SOLUTIONS

SENIOR SECONDARY EXAMINATION 2024-25

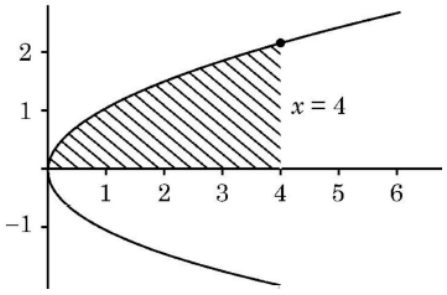
MATHEMATICS (Code–041)

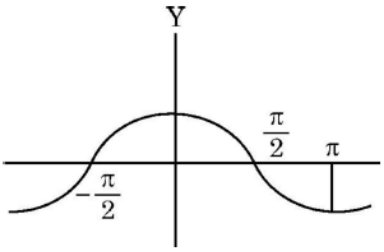
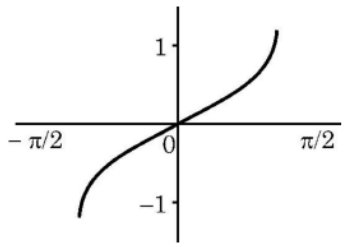
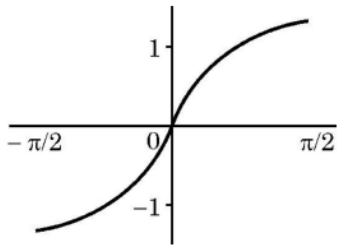
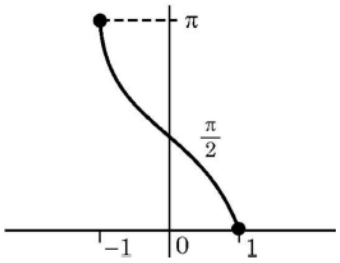
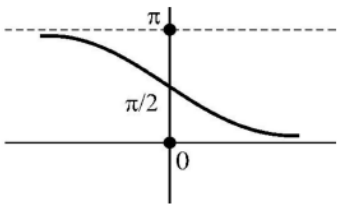
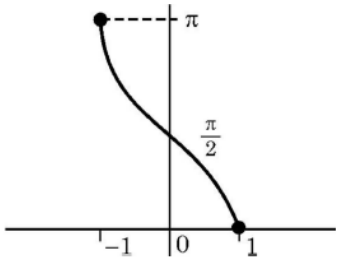
[Paper Code: 65/1/1]

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is (A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
Ans	(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	1
Q2.	If vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct ? (A) $\vec{a} \parallel \vec{b}$ (B) $\vec{a} \perp \vec{b}$ (C) $\lvert \vec{b} \rvert > \lvert \vec{a} \rvert$ (D) $\lvert \vec{a} \rvert = \lvert \vec{b} \rvert$	
Ans	(B) $\vec{a} \perp \vec{b}$	1
Q3.	$\int_{-1}^{1} \frac{\lvert x \rvert}{x} dx, x \neq 0$ is equal to (A) -1 (B) 0 (C) 1 (D) 2	
Ans	(B) 0	1

Q4.	Which of the following is <u>not</u> a homogeneous function of x and y ? (A) $y^2 - xy$ (B) $x - 3y$ (C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (D) $\tan x - \sec y$	
Ans	(D) $\tan x - \sec y$	1
Q5.	If $f(x) = \mid x \mid + \mid x - 1 \mid$, then which of the following is correct ? (A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$. (B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$. (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$. (D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.	
Ans	(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.	1
Q6.	If A is a square matrix of order 2 such that det (A) = 4, then det (4 adj A) is equal to : (A) 16 (B) 64 (C) 256 (D) 512	
Ans	(B) 64	1
Q7.	If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to : (A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{7}{9}$	
Ans	(C) $\frac{2}{3}$	1
Q8.	The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is : (A) 0 (B) 2 (C) 4 (D) 5	
Ans	(C) 4	1

Q9.	Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, C = [9 8 7], which of the following is defined ? (A) Only AB (B) Only AC (C) Only BA (D) All AB, AC and BA	
Ans	(A) Only AB	1
Q10.	If $\int \frac{2^{\frac{1}{x}}}{x^2}\,dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to (A) $\frac{-1}{\log 2}$ (B) $-\log 2$ (C) -1 (D) $\frac{1}{2}$	
Ans	(A) $\frac{-1}{\log 2}$	1
Q11.	If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $\lvert \vec{a} \rvert = \sqrt{37}$, $\lvert \vec{b} \rvert = 3$ and $\lvert \vec{c} \rvert = 4$, then angle between $\vec{b}$ and $\vec{c}$ is (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$	
Ans	(C) $\frac{\pi}{3}$	1
Q12.	The integrating factor of differential equation $(x + 2y^3)\frac{dy}{dx} = 2y$ is (A) $e^{\frac{y^2}{2}}$ (B) $\frac{1}{\sqrt{y}}$ (C) $\frac{1}{y^2}$ (D) $e^{-\frac{1}{y^2}}$	
Ans	(B) $\frac{1}{\sqrt{y}}$	1

Q13.	If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to (A) 0 (B) 1 (C) 7 (D) ± 7	
Ans	(B) 1	1
Q14.	The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then (A) Z is maximum at (2, 72), minimum at (15, 20) (B) Z is maximum at (15, 20) minimum at (40, 15) (C) Z is maximum at (40, 15), minimum at (15, 20) (D) Z is maximum at (40, 15), minimum at (2, 72)	
Ans	(C) Z is maximum at (40, 15), minimum at (15, 20)	1
Q15.	If A and B are invertible matrices, then which of the following is <u>not</u> correct ? (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$ (C) adj (A) = $\lvert A \rvert A^{-1}$ (D) $\lvert A \rvert^{-1} = \lvert A^{-1} \rvert$	
Ans	(A) $(A + B)^{-1} = B^{-1} + A^{-1}$	1
Q16.	If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ? (A) It will only have a maximum value. (B) It will only have a minimum value. (C) It will have both maximum and minimum values. (D) It will have neither maximum nor minimum value.	
Ans	(C) It will have both maximum and minimum values.	1
Q17.	The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by  (A) $\int_0^4 x\,dx$ (B) $\int_0^2 y^2\,dy$ (C) $2\int_0^4 \sqrt{x}\,dx$ (D) $\int_0^4 \sqrt{x}\,dx$	
Ans	(D) $\int_0^4 \sqrt{x}\,dx$	1

Q18.	The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse ?  (A)  (B)  (C)  (D)  	
Ans	(C)  	1

Assertion – Reason Based Questions

Direction : Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19.	**Assertion (A) :** Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective. **Reason (R) :** A function is a bijective if it is both surjective and injective.	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1

Q20.	**Assertion (A)** : $f(x) = \begin{cases} 3x-8, & x \le 5 \\ 2k\ , & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$. **Reason (R)** : For a function f to be continuous at $x = a$, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1

SECTION B

This section comprises very short answer (VSA) type questions of **2 marks each**.

Q21.	(a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$. **OR** (b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.	
Ans(a)	$\textbf{Let } u = 2^{\cos^2 x} \Rightarrow \frac{du}{dx} = 2^{\cos^2 x}(-2\cos x \sin x)\log 2$ $\textbf{Let } v = \cos^2 x \Rightarrow \frac{dv}{dx} = -2\cos x \sin x$ $\textbf{Now } \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 2^{\cos^2 x}\log 2$	1 ½ ½
	OR	
Ans(b)	$\tan^{-1}(x^2 + y^2) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$ **Differentiate both sides wrt x,** $2x + 2y\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$	½ 1 ½

Q22.	Evaluate : $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$	
Ans	$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ $=\tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right]=\tan^{-1}\left[2\sin\frac{\pi}{3}\right]$ **1** $=\tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$ **1**	
Q23.	The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.	
Ans	$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$ **1** **Area of parallelogram** $=\frac{1}{2}\left\|\vec{a}\times\vec{b}\right\|$ $=\frac{1}{2}\sqrt{(-2)^2+3^2+7^2}=\frac{\sqrt{62}}{2}$ **1**	
Q24.	Find the intervals in which function f(x) = $5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing.	
Ans	$f(x)=5x^{3/2}-3x^{5/2}\Rightarrow f'(x)=\frac{15}{2}\sqrt{x}(1-x)$ **1** **For increasing / decreasing, put** $f'(x)=0$ $\Rightarrow x=0,1$ (i) **When** $x\in[0,1]$, $f'(x)\geq 0$. **So,** f **is increasing when** $x\in[0,1]$ ½ **(The intervals** $(0,1),[0,1)$ **or** $(0,1]$ **can also be considered.)** (ii) **When** $x\in[1,\infty)$, $f'(x)\leq 0$. **So,** f **is decreasing when** $x\in[1,\infty)$ ½ **(The interval** $(1,\infty)$ **can also be considered.)**	

Q25.	(a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings. **OR** (b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, –1, 0) respectively.	
Ans(a)	**Let the required angle between the kite strings be θ.** **Then,** $\cos\theta = \dfrac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ $\Rightarrow \cos\theta = = \dfrac{\left(3\hat{i}+\hat{j}+2\hat{k}\right)\left(2\hat{i}-2\hat{j}+4\hat{k}\right)}{\sqrt{9+1+4}\sqrt{4+4+16}} = \dfrac{12}{\sqrt{336}} = \dfrac{3}{\sqrt{21}}$ $\Rightarrow \theta = \cos^{-1}\left(\dfrac{12}{\sqrt{336}}\right)$ **or** $\cos^{-1}\left(\dfrac{3}{\sqrt{21}}\right)$	1½ ½
	OR	
Ans(b)	$\overrightarrow{BA} = -6\hat{i}+2\hat{j}+3\hat{k}$ **Required unit vector of magnitude 21** $= 21\times\left(\dfrac{-6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{36+4+9}}\right)$ $= 3\left(-6\hat{i}+2\hat{j}+3\hat{k}\right)$ **or** $-18\hat{i}+6\hat{j}+9\hat{k}$	1 ½ ½
	SECTION C This section comprises short answer (SA) type questions of **3 marks each**.	
Q26.	The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm ?	
Ans	**Let 'a' be the side of the triangle, so** $\dfrac{da}{dt} = 3\text{cm/s}$ **Now area of an equilateral triangle,** $A = \dfrac{\sqrt{3}}{4}a^2$ $\Rightarrow \dfrac{dA}{dt} = \dfrac{\sqrt{3}a}{2}\times\dfrac{da}{dt}$ $\therefore \left.\dfrac{dA}{dt}\right]_{a=15\text{cm}} = \dfrac{\sqrt{3}\times 15}{2}\times 3 = \dfrac{45\sqrt{3}}{2}\text{cm}^2/\text{s}$	½ 1½ 1

Q27. Solve the following linear programming problem graphically :

Maximise $Z = x + 2y$

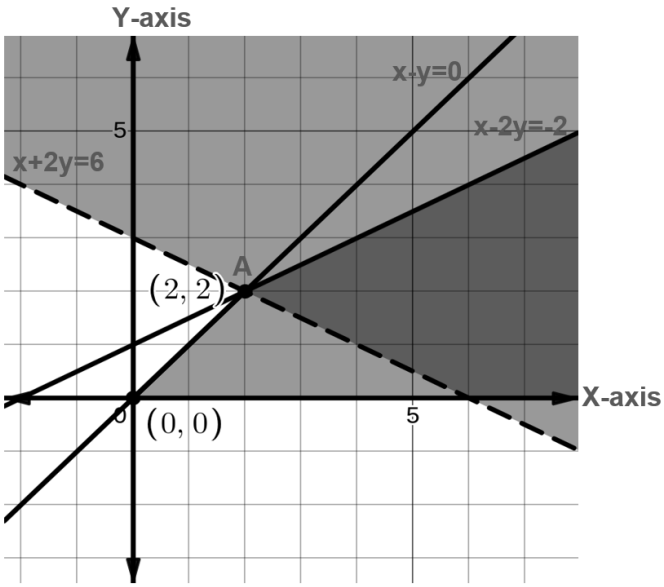
Subject to the constraints :

$x - y \geq 0$

$x - 2y \geq -2$

$x \geq 0, y \geq 0$

Ans



For correct graph and shading 1½

Corner Point	Value of $Z = x + 2y$
$O(0,0)$	0
$A(2,2)$	6

For correct table 1

Since feasible region is unbounded. Plot x + 2y > 6 which has common region with feasible region, thus Z has no maximum value.

½

Ans

(a) Find : $\int \frac{x + \sin x}{1 + \cos x}\, dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$

Ans(a)	$\int \frac{x+\sin x}{1+\cos x}dx$	
	$=\int \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$	**1**
	$=\int x\left(\frac{1}{2}\sec^2\frac{x}{2}\right)dx+\int \tan\frac{x}{2}dx$	**½**
	$=x\tan\frac{x}{2}-\int \tan\frac{x}{2}dx+\int \tan\frac{x}{2}dx$	**1**
	$=x\tan\frac{x}{2}+C$	**½**
	OR	
Ans(b)	$\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$	
	$=\frac{1}{2}\int_0^{\pi/4} \frac{dx}{\cos^4 x\sqrt{\tan x}}$	**½**
	$=\frac{1}{2}\int_0^{\pi/4} \frac{\left(1+\tan^2 x\right)\sec^2 x}{\sqrt{\tan x}}dx$	
	$\text{Put}\tan x=t\Rightarrow\sec^2 x\,dx=dt$	**½**
	$\therefore I=\frac{1}{2}\int_0^1 \frac{1+t^2}{\sqrt{t}}dt$	**½**
	$=\frac{1}{2}\int_0^1\left(\frac{1}{\sqrt{t}}+t^{3/2}\right)dt$	
	$=\frac{1}{2}\left[2\sqrt{t}+\frac{2}{5}t^{5/2}\right]_0^1$	**1**
	$=\frac{6}{5}$	**½**

Q29.	(a) Verify that lines given by $\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$ and $\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines. **OR** (b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.	
Ans(a)	**Rewriting the lines, we get** $\vec{r}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+\hat{j}-2\hat{k}\right)$ **and** $\vec{r}=\left(\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}-2\hat{k}\right)$ **Let** $\vec{a}_1=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{a}_2=\hat{i}-\hat{j}-\hat{k}$, $\vec{b}_1=-\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}_2=\hat{i}+2\hat{j}-2\hat{k}$ **Note that the dr'sof given lines are not proportional so, they are not parallel lines.** **The lines will be skew if they do not intersect each other also.** **Here** $\vec{a}_2-\vec{a}_1=\hat{j}-4\hat{k}$, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&1&-2\\1&2&-2\end{vmatrix}=2\hat{i}-4\hat{j}-3\hat{k}$ **Consider** $\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)$ $=\left(\hat{j}-4\hat{k}\right).\left(2\hat{i}-4\hat{j}-3\hat{k}\right)=8\neq0$ **Hence lines will not intersect. So the lines are skew.** **Shortest Distance** $=\dfrac{\left\|\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)\right\|}{\left\|\vec{b}_1\times\vec{b}_2\right\|}$ $=\dfrac{8}{\sqrt{4+16+9}}=\dfrac{8}{\sqrt{29}}$	½ ½ + ½ ½ 1
	OR	
Ans(b)	**Let the wicket keeper divides the line segment in ratio** $k:1$ $\therefore\overrightarrow{W}=\dfrac{k\vec{F}+1.\vec{B}}{k+1}$ B(2, 8, 0) W(6,12,0) F(12,18,0) $\Rightarrow 6\hat{i}+12\hat{j}=\left(\dfrac{12k+2}{k+1}\right)\hat{i}+\left(\dfrac{18k+8}{k+1}\right)\hat{j}$ $\Rightarrow k=\dfrac{2}{3}$ **Hence, the required ratio is** $2:3$	1 1 1

Q30.

(a) The probability distribution for the number of students being absent in a class on a Saturday is as follows :

X	0	2	4	5
P(X)	p	2p	3p	p

Where X is the number of students absent.

(i) Calculate p. **1**

(ii) Calculate the mean of the number of absent students on Saturday. **2**

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

Ans(a)

(i) **Since** $\sum P(X)=1 \Rightarrow p+2p+3p+p=1$ ½

$\Rightarrow p=\frac{1}{7}$ ½

(ii) **Mean** $=\sum X.P(X)=0(p)+2(2p)+4(3p)+5(p)$ **1**

$=21p=21\left(\frac{1}{7}\right)=3$ **1**

OR

Ans(b)

Let E_1 : The applicant is a male ½

E_2 : The applicant is a female

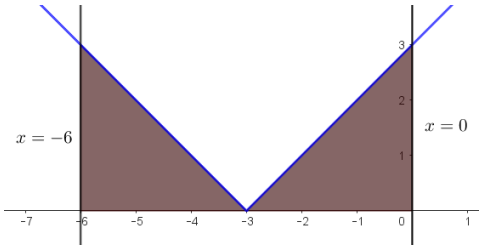
***A* : The candidate chosen will have distinction in the written test.**

$P(E_1)=\frac{1}{3}, P(E_2)=\frac{2}{3}, P(A|E_1)=0.4, P(A|E_2)=0.35$ **1**

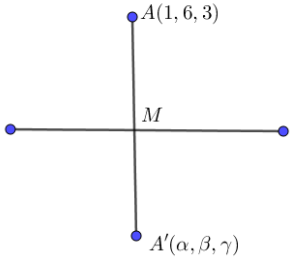
$\therefore P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$

$=\frac{1}{3}\times 0.4+\frac{2}{3}\times 0.35$ **1**

$=\frac{11}{30}$ ½

Q31.	Sketch the graph of y = | x + 3 | and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.	
Ans	 **Required Area** $= \int_{-6}^{0} y\,dx$ $= 2\int_{-3}^{0} (x+3)\,dx$ $= 2\left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$ $= 9$	**For correct graph: 1 mark** ½ ½ ½ ½
SECTION D This section comprises long answer (LA) type questions of **5 marks each**.		
Q32.	(a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. **OR** (b) If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.	
Ans(a)	**Let** $x = \sin A, y = \sin B \Rightarrow A = \sin^{-1}x,\ B = \sin^{-1}y$ $\therefore \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ $\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$ $\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\Rightarrow \cot\left(\frac{A-B}{2}\right) = a \Rightarrow A - B = 2\cot^{-1}a$ $\Rightarrow \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$ **differentiate both sides wrt** x, $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	**1** **1** **1** ½ **1½**
OR		

Ans(b)	$x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ $\Rightarrow \frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\frac{\theta}{2}} \times \sec^2\frac{\theta}{2} \times \frac{1}{2}\right)$ $= a\left(-\sin\theta + \frac{1}{\sin\theta}\right) = a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$ $\frac{dx}{d\theta} = a\cot\theta\cos\theta$ **Also,** $y = \sin\theta \Rightarrow \frac{dy}{d\theta} = \cos\theta$ $\therefore \frac{dy}{dx} = \frac{\tan\theta}{a}$ **Differetiating wrt** x**,** $\frac{d^2y}{dx^2} = \frac{\sec^2\theta}{a} \times \frac{d\theta}{dx}$ $= \frac{\sec^3\theta\tan\theta}{a^2}$ $\left.\frac{d^2y}{dx^2}\right]_{\text{at}\,\theta=\frac{\pi}{4}} = \frac{2\sqrt{2}}{a^2}$	½ ½ ½ ½ 1 1 1
Q33.	Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.	
Ans	$f(x) = 2x^3 - 15x^2 + 36x + 1$ $\Rightarrow f'(x) = 6\left(x^2 - 5x + 6\right) = 6(x-2)(x-3)$ $f'(x) = 0 \Rightarrow x = 2, 3 \in [1,5]$ **Now** $f(1) = 24$, $f(2) = 29$, $f(3) = 28$, $f(5) = 56$ **Hence, the absolute maximum value is 56 and the absolute minimum value is 24.**	1 1 2 1
Q34.	(a) Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'. **OR** (b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point Q(2, 4, –1) is 7 units. Also, find the equation of line joining P and Q.	

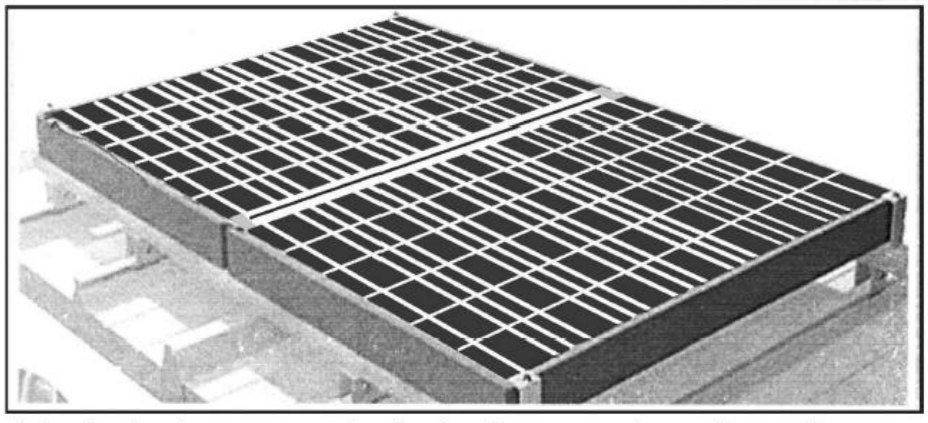
Ans(a)	**The equation of given line is** $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$  **Any arbitrary point on the line is** $M(\lambda, 2\lambda+1, 3\lambda+2)$ **dr's of** AM **are** $<\lambda-1, 2\lambda-5, 3\lambda-1>$ **Here** $1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$ $\Rightarrow \lambda=1$ $\therefore M(1,3,5)$ **is the foot perpendicular of the point A to the given line.** **Let image of point A in the line be** $A'(\alpha,\beta,\gamma)$ **Since M is the mid-point of** AA'**, so** $M\left(\frac{1+\alpha}{2},\frac{6+\beta}{2},\frac{3+\gamma}{2}\right)=M(1,3,5)$ $\Rightarrow A'(1,0,7)$ **is the image of A.** **Also, Equation of** AA' **is** $\frac{x-1}{0}=\frac{y-6}{-3}=\frac{z-3}{2}$	**1** **1** **½** **½** **1** **1**
	OR	
Ans(b)	**The given line is** $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ **and** $Q(2,4,-1)$ **Any random point on the line will be given by** $P(\lambda-5, 4\lambda-3, -9\lambda+6)$ **Since** $PQ=7\Rightarrow\sqrt{(\lambda-7)^2+(4\lambda-7)^2+(-9\lambda+7)^2}=7$ $\Rightarrow 98(\lambda^2-2\lambda+1)=0\Rightarrow\lambda=1$ **Hence, the required point is** $P(-4,1,-3)$ **The equation of line PQ is** $\frac{x+4}{6}=\frac{y-1}{3}=\frac{z+3}{2}$ **or** $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$	**1** **1** **1** **1** **1**
Q35.	A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions : • The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club. • The number of students in Music club should be 20 more than half the number of students in Sports club. • The total number of students to be allocated in all three clubs are 180. Find the number of students allocated to different clubs, using matrix method.	

Ans	**Let x, y and z be the no. of students allocated to Sports, Music and Drama clubs respectively.**	
	Here, $x = y + z, y = \frac{x}{2} + 20, x + y + z = 180$	
	$\Rightarrow x - y - z = 0, x - 2y = -40, x + y + z = 180$	**1½**
	Given equations can be written as $AX = B$	
	where, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	**½**
	$\lvert A \rvert = -4 \neq 0 \Rightarrow A^{-1}$ **exists.**	**½**
	$adjA = \begin{bmatrix} -2 & 0 & -2 \\ -1 & 2 & -1 \\ 3 & -2 & -1 \end{bmatrix}$	**1**
	$A^{-1} = \frac{1}{\lvert A \rvert} \times adjA = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}$	**½**
	$X = A^{-1}B$	
	$= \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$	**1**
	$\therefore x = 90, y = 65, z = 25$	
	Number of students allocated in sports, music and drama are 90 , 65 and 25 respectively .	

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

	Based on this information, answer the following questions : (i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.	**1**
	(ii) Write the area of the solar panel as a function of x.	**1**
	(iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.	**2**
	OR	
	(iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.	**2**
Ans	$(i)\ 2x+3y=300$	**1**
	$(ii)\ A=xy=\frac{x}{3}(300-2x)$	**1**
	$(iii)(a)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$	
	$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$	½
	For critical points, put $\frac{dA}{dx}=0\Rightarrow x=75$	½
	Also, $\frac{d^2A}{dx^2}=-\frac{4}{3}<0$. So, A is maximum at $x=75$	½
	Also, maximum area is $A=\frac{75}{3}(300-150)=3750\,\text{m}^2$	½
	OR	
	$(iii)(b)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$	
	$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$	½
	For critical points, put $\frac{dA}{dx}=0\Rightarrow x=75$	½
	As $\frac{dA}{dx}$ changes its sign from positive to negative as x passes through $x=75$ from left to right, which means $x=75$ is the point of maximum.	½
	Also, maximum area is $A=\frac{75}{3}(300-150)=3750\,\text{m}^2$	½
	Note : Full credit to be given if the student takes equation as **$2x+2y=300$ or $2x+4y=300$ or $4x+4y=300$ or $4x+3y=300$** **The solutions of sub-parts will differ and marks may be given accordingly.**	

Q37.	A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3} : $R_1 = \{(2, 3), (3, 2)\}$ $R_2 = \{(1, 2), (1, 3), (3, 2)\}$ $R_3 = \{(1, 2), (2, 1), (1, 1)\}$ $R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$ $R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$ The students are asked to answer the following questions about the above relations : (i) Identify the relation which is reflexive, transitive but not symmetric. (ii) Identify the relation which is reflexive and symmetric but not transitive. (iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. **OR** (iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?	
Ans	$(i)\ R_4$ $(ii)\ R_5$ $(iii)(a)\ R_1$ and R_3 **OR** $(iii)(b)$ Required pairs to be added to make the relation R_2 as an equivalence relation are: $(1,1),(2,2),(3,3),(2,1),(3,1)$ and $(2,3)$	**1** **1** **1+1** **2**

Q38.

A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following :

(i) What is the probability that a customer after availing the loan will default on the loan repayment ? **2**

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ? **2**

Ans	E_1 : customer avails loan on fixed rate E_2 : customer avails loan on floating rate E_3 : customer avails loan on variable rate A: the person defaults on the loan $\mathrm{P}(E_1)=\frac{1}{10},\mathrm{P}(E_2)=\frac{2}{10},\mathrm{P}(E_3)=\frac{7}{10}$ $\mathrm{P}(A\mid E_1)=\frac{5}{100},\mathrm{P}(A\mid E_2)=\frac{3}{100},\mathrm{P}(A\mid E_3)=\frac{1}{100}$ $(i)\ \mathrm{P}(A)=\mathrm{P}(E_1).\mathrm{P}(A\mid E_1)+\mathrm{P}(E_2).\mathrm{P}(A\mid E_2)+\mathrm{P}(E_3).\mathrm{P}(A\mid E_3)$ $=\frac{1}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{3}{100}+\frac{7}{10}\times\frac{1}{100}$ $=\frac{18}{1000}\text{ or }\frac{9}{500}$ $(ii)\ \mathrm{P}(E_3\mid A)=\frac{\mathrm{P}(E_3).\mathrm{P}(A\mid E_3)}{\mathrm{P}(E_1).\mathrm{P}(A\mid E_1)+\mathrm{P}(E_2).\mathrm{P}(A\mid E_2)+\mathrm{P}(E_3).\mathrm{P}(A\mid E_3)}$ $=\frac{\frac{7}{10}\times\frac{1}{100}}{\frac{18}{1000}}$ $=\frac{7}{18}$	**1** **1** **1** **1**