Rightarrow 9 + 25 + 2\,\vec{a}.\vec{b} = 49$ **1**

	$\Rightarrow 2\|\vec{a}\|\|\vec{b}\|\cos\theta = 15$	
	$\Rightarrow \cos\theta = \frac{1}{2} \therefore\ \theta = \frac{\pi}{3}$	1
	OR	
	(b) $\|\vec{a}\| = \|\vec{b}\| = 1$	½
	$\|\vec{a} - \vec{b}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2 - 2\vec{a}.\vec{b}$	1
	$= 1 + 1 - 2\|\vec{a}\|\|\vec{b}\| \cos\theta$	½
	$= 2 - 2\cos\theta$	
	$= 2\left(2\sin^2\frac{\theta}{2}\right) = 4\sin^2\frac{\theta}{2}$	½
	$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}\|\vec{a} - \vec{b}\|$	½
31.	(a) The probability that a student buys a colouring book is 0·7 and that she buys a box of colours is 0·2. The probability that she buys a colouring book, given that she buys a box of colours, is 0·3. Find the probability that the student : (i) Buys both the colouring book and the box of colours. (ii) Buys a box of colours given that she buys the colouring book. **OR** (b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find : (i) The probability distribution of the number of oranges he draws. (ii) The expectation of the random variable (number of oranges).	
Ans	**(a) Let A be the event of buying colouring book and B be the event of buying coloured box.**	½
	$P(A) = 0.7$, $P(B) = 0.2$, $P(A/B) = 0.3$	½
	(i) $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} \Rightarrow 0.3 = \frac{P(A\cap B)}{0.2}$	
	$\Rightarrow P(A \cap B) = 0.06$ or $\frac{3}{50}$	1

(ii) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{0.06}{0.7} = \frac{3}{35}$ or 0.086 — **1**

OR

(b) Let X be random variable for number of oranges.

X = 0, 1, 2, 3 — ½

Let A be the event that orange is drawn.

$P(A) = \frac{4}{10} = \frac{2}{5}$, $\quad P(\bar{A}) = 1 - \frac{2}{5} = \frac{3}{5}$ — ½

(i)

X	0	1	2	3
P(X)	$\frac{27}{125}$	$\frac{54}{125}$	$\frac{36}{125}$	$\frac{8}{125}$

— **1**

(ii) $E(X) = \sum p_i x_i = 0 \times \frac{27}{125} + 1 \times \frac{54}{125} + 2 \times \frac{36}{125} + 3 \times \frac{8}{125}$ — ½

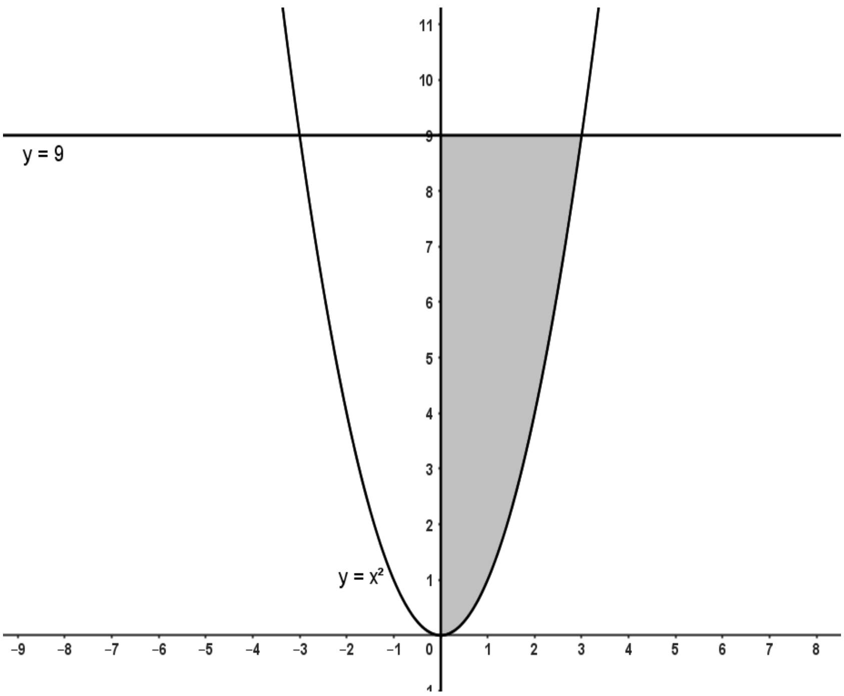
$= \frac{150}{125}$ or $\frac{6}{5}$ — ½

SECTION-D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9$, $x = 0$ and $y = x^2$.

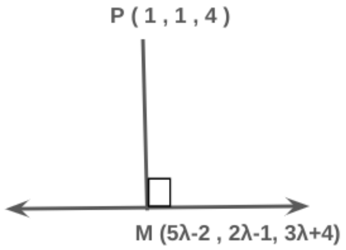
Ans



1½ for correct figure and shading

	Required area $= \int_0^9 \sqrt{y}\, dy$ $= \frac{2}{3}\left[y^{3/2}\right]_0^9$ $= 18$ **Note: If area is found in second quadrant, may be considered.**	1½ 1 1
33.	A furniture workshop produces three types of furniture – chairs, tables and beds each day. On a particular day the total number of furniture pieces produced is 45. It was also found that production of beds exceeds that of chairs by 8, while the total production of beds and chairs together is twice the production of tables. Determine the units produced of each type of furniture, using matrix method.	
Ans	**Let the numbers of chairs, tables and beds produced be x, y and z respectively.** $\therefore\ x+y+z=45;\quad -x+0.y+z=8;\quad x-2y+z=0$ **Let** $A=\begin{bmatrix}1&1&1\\-1&0&1\\1&-2&1\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}45\\8\\0\end{bmatrix}$ $\lvert A\rvert = 1(0+2)-1(-1-1)+1(2-0)=6\neq 0$ $\therefore A^{-1}$ **exists** $AX=B \Rightarrow X=A^{-1}B$ $\text{adj}(A)=\begin{bmatrix}2&-3&1\\2&0&-2\\2&3&1\end{bmatrix}$ $A^{-1}=\frac{1}{6}\begin{bmatrix}2&-3&1\\2&0&-2\\2&3&1\end{bmatrix}$ $\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{6}\begin{bmatrix}2&-3&1\\2&0&-2\\2&3&1\end{bmatrix}\begin{bmatrix}45\\8\\0\end{bmatrix}=\begin{bmatrix}11\\15\\19\end{bmatrix}$ **So,** $x=11, y=15, z=19$ **Hence the numbers of chairs, tables and beds produced are 11, 15 and 19 respectively.**	 1 ½ ½ ½ 1½ 1
34.	(a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number. **OR** (b) Find $\frac{dy}{dx}$ if $y^x+x^y+x^x=a^b$, where a and b are constants.	

Ans	(a) Let $u = a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt} = a^{t+\frac{1}{t}}.\log a.\left(1-\frac{1}{t^2}\right)$ $v = \left(t+\frac{1}{t}\right)^a \Rightarrow \frac{dv}{dt} = a\left(t+\frac{1}{t}\right)^{a-1}.\left(1-\frac{1}{t^2}\right)$ $\frac{du}{dv} = \frac{du/dt}{dv/dt} = \frac{a^{t+\frac{1}{t}}.\log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$ **OR** (b) Let $u = y^x$, $v = x^y$ and $w = x^x$ $\Rightarrow \frac{du}{dx}+\frac{dv}{dx}+\frac{dw}{dx} = 0$ ....(i) $u = y^x \Rightarrow \log u = x.\log y \Rightarrow \frac{1}{u}.\frac{du}{dx} = \frac{x}{y}.\frac{dy}{dx}+\log y$ $\Rightarrow \frac{du}{dx} = y^x\left(\frac{x}{y}.\frac{dy}{dx}+\log y\right) = xy^{x-1}\frac{dy}{dx}+y^x\log y$ $v = x^y \Rightarrow \log v = y.\log x \Rightarrow \frac{1}{v}.\frac{dv}{dx} = \frac{y}{x}+\log x.\frac{dy}{dx}$ $\Rightarrow \frac{dv}{dx} = x^y\left(\frac{y}{x}+\log x.\frac{dy}{dx}\right) = yx^{y-1}+x^y\log x\frac{dy}{dx}$ $w = x^x \Rightarrow \log w = x.\log x \Rightarrow \frac{1}{w}.\frac{dw}{dx} = 1+\log x$ $\Rightarrow \frac{dw}{dx} = x^x.(1+\log x)$ ∴ From (i), we get $xy^{x-1}.\frac{dy}{dx}+y^x.\log y+yx^{y-1}+x^y.\log x.\frac{dy}{dx}+x^x.(1+\log x) = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x^x.(1+\log x)+y^x.\log y+yx^{y-1}}{x.y^{x-1}+x^y.\log x}$	2 2 1 1 1 1 1 1
35.	(a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$. **OR** (b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point (–1, –1, 2).	
Ans	(a) Let $\frac{x+2}{5} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ Coordinate of general point on the given line are $M\ (5\lambda-2, 2\lambda-1, 3\lambda+4)$	1



	Direction Ratios of PM vector are $< 5\lambda - 3, 2\lambda - 2, 3\lambda >$	**1**
	Since, PM $\perp l$	
	$\Rightarrow 5(5\lambda - 3) + 2(2\lambda - 2) + 3(3\lambda) = 0$	**1**
	$\Rightarrow \lambda = \frac{1}{2}$	**1**
	Hence, coordinates of M are $\left(\frac{1}{2}, 0, \frac{11}{2}\right)$	**1**
	OR	
	(b) Equation of given line be $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ **(say)**	
	Coordinate of any general point on the line are P $(3\lambda + 1, 2\lambda - 1, 3\lambda + 4)$.	**1**
	Let distance of point P from $(-1, -1, 2)$ **is** $2\sqrt{2}$.	
	$\Longrightarrow \sqrt{(3\lambda + 2)^2 + (2\lambda)^2 + (3\lambda + 2)^2} = 2\sqrt{2}$	**1½**
	$\Longrightarrow 22\lambda^2 + 24\lambda = 0$	
	$\Rightarrow \lambda = 0$ **or** $\lambda = -\frac{12}{11}$	**1**
	Hence, coordinates of point P are $(1, -1, 4)$ **or** $\left(-\frac{25}{11}, -\frac{35}{11}, \frac{8}{11}\right)$	**1½**

SECTION-E

This section comprises 3 case study-based questions of 4 marks each

36.

Case Study – 1

A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions :

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant.

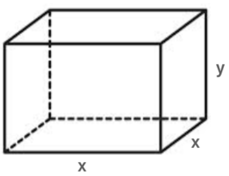
(ii) Find $\frac{dS}{dx}$.

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum.

OR

(iii) (b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for x = $\sqrt{\frac{S}{6}}$.

Ans



(i) $\mathbf{V = x^2y \;\Rightarrow\; y = \frac{V}{x^2} \ldots\ldots\ldots\ldots\ldots(i)}$	
Hence, $\mathbf{S = 2x^2 + 4xy = 2x^2 + \frac{4V}{x}}$	**1**
(ii) $\mathbf{\frac{dS}{dx} = 4\left(x - \frac{V}{x^2}\right)}$	**1**
(iii) **(a)** $\mathbf{\frac{dS}{dx} = 0 \;\Rightarrow\; V = x^3 \;\Rightarrow\; x^2y = x^3 \;\Rightarrow\; y = x}$	**1**
$\mathbf{\frac{d^2S}{dx^2} = 4\left(1 + \frac{2V}{x^3}\right) > 0 \Rightarrow}$ **S is minimum if y = x.**	**1**
OR	
(iii) **(b)** $\mathbf{V = \frac{1}{4}(Sx - 2x^3) \Rightarrow \frac{dV}{dx} = \frac{1}{4}(S - 6x^2)}$	**1**

	Put $\frac{dV}{dx} = 0 \Rightarrow x = \sqrt{\frac{S}{6}}$ $\left(\frac{d^2V}{dx^2}\right)_{x=\sqrt{\frac{S}{6}}} = -3\sqrt{\frac{S}{6}} < 0 \Rightarrow$ **Volume is maximum for** $x = \sqrt{\frac{S}{6}}$.	½ ½
37.	**Case Study – 2** Let A be the set of 30 students of class XII in a school. Let f : A → N, N is a set of natural numbers such that function f(x) = Roll Number of student x. On the basis of the given information, answer the following : (i) Is f a bijective function ? (ii) Give reasons to support your answer to (i). (iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where R = {(x, y) : x, y are Roll Numbers of students such that y = 3x}. List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer. **OR** (iii) (b) Let R be a relation defined by R = {(x, y) : x, y are Roll Numbers of students such that $y = x^3$}. List the elements of R. Is R a function ? Justify your answer.	
Ans	**(i)** **No, f is not bijective function**	1
	(ii) **Range =** $\{1, 2, 3, 4, \ldots\ldots\ldots\ldots\ldots, 30\}$ **and codomain=** N	½
	Since, Range ≠ codomain ⇒ f is not onto and hence f is not bijective.	½
	(iii) **(a)** $R = \{(1,3), (2,6), (3,9), (4,12), (5,15), (6,18), (7,21), (8,24), (9,27), (10,30)\}$	1
	Since $(1,1) \notin R \implies$ **R is not reflexive.** $(1,3) \in R$ **but** $(3,1) \notin R \implies$ **R is not symmetric** $(1,3) \in R, (3,9) \in R$ **but** $(1,9) \notin R \implies$ **R is not transitive.**	1
	OR	
	(iii) **(b)** $R = \{(1,1), (2,8), (3,27)\}$	1
	∵ elements 4, 5, 6 … 30 do not have an image. Hence the above relation is not a function.	1

38.

Case Study – 3

A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.


Radish


Cabbage


Brinjal

Based upon the above information, answer the following questions :

(i) Calculate the probability of a randomly chosen seed to germinate.

(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ?

Ans

Let A: Event that chosen seed germinates.

B: Event that Brinjal seed is chosen.

C: Event that Cabbage seed is chosen.

R: Event that Radish seed is chosen.

$P(B) = \frac{10}{30};\ P(C) = \frac{12}{30};\ P(R) = \frac{8}{30};$

$P\left(\frac{A}{B}\right) = \frac{25}{100};\ P\left(\frac{A}{C}\right) = \frac{35}{100};\ P\left(\frac{A}{R}\right) = \frac{40}{100}$ } **1**

(i) $P(A) = P(B).P\left(\frac{A}{B}\right) + P(C).P\left(\frac{A}{C}\right) + P(R).P\left(\frac{A}{R}\right)$

$= \frac{10}{30} \times \frac{25}{100} + \frac{12}{30} \times \frac{35}{100} + \frac{8}{30} \times \frac{40}{100}$

$= \frac{990}{3000}$ **or** $\frac{33}{100}$ **1**

(ii) **(a)** $P\left(\frac{C}{A}\right) = \frac{P(C).P\left(\frac{A}{C}\right)}{P(B).P\left(\frac{A}{B}\right)+P(C).P\left(\frac{A}{C}\right)+P(R).P\left(\frac{A}{R}\right)}$

$= \frac{\frac{12}{30}\times\frac{35}{100}}{\frac{990}{3000}}$ **1**

$= \frac{42}{99}$ **or** $\frac{14}{33}$ **1**