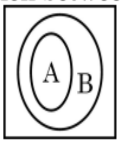
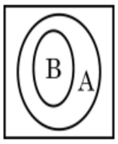
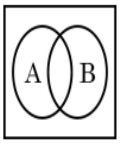
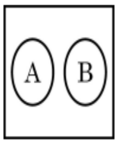
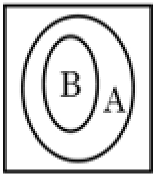
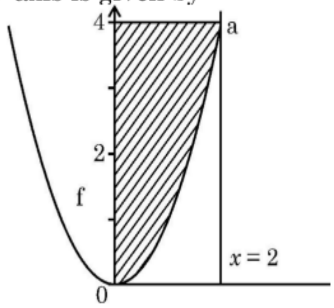
SOLUTIONS – 65/2/2

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
SECTION-A This section comprises multiple choice questions (MCQs) of 1 mark each.		
1.	The values of x for which the angle between the vectors $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse, is : (A) 0 or $\frac{1}{2}$ (B) $x > \frac{1}{2}$ (C) $\left(0, \frac{1}{2}\right)$ (D) $\left[0, \frac{1}{2}\right]$	
Ans	(C) $\left(0, \frac{1}{2}\right)$	1
2.	If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is (A) $\frac{-\pi}{3}$ only (B) $\frac{\pi}{3}$ only (C) $\frac{\pi}{6}$ (D) $\pm\frac{\pi}{3}$	
Ans	No option is correct. Full marks may be awarded for attempting the question.	1
3.	$\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to (A) $\tan x + \cot x + C$ (B) $(\tan x + \cot x)^2 + C$ (C) $\tan x - \cot x + C$ (D) $(\tan x - \cot x)^2 + C$	
Ans	(C) $\tan x - \cot x + C$	1
4.	Let P be a skew-symmetric matrix of order 3. If det(P) = α, then $(2025)^{\alpha}$ is (A) 0 (B) 1 (C) 2025 (D) $(2025)^3$	
Ans	(B) 1	1

5.	The principal value of $\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$ is (A) $\frac{-7\pi}{5}$ (B) $\frac{-\pi}{10}$ (C) $\frac{\pi}{10}$ (D) $\frac{3\pi}{5}$	
Ans	(B) $-\frac{\pi}{10}$	1
6	If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ? (A)  (B)  (C)  (D) 	
Ans	(B) 	1
7.	The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \le x \le 2$ and y-axis is given by  (A) $\int_0^2 x^2\,dx$ (B) $\int_0^2 \sqrt{y}\,dy$ (C) $\int_0^4 x^2\,dx$ (D) $\int_0^4 \sqrt{y}\,dy$	
Ans	(D) $\int_0^4 \sqrt{y}dy$	1

8.	Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4\,AB + 3(AB + BA) - 4\,BA$, where A and B are both matrices of order 2×2. It is known that $A \neq B \neq I$ and $A^{-1} \neq B$. Their answers are given as : Abhay : 6 AB Bina : 7 AB – BA Chhaya : 8 AB Devesh : 7 BA – AB Who answered it correctly ? (A) Abhay (B) Bina (C) Chhaya (D) Devesh	
Ans	(B) Bina	1
9.	If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then $(p - q)$ is (A) 0 (B) 1 (C) 2 (D) 3	
Ans	(B) 1	1
10.	The function $f(x) = x^2 - 4x + 6$ is increasing in the interval (A) $(0, 2)$ (B) $(-\infty, 2]$ (C) $[1, 2]$ (D) $[2, \infty)$	
Ans	(D) $[2, \infty)$	1

11.	In the following probability distribution, the value of p is : X: 0, 1, 2, 3 P(X): p, p, 0.3, 2p (A) $\frac{7}{40}$ (B) $\frac{1}{10}$ (C) $\frac{9}{35}$ (D) $\frac{1}{4}$	
Ans	(A) $\frac{7}{40}$	**1**
12.	If $\overrightarrow{PQ} \times \overrightarrow{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$, then the area (ΔPQR) is (A) 2 sq units (B) 4 sq units (C) 6 sq units (D) 12 sq units	
Ans	(C) 6 sq units	1
13.	If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\overline{E}/\overline{F})$ is (A) $\frac{P(\overline{E})}{P(\overline{F})}$ (B) $1 - P(\overline{E}/F)$ (C) $1 - P(E/F)$ (D) $\frac{1 - P(E \cup F)}{P(\overline{F})}$	
Ans	(D) $\frac{1-P(E\cup F)}{P(\bar{F})}$	1
14.	Which of the following can be both a symmetric and skew-symmetric matrix ? (A) Unit Matrix (B) Diagonal Matrix (C) Null Matrix (D) Row Matrix	
Ans	(C) Null Matrix	1

15.	The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point (4, –3, 7) is : (A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$ (B) $x = 3t + 4, y = t + 3, z = 2t + 7$ (C) $x = 3t + 4, y = t - 3, z = 2t + 7$ (D) $x = 3t + 4, y = -t + 3, z = 2t + 7$	
Ans	(C) $x = 3t + 4, y = t - 3, z = 2t + 7$	1
16.	If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct ? (A) A = B (B) AB = BA (C) A = 0 or B = 0 (D) A = I or B = I	
Ans	(B) AB = BA	1
17.	The line $x = 1 + 5\mu, y = -5 + \mu, z = -6 - 3\mu$ passes through which of the following point ? (A) (1, –5, 6) (B) (1, 5, 6) (C) (1, –5, –6) (D) (–1, –5, 6)	
Ans	(C) (1, -5, -6)	1
18.	A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct ? (A) The objective function maximizes the difference of the profit earned from products X and Y. (B) The objective function measures the total production of products X and Y. (C) The objective function maximizes the combined profit earned from selling X and Y. (D) The objective function ensures the company produces more of product X than product Y.	
Ans	(C) The objective function maximizes the combined profit earned from selling X and Y	1

	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	**Assertion (A)** : A = diag [3 5 2] is a scalar matrix of order 3 × 3. **Reason (R)** : If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.	
Ans	(D) Assertion (A) is false and Reason (R) is true.	1
20.	**Assertion (A)** : Every point of the feasible region of a Linear Programming Problem is an optimal solution. **Reason (R)** : The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.	
Ans	(D) Assertion (A) is false and Reason (R) is true.	1

SECTION-B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

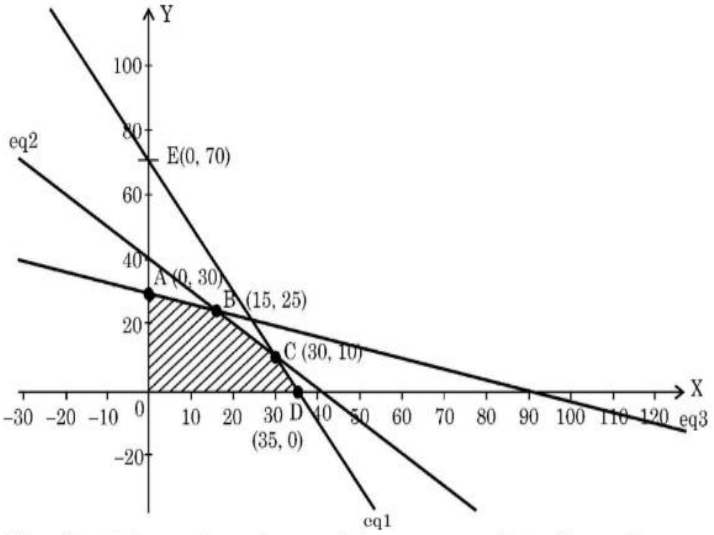
21.	Find the values of 'a' for which f(x) = sin x – ax + b is increasing on R.	
Ans	$f'(x) = cosx - a$ For f(x) to be increasing, $f'(x) \geq 0$ $i.e., cosx \geq a$ Since, $-1 \leq cosx \leq 1$ $\Rightarrow a \leq -1$ Hence, $a \in (-\infty, -1]$. (Also, accept $a \in (-\infty, -1)$)	 1 1

22.	Evaluate : $\int_0^{\pi} \frac{\sin 2px}{\sin x}\, dx$, p $\in$ N.	
Ans	$I = \int_0^{\pi} \frac{\sin 2px}{\sin x} dx$ $= \int_0^{\pi} \frac{\sin 2p(\pi - x)}{\sin(\pi - x)} dx$ $I = \int_0^{\pi} \frac{-\sin 2px}{\sin\ x} dx$ Adding, we get 2I = 0 $\therefore$ I = 0	1 1
23	(a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$. **OR** (b) If $f(x) = \begin{cases} 2x - 3 , & -3 \le x \le -2 \\ x + 1 , & -2 < x \le 0 \end{cases}$ Check the differentiability of f(x) at $x = -2$.	
23 (a) Ans	$x = e^{\frac{x}{y}}$ $\Rightarrow logx = \frac{x}{y}$ $\Rightarrow ylogx = x$ Differentiating both sides w.r.to x, we get $\frac{y}{x} + logx\frac{dy}{dx} = 1$	½ 1

	$\Rightarrow \frac{dy}{dx} = \frac{x-y}{xlogx}$ OR	½
23 (b) Ans	$Lf'(-2) = \lim_{h\to 0} \frac{f(-2-h)-f(-2)}{-h} \quad (h>0)$ $= \lim_{h\to 0} \frac{2(-2-h)-3-(-7)}{-h}$ $= \lim_{h\to 0} 2 = 2$ $Rf'(-2) = \lim_{h\to 0} \frac{f(-2+h)-f(-2)}{h} \quad (h>0)$ $= \lim_{h\to 0} \frac{(-2+h+1)-(-7)}{h}$ $= \lim_{h\to 0} \frac{6+h}{h}$, which does not exist, i.e., RHD does not exist. Therefore, the function is not differentiable at -2. Note: (1) If a student finds only RHD and concludes the result, full marks may be awarded. (2) If a student proves that the function is discontinuous at -2 and hence not differentiable at -2, full marks may be awarded.	 1 1
24.	Let $\vec{p} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\vec{q} = -3\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$. Express $\vec{r}$ in the form of $\vec{r} = \lambda\vec{p} + \mu\vec{q}$ and hence find the values of λ and μ.	
Ans	$\vec{r} = \lambda\vec{p} + \mu\vec{q}$ $\Rightarrow 1 = 2\lambda - 3\mu, 1 = -3\lambda + 4\mu, 2 = -\lambda + \mu$ $\Rightarrow \lambda = -7, \mu = -5$	 1½ ½
25	(a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$. **OR** (b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2} QP$.	

25 (a) Ans	Let α be the angle which the vector $\vec{a}$ makes with all the three axes. Then $3\cos^2\alpha = 1$ $\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$ The unit vector along the vector $\vec{a} = \frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$ $\vec{a} = 5(\hat{\imath} + \hat{\jmath} + \hat{k})$ OR	 1 ½ ½
25 (b) Ans	 $\frac{QR}{QP} = \frac{3}{2}$ Hence, R divides PQ, externally, in the ratio 1:3. The Position vector of R = $\vec{x} = \frac{\vec{\beta} - 3\vec{\alpha}}{1-3} = \frac{3\vec{\alpha} - \vec{\beta}}{2}$	 1 1
SECTION-C This section comprises 6 Short Answer (SA) type questions of 3 marks each.		
26	(a) If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$. OR (b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.	
26(a) Ans	The given function can be written as $y = 2\,log(x+1) - logx$ $\Rightarrow y_1 = \frac{2}{x+1} - \frac{1}{x} = \frac{x-1}{x(x+1)}$ $\Rightarrow (x+1)y_1 = \frac{x-1}{x} = 1 - \frac{1}{x}$ $\Rightarrow (x+1)y_2 + y_1 = \frac{1}{x^2}$	 1 1

	$\Rightarrow x(x+1)^2y_2 + x(x+1)y_1 = 1 + \frac{1}{x}$ $\Rightarrow x(x+1)^2y_2 + x(x+1)y_1 = 1 + 1 - (x+1)y_1$ $\Rightarrow x(x+1)^2y_2 + (x+1)^2y_1 = 2$ OR	1
26(b) Ans	$x\sqrt{1+y} + y\sqrt{1+x} = 0$ $\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ $\Rightarrow x^2(1+y) = y^2(1+x)$ $\Rightarrow (x-y)(x+y) + xy(x-y) = 0$ $\Rightarrow (x-y)(x+y+xy) = 0$ $x \neq y \Rightarrow x + y + xy = 0$ $\Rightarrow y = \frac{-x}{1+x}$ $\Rightarrow \frac{dy}{dx} = \frac{-1}{(x+1)^2}$	½ 1 ½ 1
27.	Prove that f : N → N defined as $f(x) = ax + b$ ($a, b \in N$) is one-one but not onto.	
Ans	Let $x_1, x_2 \in N$ (Domain) such that $f(x_1) = f(x_2)$ $\Rightarrow ax_1 + b = ax_2 + b$ $\Rightarrow x_1 = x_2$ Therefore, f is one-one. Let $y \in N$ (codomain). Then $f(x) = y$ if, $ax + b = y$ i.e., if, $x = \frac{y-b}{a}$, which may not belong to N (domain) Therefore, f is not onto.	1½ 1½

28.	 The feasible region along with corner points for a linear programming problem are shown in the graph. Write all the constraints for the given linear programing problem.	
Ans	The equation of the line AB is $y - 30 = \frac{25-30}{15-0}(x - 0)$ $\Rightarrow x + 3y = 90$ The equation of the line BC is $y - 10 = \frac{10-25}{30-15}(x - 30)$ $\Rightarrow x + y = 40$ The equation of the line CD is $y - 10 = \frac{70-10}{0-30}(x - 30)$ $\Rightarrow 2x + y = 70$ Hence, the constraints are $x + 3y \le 90, x + y \le 40, 2x + y \le 70$ $x \ge 0, y \ge 0$	 1½ 1 ½

29	(a) Solve the differential equation $2(y+3) - xy\frac{dy}{dx} = 0$; given $y(1) = -2$. **OR** (b) Solve the following differential equation : $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$.	
29(a) Ans	Given differential equation can be written as $\frac{y}{y+3}dy = \frac{2}{x}dx$ $\Rightarrow \int \left(1 - \frac{3}{y+3}\right)dy = 2\int \frac{1}{x}dx$ $\Rightarrow y - 3log\|y+3\| = 2log\|x\| + C$ y = -2, when x = 1 $\Rightarrow C = -2$ Hence, the required particular solution is $\Rightarrow y - 3log\|y+3\| = 2log\|x\| - 2$ OR	1 1½ ½
29(b) Ans	Given differential equation can be written as $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$, which is linear in y. I.F. = $e^{\int \frac{2x}{1+x^2}dx} = e^{log(1+x^2)} = 1 + x^2$ The solution is given by $y(1+x^2) = \int 4x^2 dx$ $\Rightarrow y(1+x^2) = \frac{4}{3}x^3 + C$ $or\ y = \frac{4x^3}{3(1+x^2)} + C\frac{1}{1+x^2}$, which is the required general solution	1 1 1

30

(a) A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability other numbers is equal. Find the mean of the number of times numbe appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B :
$A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.
Check if events A and B are independent or mutually exclusive.

30(a) Ans

$P(2) = \frac{3}{10}, P(\text{any other number}) = 1 - \frac{3}{10} = \frac{7}{10}$ ½

Let X represent the Random Variable "the number of 2's".

Then X = 0, 1, 2 ½

The probability distribution is

X	P(X)	XP(X)
0	$\frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$	0
1	$\frac{3}{10} \times \frac{7}{10} \times 2 = \frac{42}{100}$	$\frac{42}{100}$
2	$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$	$\frac{18}{100}$

1½

Mean = $\sum XP(X) = \frac{60}{100} = 0.6$ ½

OR

30(b) Ans

$A = \{(3,6), (4,5), (5,4), (6,3)\}$

$P(A) = \frac{4}{36} = \frac{1}{9}, P(B) = \frac{30}{36} = \frac{5}{6}$ 1

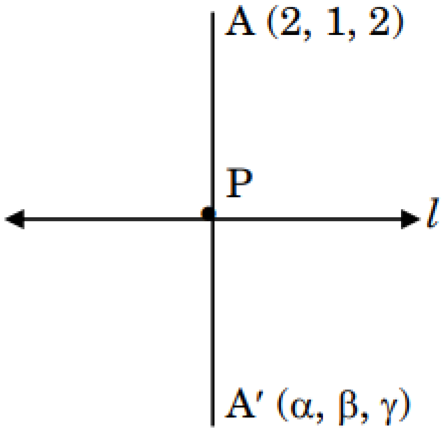
$P(A \cap B) = \frac{3}{36} = \frac{1}{12}$ ½

$P(A) \times P(B) = \frac{5}{54} \neq P(A \cap B)$ 1

Therefore, A and B are not independent.

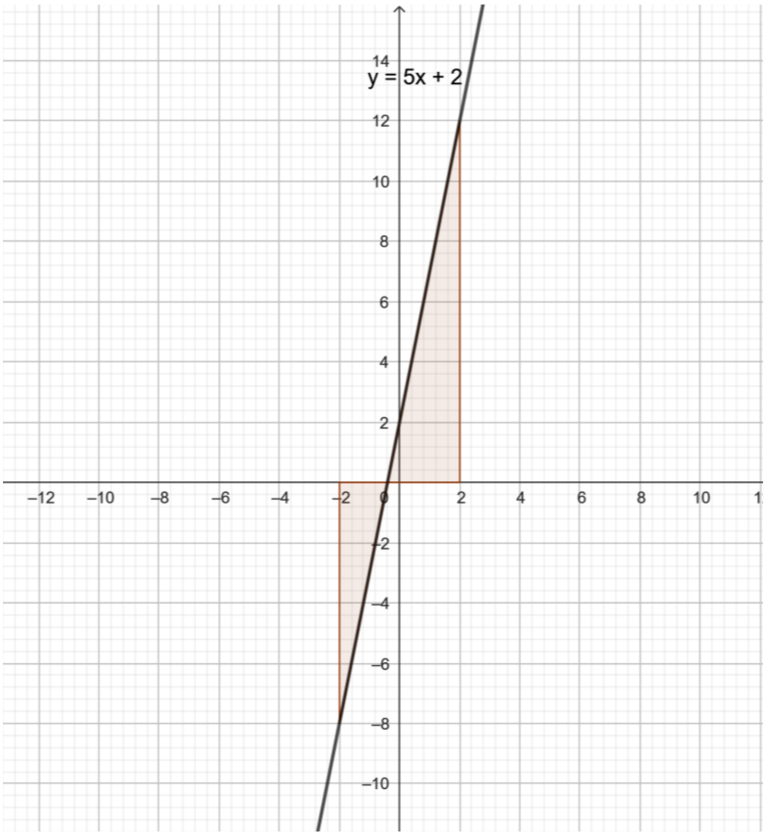
A and B are not mutually exclusive as $A \cap B \neq \emptyset$ ½

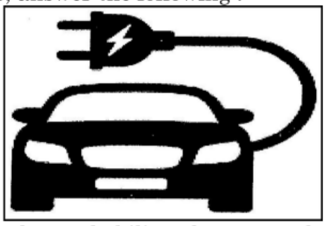
31.	f and g are continuous functions on interval [a, b]. Given that $f(a-x) = f(x)$ and $g(x) + g(a-x) = a$, show that $\int_0^a f(x)\, g(x)\, dx = \frac{a}{2}\int_0^a f(x)\, dx$.	
Ans	$I = \int_0^a f(x)g(x)dx$ $= \int_0^a f(a-x)g(a-x)dx$ $= \int_0^a f(x)[a-g(x)]dx$ $I = a\int_0^a f(x)dx - \int_0^a f(x)g(x)dx$ Adding, we get $I = \frac{a}{2}\int_0^a f(x)dx$	 1 1 1
	SECTION-D This section comprises 4 Long Answer (LA) type questions of 5 marks each.	
32	(a) Find the shortest distance between the lines : $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ and $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$. **OR** (b) Find the image A′ of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA′. Find the foot of perpendicular from point A on the line l.	
32(a) Ans	The vector equations of the lines are $\vec{r} = -\hat{\imath} + \hat{\jmath} + 9\hat{k} + \lambda(2\hat{\imath} + \hat{\jmath} - 3\hat{k})$ $\vec{r} = 3\hat{\imath} - 15\hat{\jmath} + 9\hat{k} + \mu(2\hat{\imath} - 7\hat{\jmath} + 5\hat{k})$ $\overrightarrow{a_1} = -\hat{\imath} + \hat{\jmath} + 9\hat{k},\ \overrightarrow{a_2} = 3\hat{\imath} - 15\hat{\jmath} + 9\hat{k}$ $\overrightarrow{b_1} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, $\overrightarrow{b_2} = 2\hat{\imath} - 7\hat{\jmath} + 5\hat{k}$ $\overrightarrow{a_2} - \overrightarrow{a_1} = 4\hat{\imath} - 16\hat{\jmath}$	 1 1

	$\overrightarrow{b_1}\times\overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} = -16\hat{i}-16\hat{j}-16\hat{k}$	2
	S.D. $= \frac{\lvert(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})\rvert}{\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\rvert} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ OR	1
32(b) Ans	 Let the image of A in the line be $A'(\alpha,\beta,\gamma)$ The point P, which is the point of intersection of the lines *l* and AA', will have coordinates $(\lambda+4, -\lambda+2, -\lambda+2)$ for some λ. Drs of AP are $< \lambda+2, -\lambda+1, -\lambda >$ $AP \perp l$ $(\lambda+2)-(-\lambda+1)-(-\lambda)=0$ $\Rightarrow \lambda = -\frac{1}{3}$ Therefore, the coordinates of P are $(\frac{11}{3},\frac{7}{3},\frac{7}{3})$ P is the mid-point of AA' $\Rightarrow \frac{2+\alpha}{2} = \frac{11}{3}, \frac{1+\beta}{2} = \frac{7}{3}, \frac{2+\gamma}{2} = \frac{7}{3}$ $\Rightarrow \alpha = \frac{16}{3}, \beta = \frac{11}{3}, \gamma = \frac{8}{3}$ The coordinates of the image are $(\frac{16}{3},\frac{11}{3},\frac{8}{3})$ The equation of AA' is	½ ½ 1 ½ 1½

	$\dfrac{x-2}{\frac{10}{3}} = \dfrac{y-1}{\frac{8}{3}} = \dfrac{z-2}{\frac{2}{3}}$ or, $\dfrac{3(x-2)}{5} = \dfrac{3(y-1)}{4} = \dfrac{3(z-2)}{1}$	1
33.	Find : $\int \dfrac{5x}{(x+1)(x^2+9)}\, dx.$	
Ans	$\dfrac{5x}{(x+1)(x^2+9)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+9}$ $\Rightarrow A = -\dfrac{1}{2}, B = \dfrac{1}{2}, C = \dfrac{9}{2}$ Given integral $= -\dfrac{1}{2}\int \dfrac{1}{x+1} dx + \dfrac{1}{2}\int \dfrac{x+9}{x^2+9} dx$ $= -\dfrac{1}{2}\int \dfrac{1}{x+1} dx + \dfrac{1}{4}\int \dfrac{2x}{x^2+9} dx + \dfrac{1}{4}\int \dfrac{18}{x^2+9} dx$ $= -\dfrac{1}{2}\log\lvert x+1\rvert + \dfrac{1}{4}\log(x^2+9) + \dfrac{3}{2}\tan^{-1}\dfrac{x}{3} + C$	2 1½ 1½
34	(a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equations : $x - y + z = 4$ $x - 2y - 2z = 9$ $2x + y + 3z = 1$ **OR** (b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of linear equations : $x - 2y = 10$ $2x - y - z = 8$ $-2y + z = 7$	

34(a) Ans	$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$	2
	The system of equations is equivalent to the matrix equation:	
	$BX = C$, where $C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	½
	$\Rightarrow X = B^{-1}C$ $AB = 8I$ $\Rightarrow B^{-1} = \frac{1}{8}A$	1
	$X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ $\therefore x = 3, y = -2, z = -1$	1½
	OR	
34(b) Ans	$\|A\| = 1 \neq 0 \Rightarrow A^{-1}$ exists.	1
	$adjA = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	1½
	$A^{-1} = \frac{1}{\|A\|}adjA = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	
	The given system of equations is equivalent to the matrix equation	
	$A^TX = B$, where $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	½
	$\Rightarrow X = (A^T)^{-1}B$	
	$\Rightarrow X = (A^{-1})^TB$	½
	$\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\therefore x = 0, y = -5, z = -3$	1½

35.	Using integration, find the area of the region bounded by the line $y = 5x + 2$, the $x -$ axis and the ordinates $x = -2$ and $x = 2$.	
Ans		Correct sketch and shading 2
	The required area	
	$= \left\|\int_{-2}^{-\frac{2}{5}}(5x+2)dx\right\| + \int_{-\frac{2}{5}}^{2}(5x+2)dx$	1
	$= \left\|\left[\frac{(5x+2)^2}{10}\right]_{-2}^{-\frac{2}{5}}\right\| + \left[\frac{(5x+2)^2}{10}\right]_{-\frac{2}{5}}^{2}$	1
	$= \frac{64}{10} + \frac{144}{10} = \frac{104}{5}$	1

	SECTION-E This section comprises 3 case study-based questions of 4 marks each	
36.	Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated). Based on the above, answer the following :  (i) (a) What is the probability that a randomly selected car is an electric car ? **2** **OR** (i) (b) What is the probability that a randomly selected car is a petrol car ? **2** (ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet ? **1** (iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi ? **1**	
36(i) (a) Ans	Let A = Amber manufactures the car B = Bonzi manufactures the car C = Comet manufactures the car E = The selected car is electric $P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$ $P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P(\frac{E}{C})$ $= \frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}$ $= \frac{155}{1000}\ or\ \frac{31}{200}$ OR	½ 1 ½
36(i) (b) Ans	Let A = Amber manufactures the car B = Bonzi manufactures the car C = Comet manufactures the car E = The selected car is a petrol car	

	$P(A)=\frac{60}{100}, P(B)=\frac{30}{100}, P(C)=\frac{10}{100}$ $P(E)=P(A)\times P\left(\frac{E}{A}\right)+P(B)\times P\left(\frac{E}{B}\right)+P(C)\times P(\frac{E}{C})$ $=\frac{60}{100}\times\frac{80}{100}+\frac{30}{100}\times\frac{90}{100}+\frac{10}{100}\times\frac{95}{100}$ $=\frac{845}{1000}\ or\ \frac{169}{200}$	½ 1 ½
36(ii) Ans	$P\left(\frac{C}{E}\right)=\frac{P(C)\times P(\frac{E}{C})}{P(E)}$ $=\frac{\frac{10}{100}\times\frac{5}{100}}{\frac{60}{100}\times\frac{20}{100}+\frac{30}{100}\times\frac{10}{100}+\frac{10}{100}\times\frac{5}{100}}$ $=\frac{\frac{50}{10000}}{\frac{1550}{10000}}=\frac{1}{31}$	 1
36(iii) Ans	$\mathrm{P}\left(\frac{\text{A or B}}{\text{E}}\right)=1-\mathrm{P}\left(\frac{\text{C}}{\text{E}}\right)=1-\frac{1}{31}=\frac{30}{31}$	1
37.	 A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres. Based on the above, answer the following : (i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. **2** (ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. **2**	
(i) Ans	$f'(x)=e^x(cosx+sinx)$ For critical points, $f'(x)=0$ $\Rightarrow cosx+sinx=0$ $\Rightarrow cosx=-sinx$	 ½

	For x to be a critical point $x \in (0,\pi)$, hence, $x = \frac{3\pi}{4}$ For all $x \in \left[0, \frac{3\pi}{4}\right], f'(x) \geq 0$ Hence, f is increasing in $[0, \frac{3\pi}{4}]$ Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $\left(0, \frac{3\pi}{4}\right)$ or $\left[0, \frac{3\pi}{4}\right)$ or $(0, \frac{3\pi}{4}]$ For all $x \in \left[\frac{3\pi}{4}, \pi\right], f'(x) \leq 0$ Hence, f is decreasing in $[\frac{3\pi}{4}, \pi]$ Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $(\frac{3\pi}{4}, \pi)$ or $(\frac{3\pi}{4}, \pi]$ or $[\frac{3\pi}{4}, \pi)$	½ ½ ½
37(ii) Ans	$x = \frac{3\pi}{4}$ is a critical point $f''(x) = e^x(cosx - sinx) + e^x(cosx + sinx)$ $= 2e^x cosx$ $f''\left(\frac{3\pi}{4}\right) = -ve$ Hence, $\frac{3\pi}{4}$ is a point of local maximum.	 1 ½ ½

38.	A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.  Based on the above, answer the following : (i) How many relations can be there from S to J ? **1** (ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ Check if it is bijective. **1** (iii) (a) How many one-one functions can be there from set S to set J ? **2** **OR** (iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), \{S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. **2**	
38 Ans (i)	The number of relations = $2^{4\times 3} = 2^{12}$	1
38 Ans (ii)	Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one. Hence, it is not bijective.	1
38 (iii) (a)	There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0. OR	2
38 (iii) (b)	To make R_1 reflexive and not symmetric we need to add the following ordered pairs: $(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$	2