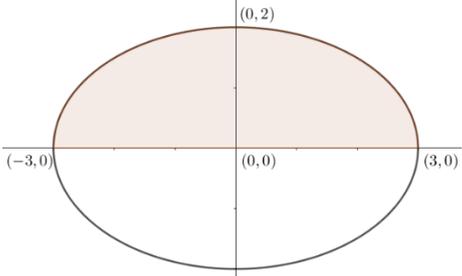


Q5.	<p>The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are :</p> <p>(A) $1 < \lambda < \sqrt{2}$ (B) $\lambda \geq 1$ (C) $\lambda \geq \sqrt{2}$ (D) $\lambda < 1$</p>	
A5.	(C) $\lambda \geq \sqrt{2}$	1
Q6.	<p>If P is a point on the line segment joining (3, 6, -1) and (6, 2, -2) and y-coordinate of P is 4, then its z-coordinate is :</p> <p>(A) $-\frac{3}{2}$ (B) 0 (C) 1 (D) $\frac{3}{2}$</p>	
A6.	(A) $-\frac{3}{2}$	1
Q7.	<p>If M and N are square matrices of order 3 such that $\det(M) = m$ and $MN = mI$, then $\det(N)$ is equal to :</p> <p>(A) -1 (B) 1 (C) $-m^2$ (D) m^2</p>	
A7.	(D) m^2	1
Q8.	<p>If $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to :</p> <p>(A) -4 (B) $-\frac{7}{2}$ (C) -2 (D) -1</p>	
A8.	(D) -1	1

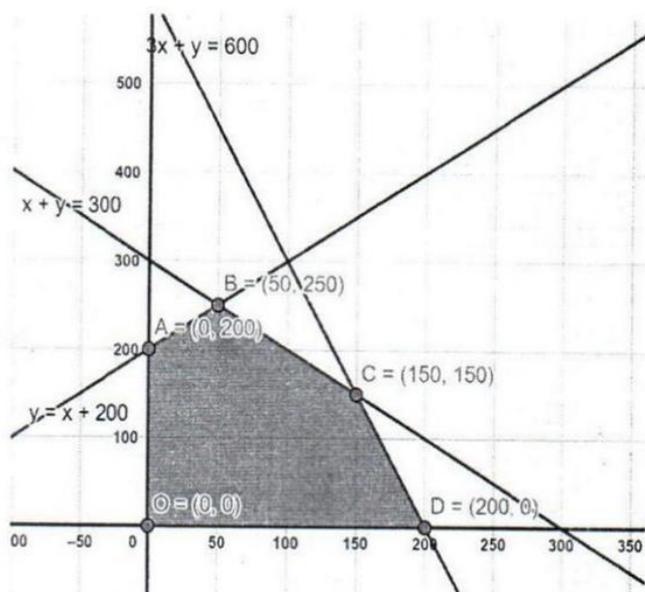
Q13.	If $f(x) = 2x + \cos x$, then $f(x)$: (A) has a maxima at $x = \pi$ (B) has a minima at $x = \pi$ (C) is an increasing function (D) is a decreasing function	
A13.	(C) is an increasing function	1
Q14.	$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to : (A) $2(\sin x + x \cos \alpha) + C$ (B) $2(\sin x - x \cos \alpha) + C$ (C) $2(\sin x + 2x \cos \alpha) + C$ (D) $2(\sin x + \sin \alpha) + C$	
A14.	(A) $2(\sin x + x \cos \alpha) + C$	1
Q15.	The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is : (A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\tan^{-1} e - \frac{\pi}{4}$ (D) $\tan^{-1} e$	
A15.	(C) $\tan^{-1} e - \frac{\pi}{4}$	1
Q16.	The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$ are : (A) order 2, degree 2 (B) order 2, degree 1 (C) order 2, degree not defined (D) order 1, degree not defined	
A16.	(C) order 2, degree not defined	1
Q17.	The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is : (A) $\frac{16}{9}$ sq. units (B) $\frac{32}{9}$ sq. units (C) $\frac{16}{3}$ sq. units (D) $\frac{32}{3}$ sq. units	
A17.	(C) $\frac{16}{3}$ sq. units	1

Q22.	<p>(a) Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on $[2, 4]$.</p> <p style="text-align: center;">OR</p> <p>(b) If $f(x) = x + \frac{1}{x}$, $x \geq 1$, show that f is an increasing function.</p>	
A22.(a)	<p>$f(x) = 2x^2 - ax + 3 \Rightarrow f'(x) = 4x - a$</p> <p>Now $2 \leq x \leq 4 \Rightarrow 8 - a \leq 4x - a \leq 16 - a$</p> <p>For f to be an increasing function, $f'(x) \geq 0$</p> <p>$\Rightarrow 8 - a \geq 0 \Rightarrow a \leq 8$</p> <p>$\therefore$ Least value of a does not exist.</p>	<p style="text-align: center;">½</p> <p style="text-align: center;">1</p> <p style="text-align: center;">½</p>
OR		
A22.(b)	<p>$f(x) = x + \frac{1}{x} \Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$</p> <p>Now $\frac{x^2 - 1}{x^2} \geq 0$ for all $x \geq 1$</p> <p>$\Rightarrow f'(x) \geq 0 \Rightarrow f$ is an increasing function.</p>	<p style="text-align: center;">1</p> <p style="text-align: center;">½</p> <p style="text-align: center;">½</p>
Q23.	<p>(a) Simplify $\sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right)$.</p> <p style="text-align: center;">OR</p> <p>(b) Find domain of $\sin^{-1} \sqrt{x-1}$.</p>	
A23.(a)	<p>Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$</p> <p>Now $\sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right)$</p> <p>$= \sin^{-1} \left(\frac{\tan \theta}{\sec \theta} \right) = \sin^{-1} (\sin \theta)$</p> <p>$= \theta = \tan^{-1} x$</p>	<p style="text-align: center;">½</p> <p style="text-align: center;">1</p> <p style="text-align: center;">½</p>
OR		
A23.(b)	<p>Here $-1 \leq \sqrt{x-1} \leq 1$</p> <p>$\Rightarrow 0 \leq x-1 \leq 1 \Rightarrow 1 \leq x \leq 2$</p> <p>Hence, domain is $x \in [1, 2]$</p>	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p>

<p>Q24.</p>	<p>Calculate the area of the region bounded by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the x-axis using integration.</p>	
<p>A24.</p>	 $A = 2 \times \frac{2}{3} \int_0^3 \sqrt{9-x^2} dx$ $= \frac{4}{3} \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \left(\frac{x}{3} \right) \right]_0^3$ $= \frac{4}{3} \left[\left(0 + \frac{9}{2} \sin^{-1} 1 \right) - 0 \right]$ $= 3\pi$	<p>(½ for correct figure)</p> <p>½</p> <p>½</p> <p>½</p>
<p>Q25.</p>	<p>For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x = 2$?</p>	
<p>A25.</p>	<p>$y = 5x - 2x^3$</p> <p>Given $\frac{dx}{dt} = 2$ units/s</p> <p>slope of the curve $= \frac{dy}{dx} = 5 - 6x^2 = m$</p> <p>$\frac{dm}{dt} = -12x \frac{dx}{dt} = -12x(2) = -24x$</p> <p>at $x = 2, \frac{dm}{dt} = -24(2) = -48$</p> <p>Hence, slope of curve is decreasing at the rate of 48</p>	<p>½</p> <p>½</p> <p>½</p> <p>½</p>
<p>SECTION C</p>		
<p>This section comprises short answer (SA) type questions of 3 marks each.</p>		
<p>Q26.</p>	<p>(a) If $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ is defined as $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$), prove that f is a bijection. (\mathbb{R}^+ is a set of all positive real numbers.)</p> <p style="text-align: center;">OR</p> <p>(b) Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.</p> <p>(i) Write all elements of R.</p> <p>(ii) Is R a function ? Justify.</p> <p>(iii) Determine domain and range of R.</p>	

A29.(b)	<p>Possible values of X are $-2, 0, 2$</p> <table border="1" data-bbox="261 244 912 423"> <tbody> <tr> <td>X</td> <td>-2</td> <td>0</td> <td>2</td> </tr> <tr> <td>$P(X)$</td> <td>$\frac{1}{4}$</td> <td>$\frac{2}{4} = \frac{1}{2}$</td> <td>$\frac{1}{4}$</td> </tr> </tbody> </table> <p>Mean = $\sum XP(X) = -2\left(\frac{1}{4}\right) + 0\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right) = 0$</p>	X	-2	0	2	$P(X)$	$\frac{1}{4}$	$\frac{2}{4} = \frac{1}{2}$	$\frac{1}{4}$	<p>$\frac{1}{2}$</p> <p>$1\frac{1}{2}$</p> <p>1</p>
X	-2	0	2							
$P(X)$	$\frac{1}{4}$	$\frac{2}{4} = \frac{1}{2}$	$\frac{1}{4}$							
Q30.	<p>Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.</p>									
A30.	<p>$l_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$</p> <p>Any point on l_1 is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$</p> <p>$l_2: \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$</p> <p>Any point on l_2 is $(5\mu + 4, 2\mu + 1, \mu)$</p> <p>For point of intersection,</p> <p>$2\lambda + 1 = 5\mu + 4, 3\lambda + 2 = 2\mu + 1$</p> <p>Solving, $\lambda = \mu = -1$</p> <p>Since, $\lambda = \mu = -1$ satisfy $4\lambda + 3 = \mu$</p> <p>\therefore Point of intersection is $(-1, -1, -1)$</p> <p>Now distance of $(-1, -5, -10)$ from $(-1, -1, -1)$ is:</p> <p>$\sqrt{(-1+1)^2 + (-1+5)^2 + (-1+10)^2} = \sqrt{97}$ units</p>	<p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>								
Q31.	<p>Solve the following Linear Programming Problem using graphical method :</p> <p>Maximise $Z = 100x + 50y$</p> <p>subject to the constraints</p> <p>$3x + y \leq 600$</p> <p>$x + y \leq 300$</p> <p>$y \leq x + 200$</p> <p>$x \geq 0, y \geq 0$</p>									

A31.



Corner Point	Value of $Z = 100x + 50y$
$O(0,0)$	0
$A(0,200)$	10000
$B(50,250)$	17500
$C(150,150)$	22500
$D(200,0)$	20000

$Z_{\max} = 22500$ when $x = 150, y = 150$

For correct graph and shading
1½

For correct table
1

½

SECTION D

This section comprises long answer (LA) type questions of **5 marks each**.

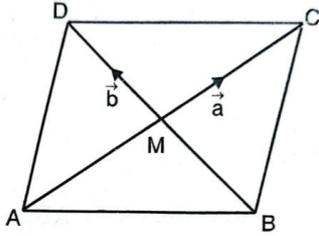
Q32.

If A is a 3×3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

<p>A32.</p>	<p>Consider $(kA)\left(\frac{1}{k}A^{-1}\right) = k \cdot \frac{1}{k}(A \cdot A^{-1}) = I$</p> <p>$\Rightarrow kA$ and $\frac{1}{k}A^{-1}$ are inverse of each other.</p> <p>$\therefore (kA)^{-1} = \frac{1}{k}A^{-1}$</p> <p>$\therefore (3A)^{-1} = \frac{1}{3}A^{-1}$</p> <p>Here, $A = 4 \neq 0 \therefore A^{-1}$ exists.</p> $adjA = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ <p>$\therefore A^{-1} = \frac{1}{ A } \cdot adjA = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$</p> <p>$\therefore (3A)^{-1} = \frac{1}{12} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$</p>	<p style="text-align: right;">}</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">2</p> <p style="text-align: right;">$\frac{1}{2}$</p> <p style="text-align: right;">$\frac{1}{2}$</p>
<p>Q33.</p>	<p>The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.</p> <p>(i) Find the rate of growth of the plant with respect to sunlight.</p> <p>(ii) In how many days will the plant attain its maximum height ? What is the maximum height ?</p>	<p style="text-align: right;">2</p> <p style="text-align: right;">3</p>
<p>A33.</p>	<p>(i) $y = 4x - \frac{1}{2}x^2 \Rightarrow \frac{dy}{dx} = (4 - x)$ cm/day</p> <p>(ii) For maximum height, $\frac{dy}{dx} = 0 \Rightarrow x = 4$ days</p> <p>as $\frac{d^2y}{dx^2} < 0$, number of days = 4</p> <p>Now, Maximum height = $y(4) = 16 - \frac{1}{2}(16) = 8$ cm</p>	<p style="text-align: right;">2</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">1</p>

<p>Q34.</p>	<p>(a) Find :</p> $\int \frac{\cos x}{(4 + \sin^2 x)(5 - 4 \cos^2 x)} dx$ <p style="text-align: center;">OR</p> <p>(b) Evaluate :</p> $\int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$	
<p>A34.(a)</p>	$I = \int \frac{\cos x}{(4 + \sin^2 x)(5 - 4 \cos^2 x)} dx$ $= \int \frac{\cos x}{(4 + \sin^2 x)(1 + 4 \sin^2 x)} dx$ <p>$\sin x = t$ gives</p> $I = \int \frac{dt}{(4 + t^2)(1 + 4t^2)}$ $= -\frac{1}{15} \int \frac{dt}{4 + t^2} + \frac{4}{15} \int \frac{dt}{1 + 4t^2} \quad (\because \text{using Partial Fraction})$ $= -\frac{1}{30} \tan^{-1}\left(\frac{t}{2}\right) + \frac{2}{15} \tan^{-1}(2t) + C$ $= -\frac{1}{30} \tan^{-1}\left(\frac{\sin x}{2}\right) + \frac{2}{15} \tan^{-1}(2 \sin x) + C$	<p>1</p> <p>1/2</p> <p>2</p> <p>1</p> <p>1/2</p>
OR		

<p>A34.(b)</p>	$I = \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ $= 2 \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ $= 2 \int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$ <p>$\tan x = t$ gives</p> $I = 2 \int_0^{\infty} \frac{dt}{a^2 + b^2 t^2}$ $= \frac{2}{b^2} \cdot \frac{b}{a} \tan^{-1} \left(\frac{bt}{a} \right) \Bigg _0^{\infty}$ $= \frac{\pi}{ab}$	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1 1/2</p> <p>1</p>
<p>Q35.</p>	<p>(a) Show that the area of a parallelogram whose diagonals are represented by \vec{a} and \vec{b} is given by $\frac{1}{2} \vec{a} \times \vec{b}$. Also find the area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.</p> <p style="text-align: center;">OR</p> <p>(b) Find the equation of a line in vector and cartesian form which passes through the point $(1, 2, -4)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$, and</p> $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}).$	
<p>A35.(a)</p>	<p>Let $ABCD$ be the parallelogram with diagonals $\overline{AC} = \vec{a}$ and $\overline{BD} = \vec{b}$.</p> $\therefore \overline{AB} = \frac{1}{2}(\vec{a} - \vec{b}) \text{ and } \overline{AD} = \frac{1}{2}(\vec{a} + \vec{b})$ <p>Area of $ABCD$</p> $= \overline{AB} \times \overline{AD} $ $= \left \frac{1}{2}(\vec{a} - \vec{b}) \times \frac{1}{2}(\vec{a} + \vec{b}) \right $ $= \frac{1}{4} \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b} $	 <p>1/2</p> <p>1/2</p> <p>1/2</p>

	$= \frac{1}{4} \vec{a} \times \vec{b} + \vec{a} \times \vec{b} \quad (\because \vec{a} \times \vec{a} = \vec{0})$ $= \frac{1}{4} 2(\vec{a} \times \vec{b}) $ $= \frac{1}{2} \vec{a} \times \vec{b} $ <p>Given $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$</p> <p>Area of parallelogram $= \frac{1}{2} \vec{a} \times \vec{b}$</p> <p>Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -2\hat{i} + 3\hat{j} + 7\hat{k}$</p> $ \vec{a} \times \vec{b} = \sqrt{62}$ <p>Area of parallelogram $= \frac{1}{2} \sqrt{62}$</p>	<p>$\frac{1}{2}$</p> <p>2</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
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OR

A35.(b)	<p>Given lines are $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$</p> <p>and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$</p> <p>The first line in vector form is $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$</p> <p>$\vec{a}_1 = 8\hat{i} - 19\hat{j} + 10\hat{k}$, $\vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k}$</p> <p>$\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$, $\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$</p> $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$ <p>\therefore Equation of line passing through $(1, 2, -4)$ and parallel to \vec{b} is</p> <p>$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + t(24\hat{i} + 36\hat{j} + 72\hat{k})$ or $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + t'(2\hat{i} + 3\hat{j} + 6\hat{k})$</p> <p>Cartesian form of line is $\frac{x-1}{24} = \frac{y-2}{36} = \frac{z+4}{72}$ or $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
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SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36.

Some students are having a misconception while comparing decimals. For example, a student may mention that $78.56 > 78.9$ as $7856 > 789$. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table :

Name of student	Distance of javelin (in meters)
Ajay	47.7
Bijoy	47.07
Kartik	43.09
Dinesh	43.9
Devesh	45.2

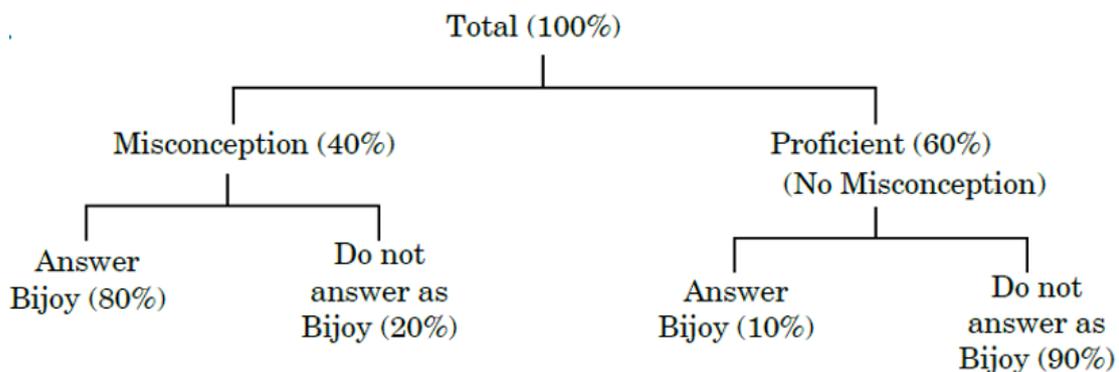
The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

- (i) What is the probability of a student not having misconception but still answers Bijoy in the test ? 1
 - (ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ? 1
 - (iii) (a) What is the probability that a student who answered as Bijoy is having misconception ? 2
- OR**
- (iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ? 2

A36.



	<p>Let E_1 : Student has a misconception E_2 : Student does not have misconception A: Student answered Bijoy as correct</p> <p>$\therefore P(E_1) = \frac{40}{100}, P(E_2) = \frac{60}{100}$</p> <p>$P(A E_1) = \frac{80}{100}, P(A E_2) = \frac{10}{100}$</p> <p>$P(\bar{A} E_1) = \frac{20}{100}, P(\bar{A} E_2) = \frac{90}{100}$</p> <p>(i) $P(A E_2) = \frac{10}{100}$ or $\frac{1}{10}$</p> <p>(ii) $P(A) = P(E_1)P(A E_1) + P(E_2)P(A E_2)$</p> $= \frac{40}{100} \times \frac{80}{100} + \frac{60}{100} \times \frac{10}{100}$ $= \frac{38}{100} \text{ or } \frac{19}{50}$ <p>(iii)(a) $P(E_1 A) = \frac{P(E_1)P(A E_1)}{P(A)}$</p> $= \frac{\frac{40}{100} \times \frac{80}{100}}{\frac{38}{100}} = \frac{16}{19}$ <p style="text-align: center;">OR</p> <p>(iii)(b) $P(E_2 A) = \frac{P(E_2)P(A E_2)}{P(A)}$</p> $= \frac{\frac{60}{100} \times \frac{10}{100}}{\frac{38}{100}} = \frac{3}{19}$	<p>1</p> <p>1</p> <p>2</p> <p>2</p>
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Q37.

An engineer is designing a new metro rail network in a city.



Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by

$$l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}, \text{ while the track for Line B is represented by } l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}.$$

Based on the above information, answer the following questions :

- (i) Find whether the two metro tracks are parallel. 1
- (ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$. 1
- (iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point $(3, 2, 1)$. Determine the equation of the pedestrian walkway. 2

OR

- (iii) (b) Find the shortest distance between Line A and Line B. 2

A37.

$$l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4} ; l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$$

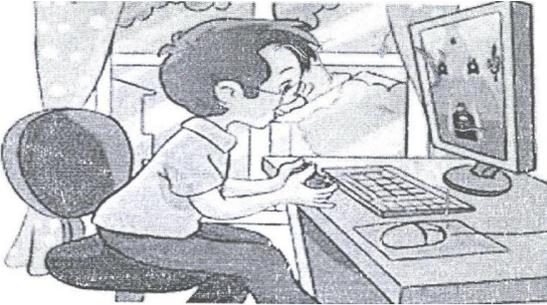
(i) Drs of l_1 are $\langle 3, -2, 4 \rangle$, Drs of l_2 are $\langle 2, 1, -3 \rangle$

as Drs are not proportional, hence l_1 is not parallel to l_2 . 1

(ii) Equations of line parallel to l_1 and passing through $(1, -2, -3)$ is

$$\frac{x-1}{3} = \frac{y+2}{-2} = \frac{z+3}{4} \text{ or } \vec{r} = (\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$$

1

	<p>(iii)(a) The pathway is perpendicular to l_1 and l_2. \therefore It is parallel to $\vec{b}_1 \times \vec{b}_2$</p> $\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = 2\hat{i} + 17\hat{j} + 7\hat{k}$ <p>\therefore Equation of pathway is $\vec{r} = (3\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 17\hat{j} + 7\hat{k})$</p> <p style="text-align: center;">OR</p> <p>(iii)(b) $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} + 3\hat{j} - 2\hat{k}$</p> $\vec{b}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}, \vec{b}_2 = 2\hat{i} + \hat{j} - 3\hat{k}$ $d = \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 }$ $= \frac{ (-\hat{i} + 4\hat{j} - 5\hat{k}) \cdot (2\hat{i} + 17\hat{j} + 7\hat{k}) }{\sqrt{4 + 289 + 49}}$ $= \frac{31}{\sqrt{342}}$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
<p>Q38.</p>	<p>During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$,</p> <p>where $T(t)$ represents the temperature of the processor at time t (in minutes) and k is a constant.</p> <div style="text-align: center;">  </div> <p>Based on the above information, answer the following questions :</p> <p>(i) Find the expression for temperature of processor, $T(t)$ given that $T(0) = 85^\circ\text{C}$. 2</p> <p>(ii) How long will it take for the processor's temperature to reach 40°C ? Given that $k = 0.03$, $\log_e 4 = 1.3863$. 2</p>	

<p>A38.</p>	<p>(i) $\frac{dT}{dt} = -k(T - 25)$</p> <p>$\Rightarrow \frac{dT}{T - 25} = -k dt$</p> <p>$\Rightarrow \int \frac{dT}{T - 25} = -k \int dt$</p> <p>$\Rightarrow \log T - 25 = -kt + C \quad \dots(a)$</p> <p>When $t = 0, T = 85$</p> <p>$\Rightarrow \log 60 = C$</p> <p>Using in equation (a), $\log T - 25 = -kt + \log 60 \quad \dots(b)$</p> <p>(ii) When $k = 0.03, \log T - 25 = -0.03t + \log 60$</p> <p>$\Rightarrow \log \left \frac{T - 25}{60} \right = -0.03t$</p> <p>$\Rightarrow T - 25 = 60.e^{-0.03t}$</p> <p>When $T = 40, t = t_1$</p> <p>$\Rightarrow \frac{15}{60} = e^{-0.03t_1}$</p> <p>$\Rightarrow e^{-0.03t_1} = \frac{1}{4} \Rightarrow -0.03t_1 = -\log 4$</p> <p>$\Rightarrow t_1 = \frac{\log 4}{0.03} = \frac{1.3863}{0.03} = 46.21 \text{ m}$</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
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