JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON WEDNESDAY 2nd APRIL 2025) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. If the image of the point P(1, 0, 3) in the line joining the points A(4, 7, 1) and B(3, 5, 3) is Q(α, β, γ), then α + β + γ is equal to

(1) $\frac{47}{3}$ (2) $\frac{46}{3}$

(3) 18 (4) 13

Ans. (2)

Sol. P(1, 0, 3)

A(4, 7, 1), B(3, 5, 3)

Line AB $\Rightarrow \frac{x-3}{1}=\frac{y-5}{2}=\frac{z-3}{-2}=\lambda$

Let foot of perpendicular of P on AB be

$R \equiv (\lambda + 3, 2\lambda + 5, -2\lambda + 3)$

$\Rightarrow (\lambda + 3 - 1)(1) + (2\lambda + 5 - 0)(2) + (-2\lambda + 3 - 3)(-2) = 0$

$\Rightarrow \lambda + 2 + 4\lambda + 10 + 4\lambda = 0$

$\Rightarrow \lambda = -\frac{4}{3}$

$\Rightarrow R \equiv \left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

$Q \equiv \left(\frac{10}{3}-1,\frac{14}{3}-0,\frac{34}{3}-3\right) \equiv \left(\frac{7}{3},\frac{14}{3},\frac{25}{3}\right)$

$\Rightarrow \alpha+\beta+\gamma = \frac{7+14+25}{3}=\frac{46}{3}$

2. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function, If $10\int_1^x f(t)dt = 5xf(x) - x^5 - 9$ for all $x \ge 1$, then the value of $f(3)$ is :

(1) 18 (2) 32

(3) 22 (4) 26

Ans. (2)

Sol. $10\frac{d}{dx}\int_1^x f(t)dt = \frac{d}{dx}\left(5xf(x)-x^5-9\right)$

$\Rightarrow 10f(x) = 5f(x) + 5x f'(x) - 5x^4$

$\Rightarrow f(x) + x^4 = x f'(x)$

$\Rightarrow y + x^4 = x\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} + y\left(-\frac{1}{x}\right) = x^3$

$\Rightarrow ye^{\int -\frac{1}{x}dx} = \int x^3 e^{\int -\frac{1}{x}dx} + c$

$\Rightarrow ye^{-\ell n|x|} = \int x^3 e^{-\ell n|x|} + c$

$\Rightarrow \frac{y}{|x|} = \int \frac{x^3}{|x|} + c$

$\Rightarrow \frac{y}{x} = \int x^2 + c$

$\Rightarrow \frac{y}{x} = \frac{x^3}{3} + c$

Put x = 1 in given equation

$\Rightarrow 0 = 5f(1) - 1 - 9 \Rightarrow f(1) = 2$

$\Rightarrow \frac{2}{1} = \frac{1}{3} + c \Rightarrow c = \frac{5}{3}$

$\Rightarrow \frac{f(3)}{3} = \frac{27}{3} + \frac{5}{3}$

$\Rightarrow f(3) = 32$

3. The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is :

(1) 4 (2) 10

(3) 6 (4) 8

Ans. (1)

Sol. $a_2 + a_4 + ... + a_n = 30$ ....(1)

$a_1 + a_3 + ... + a_{n-1} = 24$ ....(2)

(1) – (2)

$(a_2 - a_1) + (a_4 - a_3) ... (a_n - a_{n-1}) = 6$

$\Rightarrow \frac{n}{2}d = 6 \Rightarrow nd = 12$

$a_n - a_1 = (n-1)d = \frac{21}{2}$

$\Rightarrow nd - d = \frac{21}{2} \Rightarrow 12 - \frac{21}{2} = d$

$\Rightarrow d = \frac{3}{2}$, $n = 8$

Sum of odd terms $= \frac{4}{2}\left[2a + (4-1)3\right] = 24$

$\Rightarrow a = \frac{3}{2}$

A.P. $\Rightarrow \frac{3}{2}, 3, \frac{9}{2}, 6, \frac{15}{2}, 9, \frac{21}{2}, 12$

no. of integer terms = 4

4. Let A = {1, 2, 3,...., 10} and R be a relation on A such that R = {(a, b) : a = 2b + 1}. Let (a_1, a_2), (a_2, a_3), (a_3, a_4), ..., (a_k, a_{k+1}) be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k, for which such a sequence exists, is equal to :

(1) 6 (2) 7

(3) 5 (4) 8

Ans. (3)

Sol. $a = 2b + 1$

$2b = a - 1$

$R = \{(3, 1), (5, 2), ..., (99, 49)\}$

Let $(2m + 1, m)$, $(2\lambda - 1, \lambda)$ are such ordered pairs.

According to the condition

$m = 2\lambda - 1 \Rightarrow m$ = odd number

$\Rightarrow 1^{st}$ element of ordered pair (a, b)

$a = 2(2\lambda - 1) + 1 = 4\lambda - 1$

Hence $a \in \{3, 7, ..., 99\}$

$\Rightarrow \lambda \in \{1, 2, ..., 25\}$

$\Rightarrow$ set of sequence

$$\left\{(4\lambda-1, 2\lambda-1), (2\lambda-1, \lambda-1), \left(\lambda-1, \frac{\lambda-2}{2}\right), ...\right\}$$

2^{nd} element of each ordered pair $= \frac{\lambda - 2^{r-2}}{2^{r-2}}$

For maximum number of ordered pairs in such sequence

$\frac{\lambda - 2^{r-2}}{2^{r-2}} = 1$ or 2 ; $1 \le \lambda \le 25$

$\lambda = 2^{r-1}$ or $\lambda = 3.2^{r-2}$

Case-I : $\lambda = 2r - 1$

$\lambda = 2, 2^2, 2^3, 2^4$

$r = 2, 3, 4, 5$

Hence maximum value of r is 5 when $\lambda = 16$

Case-II : $\lambda = 3.2^{r-2}$

$\lambda = 3, 6, 12, 24$

$r = 2, 3, 4, 5$

Hence maximum value of r is 5 when $\lambda = 24$

5. If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is :

(1) $\frac{4}{\sqrt{17}}$ (2) $\frac{\sqrt{3}}{16}$

(3) $\frac{3}{\sqrt{19}}$ (4) $\frac{\sqrt{5}}{7}$

Ans. (1)

Sol. $2b = \frac{1}{4}(2ae)$

$\frac{b}{a} = \frac{e}{4}$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{e^2}{16}}$

$e^2\left(1 + \frac{1}{16}\right) = 1$

$e = \frac{4}{\sqrt{17}}$

6. The line L_1 is parallel to the vector $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point (7, 6, 2) and the line L_2 is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point (5, 3, 4). The shortest distance between the lines L_1 and L_2 is :

(1) $\frac{23}{\sqrt{38}}$ (2) $\frac{21}{\sqrt{57}}$

(3) $\frac{23}{\sqrt{57}}$ (4) $\frac{21}{\sqrt{38}}$

Ans. (1)

Sol. $L_1 : \left(7\hat{i} + 6\hat{j} + 2k\right) + \lambda\left(-3\hat{i} + 2\hat{j} + 4k\right)$

$L_2 : \left(5\hat{i} + 3\hat{j} + 4k\right) + \lambda\left(2\hat{i} + \hat{j} + 3k\right)$

Distance between skew lines

$$= \frac{\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right).\left(2\hat{i} + 17\hat{j} - 7\hat{k}\right)}{\sqrt{342}}$$

$$= \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$$

7. Let (a, b) be the point of intersection of the curve $x^2 = 2y$ and the straight line $y - 2x - 6 = 0$ in the second quadrant. Then the integral $I = \int_a^b \frac{9x^2}{1+5^x} dx$ is equal to :

(1) 24 (2) 27

(3) 18 (4) 21

Ans. (1)

Sol. $x^2 = 2y$ & $y = 2x + 6$

$x^2 = 4x + 12$

$x^2 - 4x - 12 = 0 \Rightarrow \begin{array}{c|c} x = 6 & \text{if } x = -2 \\ y = 18 & y = 2 \end{array}$

$\therefore$ (6, 18) & (–2, 2)

Here (6, 18) Rejected because (a, b) lies in 2nd quadrant

$\therefore$ a = –2 & b = 2

$$\therefore I = \int_{-2}^{2} \frac{9x^2}{1+5^x} dx = \int_{-2}^{2} \frac{9.5^x.x^2}{1+5^x} dx$$

$$\therefore 2I = \int_{-2}^{2} 9x^2 dx = 18\int_0^2 x^2 dx = 18\left(\frac{x^3}{3}\right)_0^2$$

$2I = 48$

$\therefore \boxed{I = 24}$

8. If the system of equation

$2x + \lambda y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \mu z = 9$

has infinitely many solutions, then $(\lambda^2 + \mu^2)$ is equal to :

(1) 22 (2) 18

(3) 26 (4) 30

Ans. (3)

Sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$

$\Rightarrow 2(2\mu + 5) + \lambda(-4 - 3\mu) + 3(7) = 0$

$\Rightarrow 4\mu - 3\lambda\mu - 4\lambda + 31 = 0$ ......(1)

$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0$

$\Rightarrow 2(-17) + \lambda(1) + 5(7) = 0$

$\Rightarrow \lambda = -1$

from equation (1)

$4\mu + 3\mu + 4 + 31 = 0 \Rightarrow \boxed{\mu = -5}$

$\therefore \boxed{\lambda^2 + \mu^2 = 26}$

9. If $\theta \in \left[-\frac{7\pi}{6}, \frac{4\pi}{3}\right]$, then the number of solutions of $\sqrt{3}\operatorname{cosec}^2\theta - 2\left(\sqrt{3} - 1\right)\operatorname{cosec}\theta - 4 = 0$, is equal to

(1) 6 (2) 8

(3) 10 (4) 7

Ans. (1)

Sol. $$\operatorname{cosec}\theta = \frac{2\left(\sqrt{3} - 1\right) \pm \sqrt{4\left(3 + 1 - 2\sqrt{3}\right) + 16\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3} - 1\right) \pm \sqrt{16 + 8\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3} - 1\right) \pm \left(2 + 2\sqrt{3}\right)}{2\sqrt{3}}$$

$\text{cosec}\theta = 2$ or $\frac{-2}{\sqrt{3}}$

$\therefore \sin\theta = \frac{1}{2}$ or $\frac{-\sqrt{3}}{2}$

$\therefore \sin\theta = \frac{1}{2}$ has 3 solutions & also $\sin\theta = \frac{-\sqrt{3}}{2}$ has 3 solutions in $\left[\frac{-7\pi}{6}, \frac{4\pi}{3}\right]$

10. Given three indentical bags each containing 10 balls, whose colours are as follows :

	Red	**Blue**	**Green**
Bag I	3	2	5
Bag II	4	3	3
Bag III	5	1	4

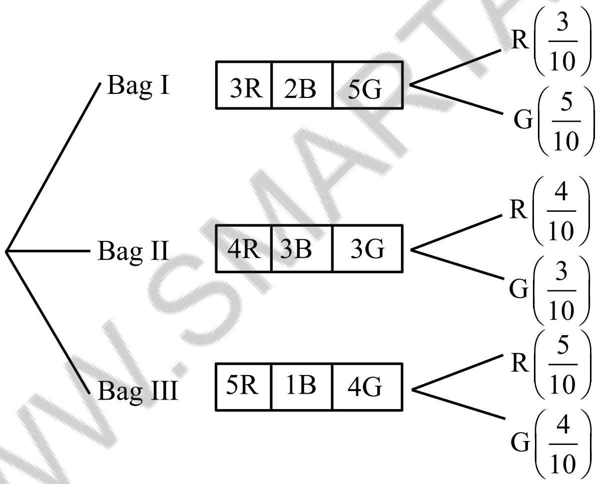
A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from bag I is p and if the balls is Green, the probability that it is from bag III is q, then the value of $\left(\frac{1}{p} + \frac{1}{q}\right)$ is :

(1) 6 (2) 9

(3) 7 (4) 8

Ans. (3)

Sol.



$$p = P\left(\frac{B_I}{R}\right) = \frac{\frac{1}{3}\left(\frac{3}{10}\right)}{\frac{1}{3}\left(\frac{3}{10} + \frac{4}{10} + \frac{5}{10}\right)} = \frac{1}{4}$$

$$q = \left(\frac{B_{III}}{G}\right) = \frac{\frac{1}{3}\left(\frac{4}{10}\right)}{\frac{1}{3}\left(\frac{5}{10} + \frac{3}{10} + \frac{4}{10}\right)} = \frac{1}{3}$$

$$\therefore \frac{1}{p} + \frac{1}{q} = 7$$

11. If the mean and the variance of 6, 4, a, 8, b, 12, 10, 13 are 9 and 9.25 respectively, then a + b + ab is equal to :

(1) 105 (2) 103

(3) 100 (4) 106

Ans. (2)

Sol. $\because$ mean = 9

$\therefore 53 + a + b = 72$

$\Rightarrow a + b = 19$

$\because \sigma^2 = \frac{37}{4}$ and $(\bar{X})^2 + \sigma^2 = \frac{\sum x_1^2}{N}$

$\Rightarrow 81 + \frac{37}{4} = \frac{529 + a^2 + b^2}{8}$

$\Rightarrow 648 + 74 = 529 + a^2 + b^2$

$\Rightarrow a^2 + b^2 = 193$

$\because a + b = 19 \Rightarrow a^2 + b^2 + 2ab = 361$

$\Rightarrow 2ab = 168$

$\Rightarrow ab = 84$

$\therefore a + b + ab = 103$

12. If the domain of the function $f(x) = \frac{1}{\sqrt{10 + 3x - x^2}} + \frac{1}{\sqrt{x + |x|}}$ is (a, b), then $(1 + a)^2 + b^2$ is equal to :

(1) 26 (2) 29

(3) 25 (4) 30

Ans. (1)

Sol. $x + |x| > 0 \Rightarrow x \in (0, \infty)$ ....(1)

$\& \ 10 + 3x - x^2 > 0$

$\Rightarrow x^2 - 3x - 10 < 0$

$\Rightarrow x \in (-2, 5)$ ....(2)

from (1) & (2) $x \in (0, 5)$

$\therefore a = 0 \ \& \ b = 5$

$\therefore (1 + a^2) + b^2 = 1 + 25 = 26$

13. $4\int_0^1\left(\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}\right)dx - 3\log_e\left(\sqrt{3}\right)$ is equal to :

(1) $2+\sqrt{2}+\log_e\left(1+\sqrt{2}\right)$

(2) $2-\sqrt{2}-\log_e\left(1+\sqrt{2}\right)$

(3) $2+\sqrt{2}-\log_e\left(1+\sqrt{2}\right)$

(4) $2-\sqrt{2}+\log_e\left(1+\sqrt{2}\right)$

Ans. (2)

Sol. $4\int_0^1\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}dx - 3ln\sqrt{3}$

$= 4\int_0^1\frac{\sqrt{3+x^2}-\sqrt{1+x^2}}{(3+x^2)-(1-x^2)}dx - \frac{3}{2}ln\,3$

$= 2\left[\left\{\frac{x}{2}\sqrt{3+x^2}+\frac{3}{2}ln\left(x+\sqrt{3+x^2}\right)\right\}_0^1\right.$

$\left.-\left\{\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}ln(x+\sqrt{1+x^2})\right\}_0^1\right]-\frac{3}{2}ln3$

$= 2\left[\left\{\frac{1}{2}\sqrt{4}+\frac{3}{2}ln(1+\sqrt{4})\right\}-\left\{0+\frac{3}{2}ln\sqrt{3}\right\}\right.$

$\left.-\left\{\frac{1}{2}\sqrt{2}+\frac{1}{2}ln(1+\sqrt{2})\right\}+\left\{0+\frac{1}{2}(0)\right\}\right]-\frac{3}{2}ln3$

$=2\left[1+\frac{3}{2}ln3-\frac{3}{4}ln3-\frac{1}{\sqrt{2}}-\frac{1}{2}ln(1+\sqrt{2})\right]-\frac{3}{2}ln3$

$=2+3\,ln\,3-\frac{3}{2}\,ln\,3-\sqrt{2}-ln\,(1+\sqrt{2})-\frac{3}{2}ln\,3$

$=2-\sqrt{2}-ln(1+\sqrt{2})$

14. If $\lim_{x\to0}\frac{\cos(2x)+a\cos(4x)-b}{x^4}$ is finite, then (a+b) is equal to :

(1) $\frac{1}{2}$ (2) 0

(3) $\frac{3}{4}$ (4) –1

Ans. (1)

Sol. $\lim_{x\to0}\frac{\cos2x+a\cos4x-b}{x^4}$ = finite

$$L=\frac{\left\{1-\frac{(2x)^2}{\underline{|2}}+\frac{(2x)^4}{\underline{|4}}....\right\}+a\left\{1-\frac{(4x)^2}{\underline{|2}}+\frac{(4x)^4}{\underline{|4}}....\right\}-b}{x^4}$$

$$L=\frac{(1+a-b)-x^2(2+8a)+x^4\left(\frac{2}{3}+\frac{32}{3}a\right)+x^6(\,)....}{x^4}$$

$\therefore$ 1 + a – b = 0 and 2 + 8a = 0 $\Rightarrow a=-\frac{1}{4}$

b = a + 1

$=-\frac{1}{4}+1=\frac{3}{4}$

$\therefore a+b=-\frac{1}{4}+\frac{3}{4}=\frac{1}{2}$

15. If $\sum_{r=0}^{10}\left(\frac{10^{r+1}-1}{10^r}\right).{}^{11}C_{r+1}=\frac{\alpha^{11}-11^{11}}{10^{10}}$, then α is equal to :

(1) 15 (2) 11

(3) 24 (4) 20

Ans. (4)

Sol. $\sum_{r=0}^{10}\left(\frac{10^{r-1}-1}{10^r}\right){}^{11}C_{r+1}$

$=\sum_{r=0}^{10}\left(10-\frac{1}{10^r}\right){}^{11}C_{r+1}$

$=10\sum_{r=0}^{10}{}^{11}C_{r+1}-10\sum\left({}^{11}C_{r+1}\left(\frac{1}{10}\right)^{r+1}\right)$

$=10\left[{}^{11}C_1+{}^{11}C_2+.....+{}^{11}C_{11}\right]$

$-10\left[{}^{11}C_1\left(\frac{1}{10}\right)^1+{}^{11}C_2\left(\frac{1}{10}\right)^2+.....+{}^{11}C_{11}\left(\frac{1}{10}\right)^{11}\right]$

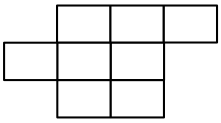
$=10\left[2^{11}-1\right]-10\left[\left(1+\frac{1}{10}\right)^{11}-1\right]$

$=10(2)^{11}-10-\frac{11^{11}}{10^{10}}+10$

$=\frac{(20)^{11}-11^{11}}{10^{10}}$

$\therefore \alpha=20$

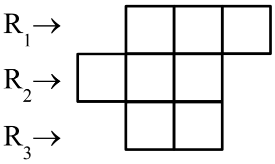
16. The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is :



(1) 5880 (2) 960
(3) 840 (4) 5760

Ans. (4)

Sol.



= Total – [(All in R_1 and R_3) + (All in R_2 and R_3) + (All in R_1 and R_2)]

$= {}^8C_5 \cdot \lfloor 5 - \{\lfloor 5 + \lfloor 5 + {}^6C_5 \cdot \lfloor 5\}$

$= \lfloor 5(56-1-1-6) = 120(48)$

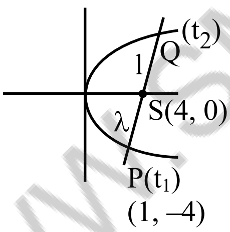
= 5760

17. Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be (1, –4). If the focus of the parabola divides the chord PQ in the ratio m : n, gcd(m, n) = 1, then $m^2 + n^2$ is equal to :

(1) 17 (2) 10
(3) 37 (4) 26

Ans. (1)

Sol. $y^2 = 16x$; a = 4 focus S ≡ (4, 0)



$2at_1 = -4$

$\Rightarrow 2(4)t_1 = -4$

$\Rightarrow t_1 = -\frac{1}{2}$

$\because t_1t_2 = -1$

$\Rightarrow t_2 = 2$

$\therefore Q\left(at_2^2, 2at_2\right) = (16, 16)$

Let, S divides PQ internally in λ : 1 ratio

$\therefore \frac{16\lambda - 4}{\lambda + 1} = 0$

$\lambda = \frac{1}{4} = \frac{m}{n}$

$\therefore m^2 + n^2 = 1 + 16 = 17$

18. Let $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and a vector $\vec{c}$ be such that $\left(\vec{a} - \vec{c}\right) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$ and $\vec{a}.\vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then $|\vec{a}.\vec{d}|$ is equal to :

(1) 18 (2) 12
(3) 9 (4) 15

Ans. (4)

Sol. $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 3 & 2 & 5 \end{vmatrix}$$

$= -17\hat{i} - 7\hat{j} + 13\hat{k}$

$(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3j + 12\hat{k}$

$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{c} \times \vec{b}) = -18\hat{i} - 3\hat{j} + 12\hat{k}$

$\Rightarrow \vec{b} \times \vec{c} = (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (\vec{a} \times \vec{b})$

$= (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (-17i - 7\hat{j} + 13\hat{k})$

$\vec{b} \times \vec{c} = -\hat{i} + 4\hat{j} - \hat{k}$

$\therefore \vec{a} \cdot \vec{d} = \vec{a} \cdot (\vec{b} \times \vec{c}) = (2i - 3j + k) \cdot (-\hat{i} + 4\hat{j} - \hat{k})$

= –2 – 12 – 1 = –15

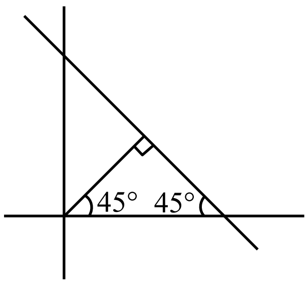
$\therefore |\vec{a} \cdot \vec{d}| = 15$

19. Let the area of the triangle formed by a straight Line L : x + by + c = 0 with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x-axis, then the value of b^2+c^2 is:

(1) 90 (2) 93
(3) 97 (4) 83

Ans. (3)

Sol. $\frac{x}{-c}+\frac{y}{-c/b}=1$



$\therefore$ area of triangle $=\frac{1}{2}\left|\frac{c^2}{b}\right|=48$

$\left|\frac{c^2}{b}\right|=96$

$\because -c=-\frac{c}{b}$

$\Rightarrow b = 1 \quad \therefore c^2 = 96$

$\therefore b^2 + c^2 = 97$

20. Let A be a 3 × 3 real matrix such that $A^2(A - 2I) - 4(A - I) = O$, where I and O are the identity and null matrices, respectively. If $A^5 = \alpha A^2 + \beta A + \gamma I$, where α, β and γ are real constants, then $\alpha + \beta + \gamma$ is equal to:

(1) 12 (2) 20

(3) 76 (4) 4

Ans. **(1)**

Sol. $A^3 - 2A^2 - 4A + 4I = 0$

$A^3 = 2A^2 + 4A - 4I$

$A^4 = 2A^3 + 4A^2 - 4A$

$= 2(2A^2 + 4A - 4I) + 4A^2 - 4A$

$A^4 = 8A^2 + 4A - 8I$

$A^5 = 8A^3 + 4A^2 - 8A$

$= 8(2A^2 + 4A - 4I) + 4A^2 - 8A$

$A^5 = 20A^2 + 24A - 32I$

$\therefore \alpha = 20, \beta = 24, \gamma = -32$

$\therefore \alpha + \beta + \gamma = 12$

SECTION-B

21. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}+2y\sec^2 x = 2\sec^2 x + 3\tan x.\sec^2 x$ such that $y(0)=\frac{5}{4}$. Then $12\left(y\left(\frac{\pi}{4}\right)-e^{-2}\right)$ is equal to __________.

Ans. **(21)**

Sol. $\text{I.F.} = e^{\int 2\sec^2 x dx}$

$= e^{2\tan x}$

Solution of diff. eq.

$$y.e^{2\tan x}=\int e^{2\tan x}(2\sec^2 x+3\tan x.\sec^2 x)dx$$

$$y.e^{2\tan x}=\int e^{2\tan x}.(2\sec^2 x)dx+\int e^{2\tan x}.(3\tan x.\sec^2 x)dx$$

$$y.e^{2\tan x}=e^{2\tan x}.2\tan x-\int e^{2\tan x}.2\sec^2 x\times 2\tan x dx+\int e^{2\tan x}.3\tan x.\sec^2 x dx$$

$$y.e^{2\tan x}=2\tan x.e^{2\tan x}-\int e^{2\tan x}.\tan x\sec^2 x dx$$

$$y.e^{2\tan x}=2\tan x.e^{2\tan x}-\frac{\tan x.e^{2\tan x}}{2}+\frac{e^{2\tan x}}{4}+C$$

$$y=2\tan x-\frac{\tan x}{2}+\frac{1}{4}+Ce^{-2\tan x}$$

$x = 0, y = \frac{5}{4}$

$c = 1$

$y\left(\frac{\pi}{4}\right)=\frac{7}{4}+e^{-2}$

Then $12\left(y\left(\frac{\pi}{4}\right)-e^{-2}\right)=12\left(\frac{7}{4}\right)=21$

22. If the sum of the first 10 terms of the series $\frac{4.1}{1+4.1^4}+\frac{4.2}{1+4.2^4}+\frac{4.3}{1+4.3^4}+\ldots$ is $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to ________.

Ans. **(441)**

Sol. $T_r=\frac{4.r}{1+4.r^4}$

$$T_r=\frac{4.r}{(2r^2+2r+1)(2r^2-2r+1)}$$

$$T_r=\frac{(2r^2+2r+1)-(2r^2-2r+1)}{(2r^2+2r+1)(2r^2-2r+1)}$$

$$T_r = \frac{1}{2r^2 - 2r + 1} - \frac{1}{2r^2 + 2r + 1}$$

$$T_1 = \frac{1}{1} - \frac{1}{5}$$

$$T_2 = \frac{1}{5} - \frac{1}{13}$$

$\vdots$

$$T_{10} = \frac{1}{181} - \frac{1}{221}$$

$$S_{10} = 1 - \frac{1}{221} = \frac{220}{221} = \frac{m}{n}$$

$m + n = 441$

23. If $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$, then $(x - y)^2 + 3y^2$ is equal to __________.

Ans. (3)

Sol. $y = \cos\left(\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{x}{2}\right)$

$$y = \frac{1}{2} \times \frac{x}{2} - \sqrt{1 - \frac{1}{4}}\sqrt{1 - \frac{x^2}{4}}$$

$4y = x - \sqrt{3}\sqrt{4 - x^2}$

$3(4 - x^2) = x^2 + 16y^2 - 8xy$

$12 - 3x^2 = x^2 + 16y^2 - 8xy$

$4x^2 + 16y^2 - 8xy = 12$

$x^2 + 4y^2 - 2xy = 3$

$x^2 + y^2 - 2xy - 3y^2 = 3$

$(x - y)^2 + 3y^2 = 3$

24. Let A(4, –2), B(1, 1) and C(9, –3) be the vertices of a triangle ABC. Then the maximum area of the parallelogram AFDE, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is __________.

Ans. (3)

Sol. Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & -3 & 1 \end{vmatrix}$

= 6 square units

Maximum area of AFDE = $\frac{1}{2} \times 6$ = 3 sq. units

25. If the set of all $a \in R - \{1\}$, for which the roots of the equation $(1 - a)x^2 + 2(a - 3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to _________.

Ans. (7)

Sol. Both the roots are positive

$D \geq 0$

$4(a - 3)^2 - 4 \times 9(1 - a) \geq 0$

$a^2 - 6a + 9 - 9 + 9a \geq 0$

$a^2 + 3a \geq 0$

$a(a + 3) \geq 0$

$a \in (-\infty, -3] \cup [0, \infty)$ ……(i)

$$-\frac{b}{2a} > 0$$

$$\frac{2(a-3)}{2(a-1)} > 0$$

$a \in (-\infty, 1) \cup (3, \infty)$ ……..(ii)

$f(0) = 9 > 0$

Equation (i) $\cap$ (ii)

$a \in (-\infty, -3] \cup [0, 1)$

$2\alpha + \beta + \gamma - 6 + 0 + 1 = 7$

JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON WEDNESDAY 2nd APRIL 2025) **TIME : 3:00 PM TO 6:00 PM**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

26. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Net dipole moment of a polar linear isotropic dielectric substance is not zero even in the absence of an external electric field.

Reason (R) : In absence of an external electric field, the different permanent dipoles of a polar dielectric substance are oriented in random directions.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **(A)** is correct but **(R)** is not correct

(2) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**

(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(4) **(A)** is not correct but **(R)** is correct

Ans. (4)

Sol. A : Since polar dielectrics are randomly oriental $\vec{P}_{net} = \vec{0}$.

R : If $\vec{E}$ is absent, polar dielectric remain polar & are randomly oriented.

27. In a moving coil galvanometer, two moving coils M_1 and M_2 have the following particulars :

$R_1 = 5\ \Omega$, $N_1 = 15$, $A_1 = 3.6 \times 10^{-3}\ m^2$, $B_1 = 0.25$ T

$R_2 = 7\ \Omega$, $N_2 = 21$, $A_2 = 1.8 \times 10^{-3}\ m^2$, $B_2 = 0.50$ T

Assuming that torsional constant of the springs are same for both coils, what will be the ratio of voltage sensitivity of M_1 and M_2 ?

(1) 1 : 1 (2) 1 : 4
(3) 1 : 3 (4) 1 : 2

Ans. (1)

Sol. Voltage sensitivity $= \frac{\theta}{V} = \frac{NAB}{cR}$

$$\text{Ratio} = \left(\frac{N_1A_1B_1}{N_2A_2B_2}\right)\frac{R_2}{R_1} = \frac{15\times3.6\times0.25}{21\times1.8\times0.5}\times\frac{7}{5} = \frac{1}{1}$$

28. The moment of inertia of a circular ring of mass M and diameter r about a tangential axis lying in the plane of the ring is :

(1) $\frac{1}{2}Mr^2$ (2) $\frac{3}{8}Mr^2$

(3) $\frac{3}{2}Mr^2$ (4) $2\ Mr^2$

Ans. (2)

Sol. Diameter is given as R.

$\therefore$ Radius = R/2

$$I_{\tan gent} = \frac{3}{2}m\left(\frac{R}{2}\right)^2 = \frac{3}{8}mR^2$$

29. Two water drops each of radius 'r' coalesce to from a bigger drop. If 'T' is the surface tension, the surface energy released in this process is :

(1) $4\pi r^2T\left[2-2^{\frac{2}{3}}\right]$ (2) $4\pi r^2T\left[2-2^{\frac{1}{3}}\right]$

(3) $4\pi r^2T\left[1+\sqrt{2}\right]$ (4) $4\pi r^2T\left[\sqrt{2}-1\right]$

Ans. (1)

Sol. $2\times\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \Rightarrow r = 2^{1/3}R$

$U_i = 2 \times 4\pi R^2T$

$U_f = 4\pi r^2T = 4\pi R^2T2^{2/3}$

$\therefore$ Heat lost $= u_i - u_f = 4\pi R^2T[2-2^{2/3}]$

30. An electron with mass 'm' with an initial velocity (t = 0) $\vec{v} = v_0\hat{i}$ ($v_0 > 0$) enters a magnetic field $\vec{B} = B_0\hat{j}$. If the initial de-Broglie wavelength at t = 0 is λ_0 then its value after time 't' would be :

(1) $\frac{\lambda_0}{\sqrt{1-\frac{e^2B_0^2t^2}{m^2}}}$ (2) $\frac{\lambda_0}{\sqrt{1+\frac{e^2B_0^2t^2}{m^2}}}$

(3) $\lambda_0\sqrt{1+\frac{e^2B_0^2t^2}{m^2}}$ (4) λ_0

Ans. (4)

Sol. Magnetic field does not work

$\therefore$ Speed will not charge, so De-Broglie wavelength remains same.

31. A sinusoidal wave of wavelength 7.5 cm travels a distance of 1.2 cm along the x-direction in 0.3 sec. The crest P is at x = 0 at t = 0 sec and maximum displacement of the wave is 2 cm. Which equation correctly represents this wave ?

(1) $y = 2\cos(0.83x - 3.35t)$ cm

(2) $y = 2\sin(0.83x - 3.5t)$ cm

(3) $y = 2\cos(3.35x - 0.83t)$ cm

(4) $y = 2\cos(0.13x - 0.5t)$ cm

Ans. (1)

Sol. $v = \dfrac{\text{distance}}{\text{time}}$

$v = \dfrac{12}{0.3} = 4 \text{ cm/s}$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{7.5} = \dfrac{4\pi}{15} = 0.83$

$v = \dfrac{\omega}{k} \Rightarrow \omega = vk = 4 \times \dfrac{4\pi}{15} = 3.35$

So $y = A\cos(kx - \omega t)$

32. Given a charge q, current I and permeability of vacuum μ_0. Which of the following quantity has the dimension of momentum ?

(1) qI/μ_0 (2) $q\mu_0 I$

(3) $q^2\mu_0 I$ (4) $q\mu_0/I$

Ans. (2)

Sol. $Q = AT$

$I = A$

$\mu_0 = MLT^{-2}A^{-2}$

$P = Q^x \mu_0^y I^z = [AT]^x [MLT^{-2}A^{-2}]^y [A]^z$

$MLT^{-1} = M^y L^y T^{x-2y} A^{-2y+z+x}$

Now; $y = 1$

$x - 2y = -1$

$-2y + z = 0$

$\therefore x = y = z = 1$

33. A solenoid having area A and length 'l' is filled with a material having relative permeability 2. The magnetic energy stored in the solenoid is :

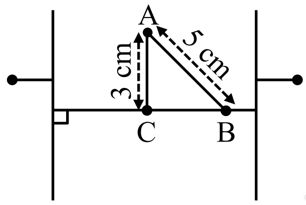
(1) $\dfrac{B^2Al}{\mu_0}$ (2) $\dfrac{B^2Al}{2\mu_0}$

(3) B^2Al (4) $\dfrac{B^2Al}{4\mu_0}$

Ans. (4)

Sol. $\dfrac{U}{V} = \dfrac{B^2}{2\mu_r u_0} \Rightarrow U = \dfrac{B^2}{4\mu_0}V = \dfrac{B^2}{4\mu_0}A\ell$

34. Two large plane parallel conducting plates are kept 10 cm apart as shown in figure. The potential difference between them is V. The potential difference between the points A and B (shown in the figure) is :

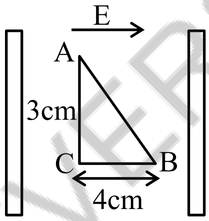


(1) $\dfrac{1}{4}$V (2) $\dfrac{2}{5}$V

(3) $\dfrac{3}{4}$V (4) 1 V

Ans. (2)

Sol.



Using $\Delta V = E(\Delta d)$

$V = E(10)$

$V_{AB} = E.4 = \dfrac{V}{10} \times 4 = \dfrac{2V}{5}$

35. Identify the characteristics of an adiabatic process in a monoatomic gas.

(A) Internal energy is constant.

(B) Work done in the process is equal to the charge in internal energy.

(C) The product of temperature and volume is a constant.

(D) The product of pressure and volume is a constant.

(E) The work done to change the temperature from T_1 to T_2 is proportional to $(T_2 - T_1)$

Choose the **correct** answer from the options given below :

(1) (A), (C), (D) only (2) (A), (C), (E) only

(3) (B), (E) only (4) (B), (D) only

Ans. (3)

Sol. $Q = \Delta U + W = 0 \Rightarrow -\Delta U = W$

$WD = -nC_v\Delta T \Rightarrow |WD| = nC_v\Delta T \propto T_2 - T_1$

$\therefore$ B & E [Only possibility]

36. Assuming the validity of Bohr's atomic model for hydrogen like ions the radius of Li^{++} ion in its ground state is given by $\frac{1}{X}a_0$, where X = ______.

(Where a_0 is the first Bohr's radius.)

(1) 2 (2) 1

(3) 3 (4) 9

Ans. **(3)**

Sol. $r = r_0 \frac{n^2}{z}$ & z = 3 for Li^{+2} and n = 1

$\therefore r = r_0 \frac{1^2}{3} = \frac{r_0}{3}$ $\therefore x = 3$

37. Energy released when two deuterons ($_1H^2$) fuse to form a helium nucleus ($_2He^4$) is :

(Given : Binding energy per nucleon of $_1H^2$ = 1.1 MeV and binding energy per nucleon of $_2He^4$ = 7.0 MeV)

(1) 8.1 MeV (2) 5.9 MeV

(3) 23.6 MeV (4) 26.8 MeV

Ans. **(3)**

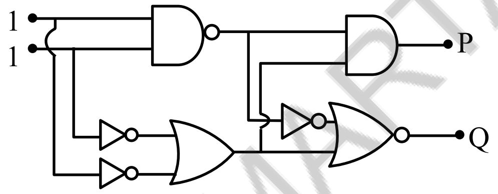
Sol. $\underset{1.1\text{MeV}}{_1H^2} + {_1H^2} \rightarrow \underset{7.0\text{MeV}}{_1He^4}$

$E_B = BE_{reactant} - BE_{product}$

$= 1.1 \times 2 + 1.1 \times 2 - 7 \times 4 = -23.6$ MeV

= Q = 23.6 MeV

38. In the digital circuit shown in the figure, for the given inputs the P and Q values are :

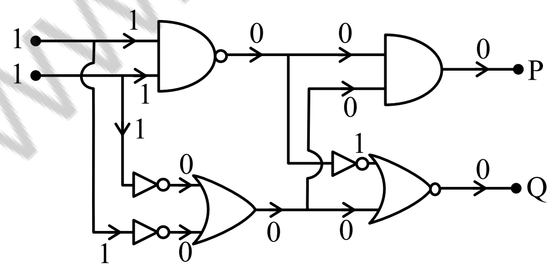


(1) P = 1, Q = 1 (2) P = 0, Q = 0

(3) P = 0, Q = 1 (4) P = 1, Q = 0

Ans. **(2)**

Sol.



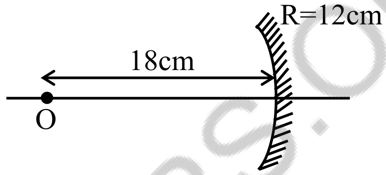
39. Two identical objects are placed in front of convex mirror and concave mirror having same radii of curvature of 12 cm, at same distance of 18 cm from the respective mirrors. The ratio of sizes of the images formed by convex mirror and by concave mirror is :

(1) 1/2 (2) 2

(3) 3 (4) 1/3

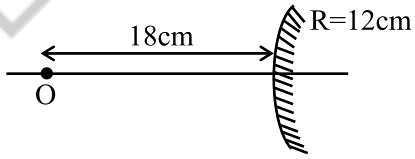
Ans. **(1)**

Sol.



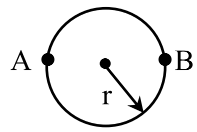
Using $m = \frac{f}{u - f}$

$m_1 = \frac{6}{18 - 6} = \frac{1}{2}$



$m_2 = \frac{6}{18 + 6} = \frac{1}{4}$ $\therefore \frac{m_2}{m_1} = \frac{1}{2}$

40. A sportsman runs around a circular track of radius r such that he traverses the path ABAB. The distance travelled and displacement, respectively, are



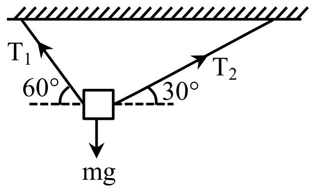
(1) 2r, $3\pi r$ (2) $3\pi r$, πr

(3) πr, 3r (4) $3\pi r$, 2r

Ans. **(4)**

Sol. Displacement = 2r

Distance = $2\pi r + \pi r = 3\pi r$

41. 

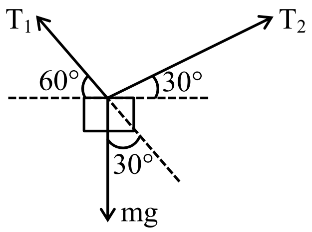
A body of mass 1kg is suspended with the help of two strings making angles as shown in figure. Magnitude of tensions T_1 and T_2, respectively, are (in N) :

(1) $5,\ 5\sqrt{3}$ (2) $5\sqrt{3}$, 5

(3) $5\sqrt{3}$, $5\sqrt{3}$ (4) 5, 5

Ans. **(2)**

Sol.



$T_1 = mg\cos 30°$

$T_2 = mg\sin 30°$

42. A bi–convex lens has radius of curvature of both the surfaces same as 1/6 cm. If this lens is required to be replaced by another convex lens having different radii of curvatures on both sides ($R_1 \neq R_2$), without any change in lens power then possible combination of R_1 and R_2 is :

(1) $\frac{1}{3}$cm and $\frac{1}{3}$cm (2) $\frac{1}{5}$cm and $\frac{1}{7}$cm

(3) $\frac{1}{3}$cm and $\frac{1}{7}$cm (4) $\frac{1}{6}$cm and $\frac{1}{9}$cm

Ans. **(2)**

Sol. This will happen when

$$\frac{1}{f_1} = \frac{1}{f_2}$$

$$(\mu-1)\left(\frac{1}{R_1} - \frac{1}{-R_2}\right) = (\mu-1)\left(\frac{2}{R}\right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}$$

43. If μ_0 and ε_0 are the permeability and permittivity of free space, respectively, then the dimension of $\left(\frac{1}{\mu_0\varepsilon_0}\right)$ is :

(1) L/T^2 (2) L^2/T^2

(3) T^2/L (4) T^2/L^2

Ans. **(2)**

Sol. $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \frac{1}{\mu_0\varepsilon_0} = C^2 = L^2T^{-2}$

44. Match **List-I with List-II**.

List-I	List-II
(A) Heat capacity of body	(I) $J\ kg^{-1}$
(B) Specific heat capacity of body	(II) JK^{-1}
(C) Latent heat	(III) $J\ kg^{-1}\ K^{-1}$
(D)Thermal conductivity	(IV) $Jm^{-1}K^{-1}s^{-1}$

Choose the **correct** answer from the options given below :

(1) (A)–(III), (B)–(I), (C)–(II), (D)–(IV)

(2) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)

(3) (A)–(III), (B)–(IV), (C)–(I), (D)–(II)

(4) (A)–(II), (B)–(III), (C)–(I), (D)–(IV)

Ans. **(4)**

Sol. $C' = \frac{\Delta Q}{\Delta T} = JK^{-1}$

$$S = \frac{\Delta Q}{m\Delta T} = Jkg^{-1}K^{-1}$$

$$L = \frac{\Delta Q}{m} = Jkg^{-1}$$

$$\Delta Q = \frac{KA\Delta T}{L} \Rightarrow K = \frac{\Delta Q(L)}{A\Delta T} = Jm^{-1}K^{-1}s^{-1}$$

45. Consider a circular loop that is uniformly charged and has a radius $a\sqrt{2}$. Find the position along the positive z–axis of the cartesian coordinate system where the electric field is maximum if the ring was assumed to be placed in xy–plane at the origin :

(1) $\frac{a}{\sqrt{2}}$ (2) $\frac{a}{2}$

(3) a (4) 0

Ans. (3)

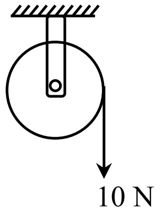
Sol. $E = \dfrac{KQr}{(x^2 + R^2)^{3/2}}$

$\dfrac{dE}{dx} = 0$

$\therefore x = \dfrac{R}{\sqrt{2}} = \dfrac{\sqrt{2}a}{\sqrt{2}} = a$

SECTION-B

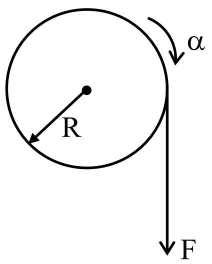
46.



A wheel of radius 0.2 m rotates freely about its center when a string that is wrapped over its rim is pulled by force of 10 N as shown in figure. The established torque produces an angular acceleration of 2 rad/s^2. Moment of inertia of the wheel is ________ kg m^2.

(Acceleration due to gravity = 10 m/s^2)

Ans. (1)



Sol.

$FR = I\alpha$

$\Rightarrow I = \dfrac{FR}{\alpha} = \dfrac{10 \times 0.2}{2} = 1$ kg–m^2

47. The internal energy of air in 4 m × 4 m × 3 m sized room at 1 atmospheric pressure will be ___ × 10^6 J. (Consider air as diatomic molecule)

Ans. (12)

Sol.

To find the internal energy of gas in the room.

$U = nC_vT = n\dfrac{5RT}{2}$

$= \dfrac{5}{2}PV = \dfrac{5}{2} \times 10^5 \times 48 = 12 \times 10^6\,J$

48. A ray of light suffers minimum deviation when incident on a prism having angle of the prism equal to 60°. The refractive index of the prism material is $\sqrt{2}$. The angle of incidence (in degrees) is _____.

Ans. (45)

Sol. $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$, since A = 60° $\quad \therefore \delta m = 30°$

$\delta_m = 2i - A$ [as i = e]

$\Rightarrow i = 45°$

49. The length of a light string is 1.4 m when the tension on it is 5 N. If the tension increases to 7 N, the length of the string is 1.56 m. The original length of the string is _____ m.

Ans. (1)

Sol. $T = K(\ell - \ell_0)$

$\Rightarrow 5 = K(1.4 - \ell_0)$

$\Rightarrow 7 = K(1.56 - \ell_0)$

$\Rightarrow \dfrac{5}{1.4 - \ell_0} = \dfrac{7}{1.56 - \ell_0}$

$\therefore \ell_0 = 1m$

50. A satellite of mass 1000 kg is launched to revolve around the earth in an orbit at a height of 270 km from the earth's surface. Kinetic energy of the satellite in this orbit is ________ × 10^{10} J.

(Mass of earth = 6 × 10^{24} kg, Radius of earth = 6.4 × 10^6 m, Gravitational constant = 6.67 × 10^{-11} Nm^2 kg^{-2})

Ans. (3)

Sol. $KE = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\dfrac{GM_e}{r} = \dfrac{GM_em}{2r} = \dfrac{GM_em}{2(R_E + h)}$

$= \dfrac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 6.4 \times 10^6}{2(6.4 \times 10^6 + 2.7 \times 10^5)} = 3 \times 10^{10}\,J$